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Mathematical Modelling for the Effects of Thermophoresis and Heat Generation/Absorption on MHD Convective Flow along an Inclined Stretching Sheet in the Presence of Dufour-Soret Effects

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ABSTRACT

In this paper, a numerical analysis has been carried out to investigate the effects of heat generation or absorption and thermophoresis on hydromagnetic free convective and mass transfer steady laminar boundary-layer flow over an inclined permeable stretching sheet in the presence of Dufour and Soret effects. The governing non-linear partial differential equations are transformed into a set of coupled ordinary differential equations by using similarity transformation. The resulting similarity equations are solved numerically by applying sixth-order Runge-Kutta method with Nachtsheim-Swigert shooting iteration technique. The effects of the angle of inclination to vertical α, suction or blowing parameter f_w, magnetic field parameter M, heat generation or absorption parameter Q, heat flux exponent n, Dufour number Df, Soret number So and thermophoretic parameter τ have been examined on the flow field of a hydrogen-air mixture as a non-chemical reacting fluid pair. The numerical results have shown that the above-mentioned effects have to be taken into consideration in the fluid heat, heat and mass transfer processes.

Keywords: Modelling, Hydromagnetic, Thermophoresis, Heat and mass transfer, Inclined stretching sheet, Heat generation/absorption, Dufour-Soret effects.

1. INTRODUCTION

The study of magnetohydrodynamic (MHD) heat and mass transfer flows has become more important in recent years because of its applications in geophysical and industrial fields. For example, many metallurgical processes, which involve cooling of continuous strips or filaments, these elements, are drawn through a quiescent fluid. During this process, these strips are sometimes stretched. The properties of the final product depend to a great extent on the rate of cooling. This rate of cooling and therefore, the desired properties of the end product can be controlled by the use of electrically conducting fluids and the applications of magnetic fields. The use of magnetic fields has been also used in the process of purification of molten metals from non-metallic inclusions. In light of these applications, Chiam [1] studied the hydromagnetic flow over a surface stretching with a power-law velocity. Chandran et al. [2] analyzed the hydromagnetic flow and heat transfer past a continuously moving porous boundary. Pop and Na [3] studied a note on MHD flow over a stretching permeable surface. Khan et al.[4] studied the visco-elastic MHD flow; heat and mass transfer over a porous stretching sheet with dissipation of energy and stress work. Recently, Mukhopadhyay et al. [5] investigated MHD boundary layer flow over a heated stretching sheet with variable viscosity.

Thermophoresis, the motion of suspended particles in a fluid induced by a high temperature gradient, is of practical importance in a variety of industrial and engineering applications such as design of thermal precipitators, study on the behavior of soot or seeding particles in combustion systems, nuclear reactor safety, gas cleaning, chemical or physical vapor deposition and micro contamination control, etc. In view of these various important applications, Chiou [6] studied the effect of thermophoresis on submicron particle deposition from a forced laminar boundary layer flow onto an isothermal moving plate. Tsai and Liang [7] developed a correlation for thermophoretic deposition of aerosol particles onto cold plates. Walsh et al. [8] studied the thermophoretic deposition of aerosol particles in laminar tube flow with mixed convection. Alam et al. [9] studied the effects of variable suction and thermophoresis on steady MHD free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined flat plate in the presence of thermal radiation. Recently, Alam and Rahman [10] investigated the effectiveness of variable heat and mass fluxes on hydromagnetic free convection and mass transfer flow along an inclined permeable stretching surface with thermophoresis.

In all the above studies, thermal-diffusion (Soret) and diffusion-thermo (Dufour) effects were neglected, on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. There are however, exceptions. The thermal-diffusion effect, for instance, has been utilized for isotope separation, and in mixture between gases with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air) the diffusion-thermo effect was found to be of considerable magnitude such that it cannot be neglected (Eckert and Drake [11]). In view of the importance of these Soret-Dufour effects, Kafoussias and Williams [12] studied numerically the thermal-diffusion and diffusion-thermo effects on steady combined (free-forced) convection and mass transfer boundary layer flow with temperature dependent viscosity for a hydrogen-air mixture as non-chemical reacting fluid pair. Anghel et al.[13] investigated the Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium. Postelnicu [14] studied the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in a porous media considering Soret and Dufour effects. Alam and Rahman [15] studied numerically the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction.

Therefore, the objective of the present paper is to investigate the effects of heat generation or absorption and thermophoresis on hydromagnetic free convection and mass transfer steady laminar boundary-layer flow over an inclined permeable stretching sheet in the presence of Dufour and Soret effects with variable heat and mass fluxes.

2. MATHEMATICAL MODELLING

Consider a steady two-dimensional; laminar MHD free convection and mass transfer flow of a viscous and incompressible fluid along a linearly stretching semi-infinite sheet that is inclined from the vertical with an acute angle α. The surface is assumed to be permeable and moving with velocity, $u_w(x) = bx$ (where b is constant called stretching rate). Fluid suction or blowing is imposed at the stretching surface. The fluid is assumed to be Newtonian, electrically conducting and heat generating or absorbing. The x-axis runs along the stretching surface in the direction of motion with the slot as the origin and the y-axis is measured normally from the sheet to the fluid. A magnetic field of uniform strength B_0 is applied normal to the sheet in the y-direction, which produces magnetic effect in the x-direction. We further assume that (a) due to the boundary layer behavior the temperature gradient in the y-direction is much larger than that in the x-direction and hence only the thermophoretic velocity component which is normal to the surface is of importance, (b) the fluid has constant kinematic viscosity and thermal diffusivity, and that the Boussinesq approximation may be adopted for steady laminar flow and (c) the magnetic Reynolds number is small so that the induced magnetic field can be neglected. The configuration and co-ordinate system are shown in Figure 1.

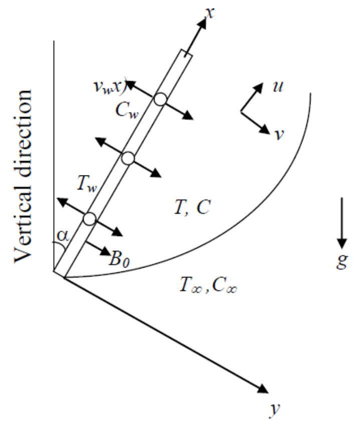

Figure 1. Flow configurations and coordinate system

Under the above assumptions the governing equations describing the conservation of mass, momentum, energy and concentration respectively can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)\cos\alpha + g\beta^*(C - C_\infty)\cos\alpha - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\lambda}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} + \frac{Q_0}{\rho c_p}\left(T - T_\infty\right) \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} - \frac{\partial}{\partial y}[V_T(C - C_\infty)] \tag{4}$$

where the the thermophoretic deposition velocity in the y-direction is given by

$$V_T = -k\upsilon\frac{\nabla T}{T_r} = -\frac{k\upsilon}{T_r}\frac{\partial T}{\partial y} \tag{5}$$

where k is the thermophoretic coefficient and T_r is some reference temperature.

The boundary conditions suggested by the physics of the problem are:

$$u = u_w(x) = bx, v = \pm v_w(x), -\lambda\frac{\partial T}{\partial y} = q_w = A_1 x^n, \quad -D\frac{\partial C}{\partial y} = M_w = A_2 x^n \quad \text{at } y = 0 \tag{6a}$$

$$u = 0, \ T = T_\infty, \quad C = C_\infty \ \text{ as } y \to \infty, \tag{6b}$$

where b is a constant called stretching rate and A_1, A_2 are proportionality constants and $v_w(x)$ represents the permeability of the porous surface where its sign indicates suction (< 0) or injection (> 0). Here n is the heat flux exponent parameter. For $n = 0$, the accelerating sheet is subject to uniform heat flux.

3. DIMENSIONAL ANALYSIS

Dimensional analysis is one of the most important mathematical tools in the study of fluid mechanics. To describe several transport mechanisms in nanofluids, it is meaningful to make the conservation equations into non-dimensional form. The advantages of non-dimensionalization are as follows: (i) non-dimensionalization gives us freedom to analysis for any system irrespective of their material properties. (ii) one can easily understand the controlling flow parameters of the system, (iii) make a generalization of the size and shape of the geometry, and (iv) before doing experiment one can get insight of the physical problem. These aims can be achieved through the appropriate choice of scales.Therefore in order to obtain the dimensionless form of the governing equations (1)-(4) together with the boundary conditions (6) we introduce the following non-dimensional variables (see also Acharya et al. [16]):

$$\left.\begin{aligned} & u=\frac{\partial\psi}{\partial y},\, v=-\frac{\partial\psi}{\partial x} \\ & \psi=(\upsilon b)^{1/2}\, xf(\eta),\, \eta=(b/\upsilon)^{1/2}\, y, \\ & T-T_\infty=\frac{A_1x^n}{\lambda}(\upsilon/b)^{1/2}\,\theta(\eta), \\ & C-C_\infty=\frac{A_2x^n}{D}(\upsilon/b)^{1/2}\,\varphi(\eta) \end{aligned}\right\} \tag{7}$$

Now employing the relation (7) into equation (1)-(4), we obtain the following nonlinear ordinary differential equations:

$$f'''+ff''-(f')^2+g_s\theta\cos\alpha+g_c\varphi\cos\alpha-Mf'=0 \tag{8}$$

$$\theta''-n\Pr f'\theta+\Pr f\theta'+\Pr Df\varphi''+\Pr Q\theta=0 \tag{9}$$

$$\varphi''-nScf'\varphi+Sc(f+\tau\theta')\varphi'+Sc\tau\varphi\theta''+ScSo\theta''=0 \tag{10}$$

The boundary conditions (6) then turn into

$$f=f_w,\, f'=1, \theta'=-1, \varphi'=-1 \text{ at } \eta=0 \tag{11a}$$

$$f'=0,\, \theta=0,\, \varphi=0 \quad\text{as}\quad \eta\to\infty \tag{11b}$$

where $f_w=-v_w/(b\upsilon)^{1/2}$ is the dimensionless wall mass transfer coefficient such that $f_w>0$ indicates wall suction and $f_w<0$ indicates wall injection.

The dimensionless parameters introduced in the above equations are defined as follows: $M=\frac{\sigma B_0^2 x}{\rho u_w(x)}$ is the local Magnetic field parameter, $Gr=\frac{g\beta q_w(x)\, x^4}{\lambda\upsilon^2}$ is the local Grashof number, $Gm=\frac{g\beta^* M_w(x)x^4}{D\upsilon^2}$ is the local modified Grashof number, $\mathrm{Re}_x=\frac{u_w(x)x}{\upsilon}$ is the local Reynolds number, $g_s=\frac{Gr}{\mathrm{Re}_x^{5/2}}$ is the temperature buoyancy parameter, $g_c=\frac{Gm}{\mathrm{Re}_x^{5/2}}$ is the mass buoyancy parameter, $Df=\frac{D_mM_w(x)k_T}{\upsilon c_s c_p q_w(x)}$ is the Dufour number, $So=\frac{D_m q_w(x)k_T}{\upsilon M_w(x)T_m}$ is the Soret number, $Q=\frac{Q_0x}{\rho c_p u_w(x)}$ is the heat generation/absorption parameter, $\Pr=\frac{\upsilon\rho c_P}{\lambda}$ is the Prandtl number, $Sc=\frac{\upsilon}{D}$ is the Schmidt number and $\tau=\frac{k(\upsilon/b)^{1/2}\, q_w(x)}{\lambda T_r}$ is the thermophoretic parameter.

The parameter of engineering interest for the present problem is local Nusselt number which is obtained from the following expressions:

$$Nu_x(\mathrm{Re}_x)^{-\frac{1}{2}}=\frac{1}{\theta(0)} \tag{12}$$

4. METHOD OF SOLUTION

The locally similar and nonlinear ordinary differential equations (8)-(10) with boundary conditions (11) have been solved numerically by using sixth order Runge-Kutta method along with Nachtsheim-Swigert [17] shooting iteration technique (for detailed discussion of this method see Alam et al. [18]) with K, f_w , g_s, g_c, α, Sc ,n, Q, Sc,τ, Pr and M as prescribed parameter. The computations were done by a program, which uses a symbolic and computational computer language FORTRAN LAHEY. A step size of $\Delta\eta=0.01$ was selected to be satisfactory for a convergence criterion of 10^{-6}. The value of η_∞ was found to each iteration loop by the statement $\eta_\infty=\eta_\infty+\Delta\eta$. The maximum value of η_∞ was determined when the value of the unknown boundary conditions at $\eta=0$ does not change in the successful loop with an error less than 10^{-6}.

5. TESTING OF CODE

To assess the accuracy of the present numerical method, we have compared our local skin-friction coefficients with those of Andersson et al. [19] and Chen [20] in Table 1 and we see that excellent agreement among the results exist. Also the rate of heat transfer $1/\theta(0)$ obtained in the present study is compared with those of Elbashbeshy [21] and Chen [20] for their Newtonian fluid case in the absence of magnetic field in Table 2 and we found excellent agreement among the results.

Table 1. Comparison of $-f''(0)$ with Andersson et al. [19] and Chen [20]for their Newtonian fluid case

M	Andersson et al. [19]	Chen [20]	Present study
0.0	1.00	1.00000	0.9999000
1.0	1.414	1.41421	1.4142136
2.0	1.732	1.73205	1.7320508

Table 2. Comparison of $1/\theta(0)$ with Elbashbeshy [21] and Chen [20] for their Newtonian fluid case

Pr	Elbashbeshy [21]	Chen [20]	Present study
0.72	0.7711	0.76217	0.7622729
1.00	1.0060	1.00616	1.0062684
10.0	7.0921	7.09205	7.0939394

6. NUMERICAL RESULTS AND DISCUSSION

In order to get an insight into the physical situation of the problem, we have computed numerical values of the velocity, temperature and concentration. The velocity, temperature and concentration distributions are found for the different values of the various parameters occurring in the problem. Solutions were obtained for Pr = 0.70 (air), Sc = 0.22 (hydrogen), g_s =12; g_c = 6 (due to free convection problem), while the values of the Dufour number Df and Soret number So are taken in such a way that their product has a constant value, let us say $Sr\times Df$ = 0.06. So under the above assumptions our numerical results are shown in Figs. 2-8 and Tables 3-6.

The effects of angle of inclination to the vertical direction on the velocity, temperature and concentration fields are displayed in Figs. 2(a)-(c) respectively. It is revealed from Figure 2(a) that increasing the angle of inclination decreases the velocity. The fact is that as the angle of inclination increases the effect of the buoyancy force due to thermal diffusion decrease by a factor of $\cos\alpha$. Consequently the driving force to the fluid decreases as a result velocity profiles decrease. From Figs.2 (b)-(c) we also see that both the thermal and concentration boundary layer thickness increase as the angle of inclination increases.

Figs. 3(a)-(c) illustrate the influence of the suction parameter f_w on the velocity, temperature and concentration profiles, respectively. The imposition of wall fluid suction (f_w>0) for this problem has the effect of reducing all the hydrodynamic, thermal and concentration boundary layers causing the fluid velocity and its concentration to increase while decreasing its temperature. But imposition of wall fluid injection or blowing (f_w<0) produces the opposite effect, namely decreases the fluid velocity and concentration and increases its temperature. The decreasing thickness of the concentration layer is caused by two effects; (i) the direct action of suction, and (ii) the indirect action of suction causing a thinner thermal boundary layer, which corresponds to higher temperature gradient, a consequent increase in the thermophoretic force and higher concentration gradient.

Figs. 4(a)-(c) represent respectively, the dimensionless velocity, temperature and concentration for various values of the magnetic field parameter (M). The presence of a magnetic field normal to the flow in an electrically conducting fluid produces a Lorentz force, which acts against the flow. This resistive force tends to slow down the flow and hence the fluid velocity decreases with the increase of the magnetic field parameter as observed in Figure 4(a). From Figure 4(b) we see that the temperature profiles increase with the increase of the magnetic field parameter, which implies that the applied magnetic field tends to heat the fluid, and thus reduces the heat transfer from the wall. In Figure 4(c), the effect of an applied magnetic field is found to increase the concentration profiles, and hence increase the concentration boundary layer.

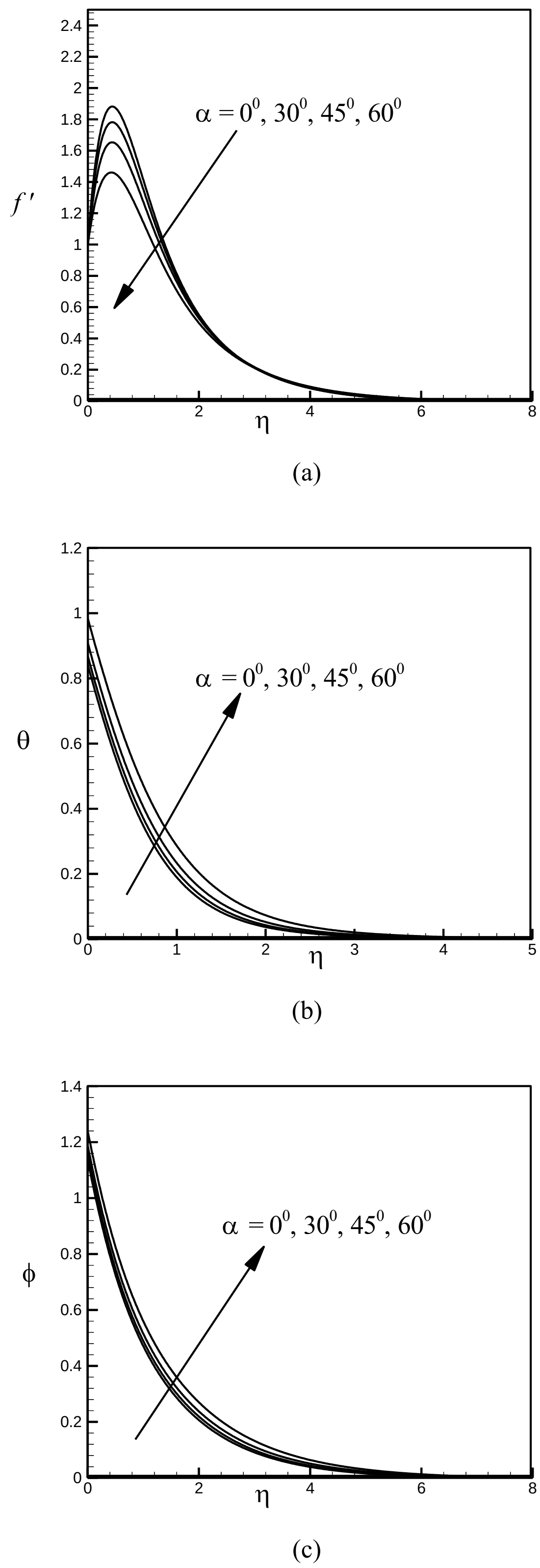

Figure 2. Dimensionless (a) velocity, (b) temperature and (c) concentration profiles for different values of α

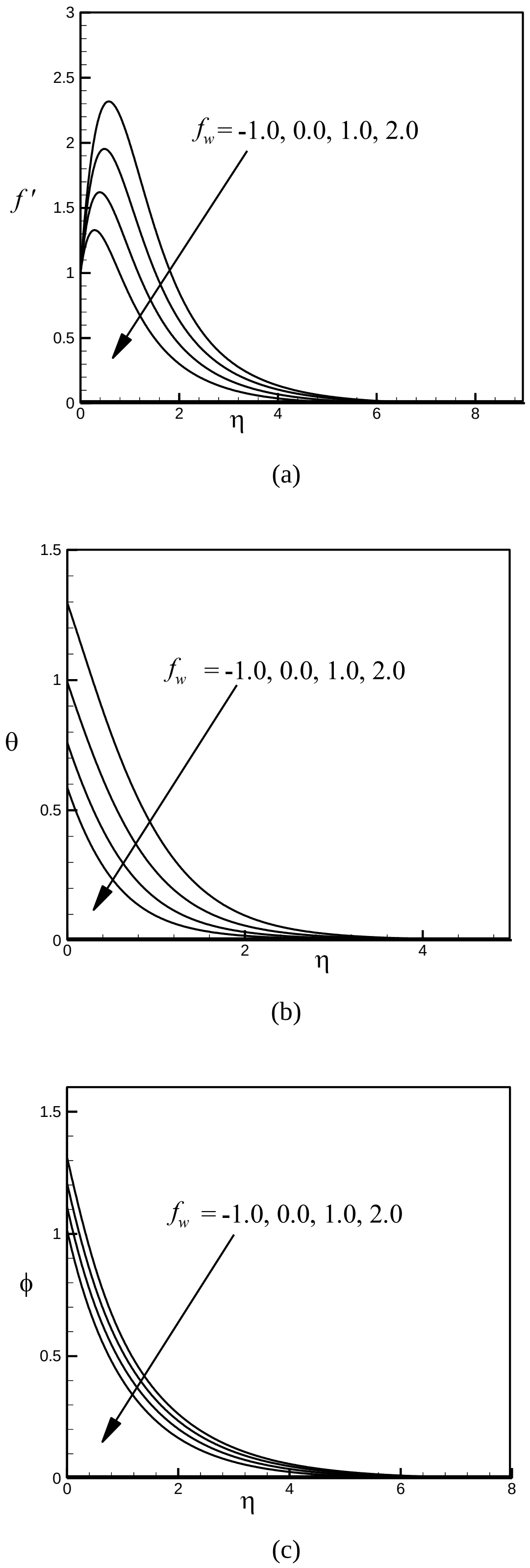

Figure 3. Dimensionless (a) velocity, (b) temperature and (c) concentration profiles for different values of f_w

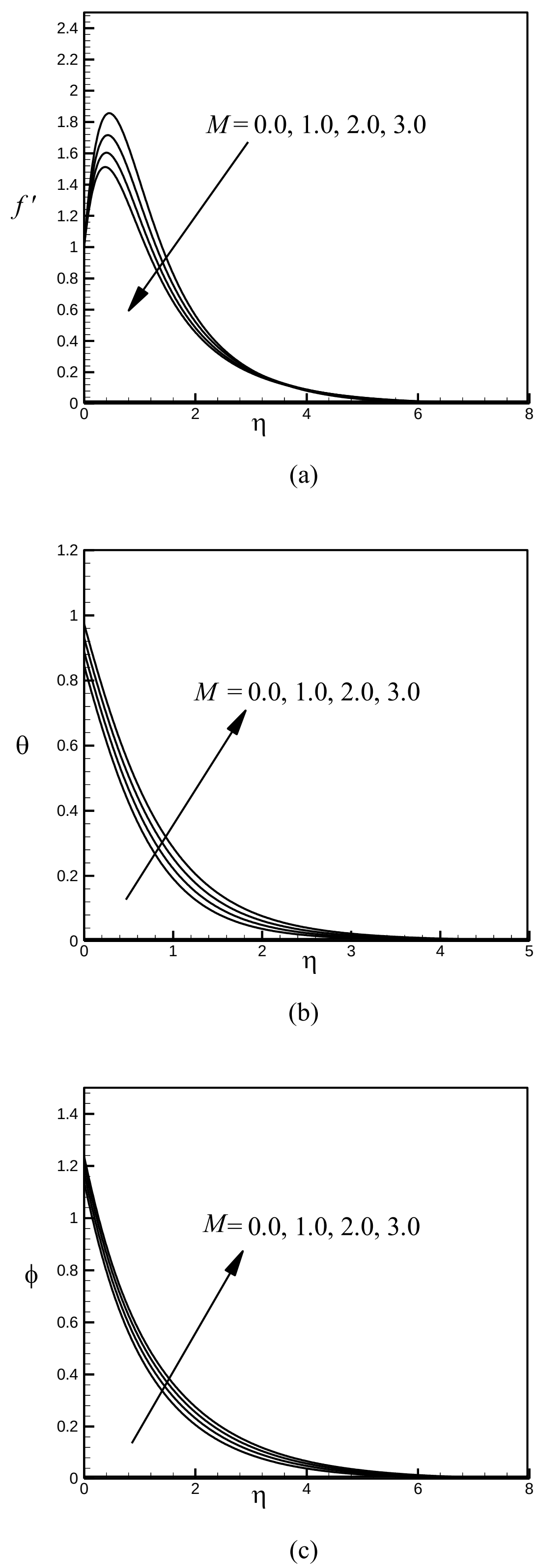

Figure 4. Dimensionless (a) velocity, (b) temperature and (c) concentration profiles for different values of M

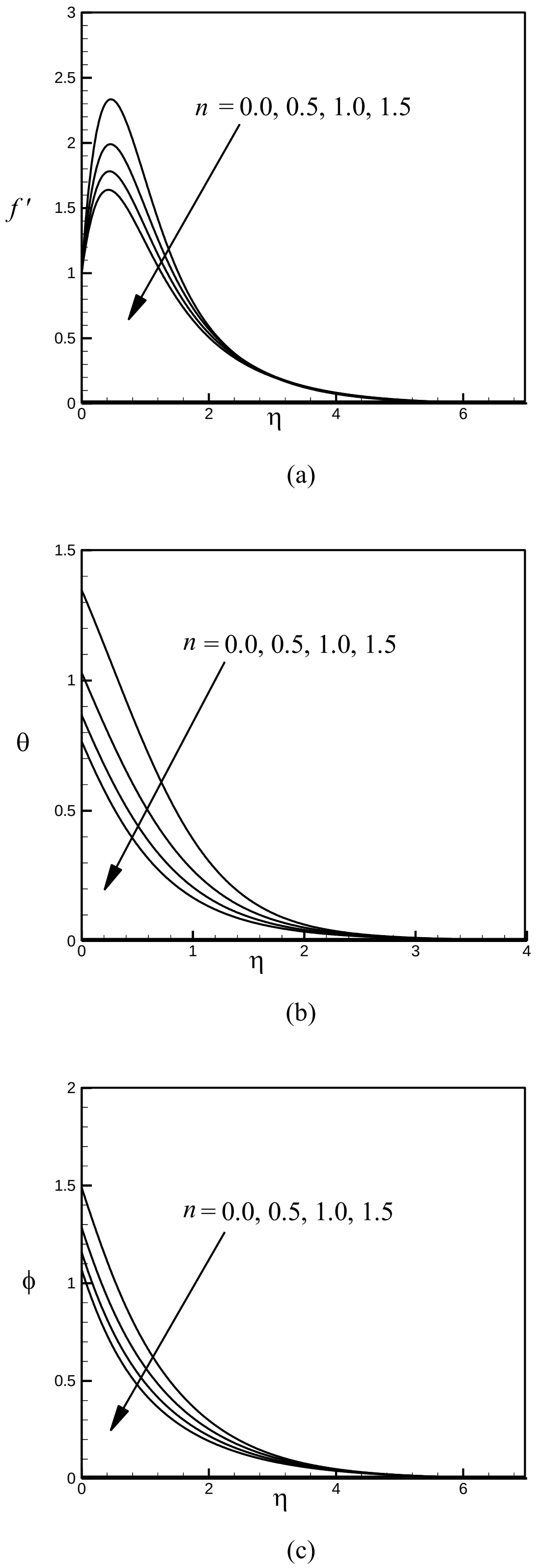

Figure 5. Dimensionless (a) velocity, (b) temperature and (c) concentration profiles for different values of *n*

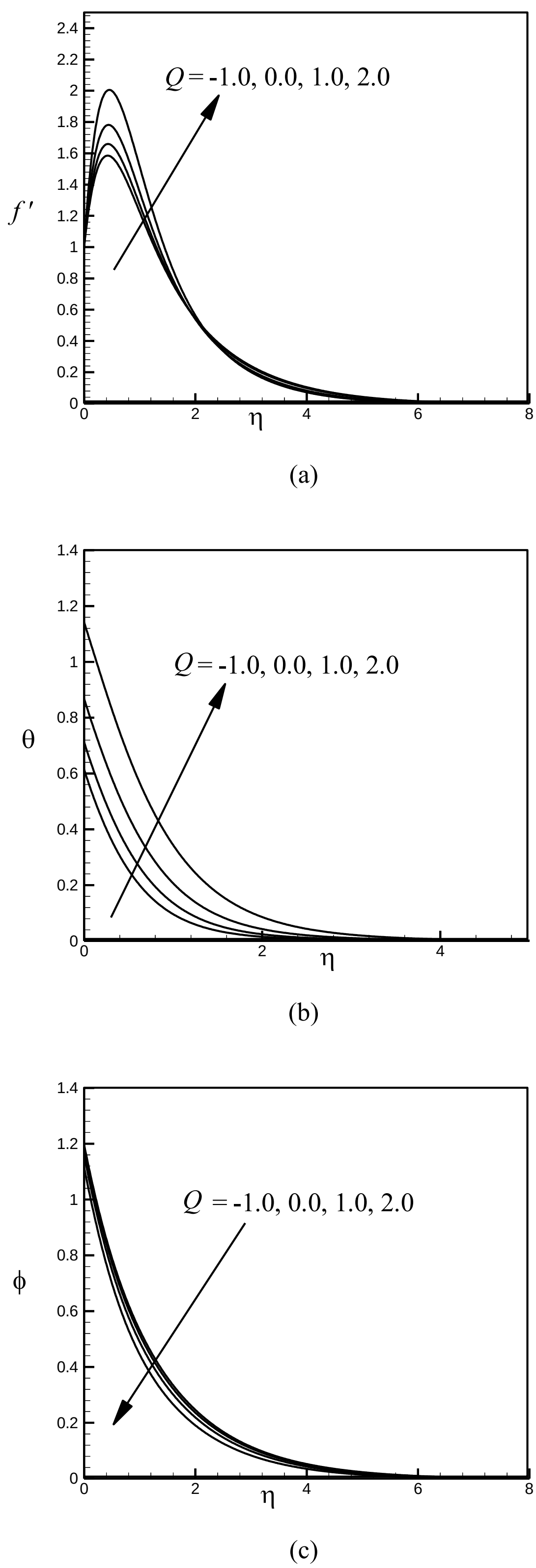

Figure 6. Dimensionless (a) velocity, (b) temperature and (c) concentration profiles for different values of *Q*

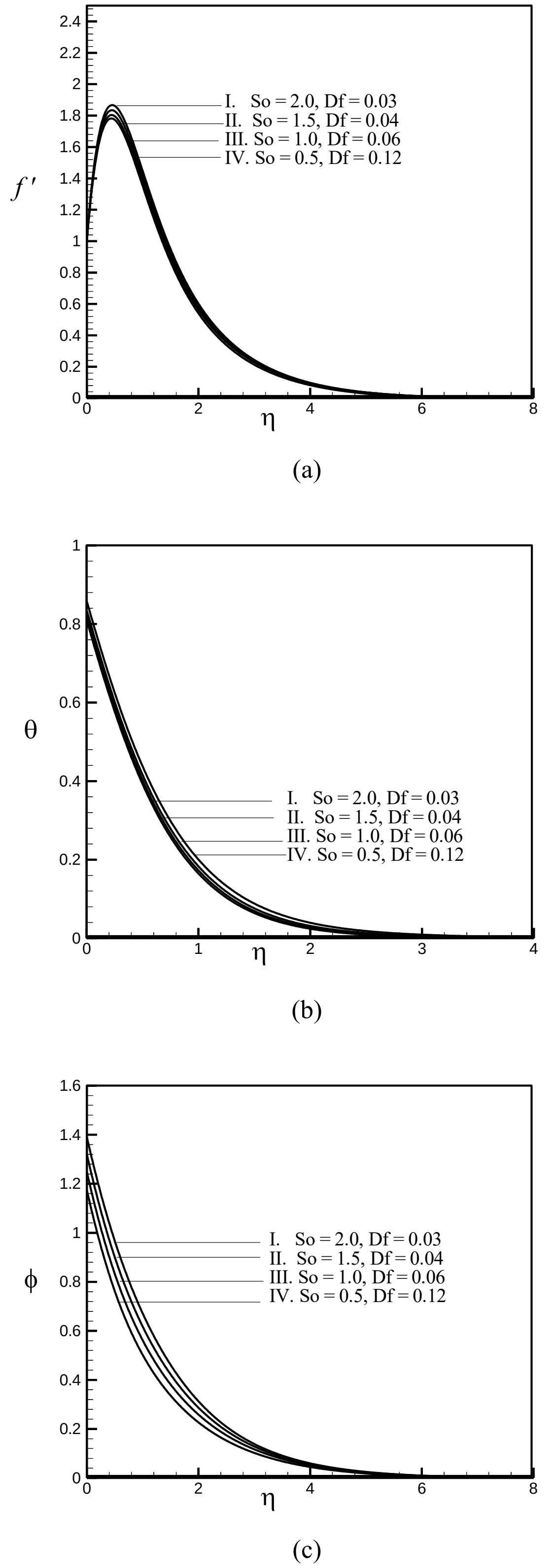

Figure 7: Dimensionless (a) velocity, (b) temperature and (c) concentration profiles for different values of *Df* and S_0

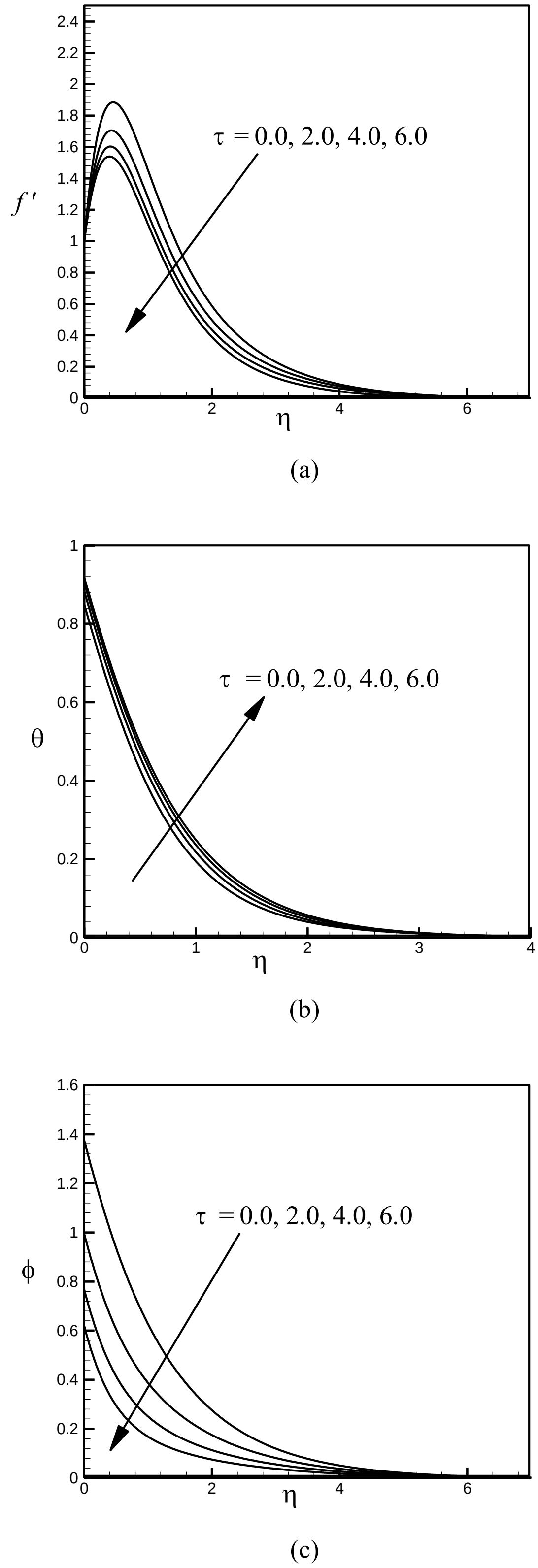

Figure 8: Dimensionless (a) velocity, (b) temperature and (c) concentration profiles for different values of τ

The effects of the surface heat flux exponent *n* on the dimensionless velocity, temperature and concentration profiles are displayed in Figs. 5(a)-(c) respectively. From Figure 5(a) it is seen that, the velocity gradient at the wall increases and hence the momentum boundary layer thickness decreases as *n* increases. Furthermore from Figure 5(b) we can see that as *n* increases, the thermal boundary layer thickness decreases and the temperature gradient at the wall increases. This means a higher value of the heat transfer rate is associated with higher values of *n*. We also observe from Figure 5(c) that the concentration boundary layer thickness decreases as the heat flux exponent *n* increases.

Figs. 6(a)-(c) depict the influence of the dimensionless heat generation or absorption parameter *Q* on the fluid velocity, temperature and concentration profiles respectively. It is seen from Figure 6(a) that when the heat is generated ($Q > 0$) the buoyancy force increases, which induces the flow rate to increase giving, rise to the increase in the velocity profiles. From Figure 6(b), we observe that when the value of the heat generation parameter *Q* increases, the temperature distribution also increases significantly which implies that owing to the presence of a heat source, the thermal state of the fluid increases causing the thermal boundary layer to increase. In the case that the strength of the heat source is relatively large, the maximum fluid temperature does not occur at the wall but rather in the fluid region close to it. We also see from Figure 6(c) that the concentration profiles increase while the concentration boundary layer decreases as the heat generation parameter *Q* increases. Conversely, the presence of a heat sink or a heat absorption ($Q < 0$) causes a reduction in the thermal state of the fluid, thus producing lower thermal boundary layer.

Table 3. Effects of α, *So* and *Df* on local Nusselt number (Nu_x) for $g_s = 12$, $g_c = 6$, $Pr = 0.70$, $Sc = 0.22$, $\tau = 1$, $M = 0.50$, $f_w = 0.50$, $Q = 1.0$ and $n = 1.0$

α	*So*	*Df*	Nu_x
0^0	2.0	0.03	1.27185037
0^0	1.0	0.06	1.23372791
0^0	0.5	0.12	1.20066537
30^0	2.0	0.03	1.23368802
30^0	1.0	0.06	1.19617439
30^0	0.5	0.12	1.16364571
60^0	2.0	0.03	1.09448479
60^0	1.0	0.06	1.05865319
60^0	0.5	0.12	1.02769177

Table 4. Effects of *Q*, *So* and *Df* on local Nusselt number (Nu_x)for $g_s = 12$, $g_c = 6$, $Pr = 0.70$, $Sc = 0.22$, $\tau = 1$, $M = 0.50$, $f_w = 0.50$, $n = 1.0$ and $\alpha = 30^0$

Q	*So*	*Df*	Nu_x
-1.0	0	0	1.666218176
-0.5	0	0	1.555645679
0.0	0	0	1.430410775
0.5	0	0	1.313764724
1.0	0	0	1.101027688
-1.0	0.5	0.12	1.630913309
-0.5	0.5	0.12	1.530252089
0.0	0.5	0.12	1.415227651
0.5	0.5	0.12	1.293156011
1.0	0.5	0.12	1.163645707
-1.0	1.0	0.06	1.670585582
-0.5	1.0	0.06	1.562124358
0.0	1.0	0.06	1.447341798
0.5	1.0	0.06	1.325530202
1.0	1.0	0.06	1.196174395

Table 5. Effects of *n*, *So* and *Df* on local Nusselt number (Nu_x)for $g_s = 12$, $g_c = 6$, $Pr = 0.70$, $Sc = 0.22$, $\tau = 1$, $M = 0.50$, $f_w = 0.50$, $Q = 1.0$ and $\alpha = 30^0$

n	*So*	*Df*	Nu_x
0	0	0	0.678911699
1	0	0	1.101921089
2	0	0	1.399236828
0	0.4	0.15	0.704524874
1	0.4	0.15	1.105737595
2	0.4	0.15	1.390492204
0	1.5	0.04	0.728881991
1	1.5	0.04	1.144690872
2	1.5	0.04	1.448519094

The influence of Soret number *So* and Dufour number *Df* on the velocity field are shown in Figure 7(a). Quantitatively, when η = 1.0 and So decreases from 2.0 to 1.5 (or *Df* increases from 0.03 to 0.04), there is 2.39% decrease in the velocity value whereas the corresponding decrease is 1.98% when *So* decreases from 1.0 to 0.5(or *Df* increases from 0.06 to 0.12). From Figure 7(b) when $\eta = 1.0$ and *So* decreases from 2.0 to 1.5 (or *Df* increases from 0.03 to 0.04), there is 4.85% increase in the temperature, whereas the corresponding increase is 9.44% when *So* decreases from 1.0 to 0.5. In Figure 7(c) when η = 1.0 and *So* decreases from 2.0 to 1.5 (or *Df* increases from 0.03 to 0.04), there is 8.02% decrease in the concentration, whereas the corresponding decrease is 10.51% when *So* decreases from 1.0 to 0.5.

Table 6. Effects of τ, *So* and *Df* on local Nusselt number (Nu_x) for $g_s = 12$, $g_c = 6$, $Pr = 0.70$, $Sc = 0.22$, $n = 1$, $M = 0.50$, $f_w = 0.50$, $Q = 1.0$ and $\alpha = 30^0$

τ	*So*	*Df*	Nu_x
0	0	0	1.221651724
2	0	0	1.148434334
4	0	0	1.101921089
0	0.5	0.12	1.191997975
2	0.5	0.12	1.141528404
4	0.5	0.12	1.118797373
0	2.0	0.03	1.278837284
2	2.0	0.03	1.203444741
4	2.0	0.03	1.158611881

The effects of thermophoretic parameter τ on the velocity, temperature as well as concentration distributions are displayed in Figs. 8(a)-(c) respectively. It is observed from these figures that an increase in the thermophoretic parameter τ leads to decrease in the velocity across the boundary layer. This is accompanied by a decrease in the concentration and a slight increase in the fluid temperature. This means that the effect of increasing τ is limited to increasing the wall slope of the convection profile without any significant effect on the concentration boundary layer.

Finally, the effects of the angle of inclination to vertical direction, surface heat flux parameter, Soret number, Dufour number, heat generation/absorption parameter and thermophoretic parameter on the local Nusselt number is shown in Tables 3-6. The behavior of these parameters is self-evident from Tables 3-6 and hence they will not discuss any further due to brevity.

7. CONCLUSIONS

In this paper, the effects of thermophoresis and heat generation/absorption on hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting fluid along an inclined permeable stretching surface with variable heat and mass fluxes in the presence of Dufour-Soret effects have been investigated numerically. The governing equations are developed and transformed using appropriate similarity transformations. The transformed similarity equations are then solved numerically by applying shooting method. The obtained results for special cases of the problem are compared with previously published work and found to be in excellent agreement. From the present numerical investigations the following conclusions may be drawn:

- The fluid velocity within the boundary layer decreases with the increasing values of the magnetic field parameter, suction parameter, angle of inclination to the vertical, heat flux parameter as well as thermophoretic parameter.
- The temperature distribution increases with the increasing values of the magnetic field parameter, angle of inclination to the vertical, heat flux parameter, heat generation parameter as well as thermophoretic parameter whereas it decreases with an increasing values of the suction parameter, heat absorption parameter as well as heat flux exponent.
- The concentration profile increases with increasing values of the magnetic field parameter, heat absorption parameter and the angle of inclination to the vertical whereas it decreases with the increasing values of the suction parameter, heat flux exponent, heat generation parameter as well as thermophoretic parameter.
- Dufour and Soret parameters have significant effects on the heat and mass transfer flow of a hydrogen-air mixture fluid.

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REFERENCES

[1] Chiam T. C., "Hydromagnetic flow over a surface stretching with a power-law velocity," *Int. J. Engng. Sci*., vol. 33, pp. 429-435, 1995.

[2] Chandran P., Sacheti N. C. and Singh A. K., "Hydromagnetic flow and heat transfer past a continuously moving porous boundary." *Int. Comm. Heat Mass Transfer*, vol. 23, pp. 889-898, 1996.

[3] Pop I. and Na T. Y., "A note on MHD flow over a stretching permeable surface," *Mech. Res. Comm*., vol. 25, pp. 263-269, 1998.

[4] Khan S. K., Abel M. S. and Sonth R. M., "Visco-elastic MHD flow; heat and mass transfer over a porous stretching sheet with dissipation of energy and stress work," *Heat Mass Transfer*., vol. 40, pp. 47-57, 2003.

[5] Mukhopadhyay S., Layek G. C. and Samad A. S., "Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity," *Int. J. Heat Mass Transfer*, vol. 48, pp. 4460-4466, 2005.

[6] Chiou, M. C., "Effect of thermophoresis on sub-micron particle deposition from a forced laminar boundary layer flow on to an isothermal moving plate," *Acta Mechanica*., vol. 129, pp. 219-229, 1998.

[7] Tsai, R. and Liang, L. J., "Correlation for thermophoretic deposition of aerosol particles onto cold plates," *J. Aerosol Sci*., vol. 32, pp. 473-487, 2001.

[8] Walsh, J. K., Weimer A. W. and Hrenya C. M., "Thermophoretic deposition of aerosol particles in laminar tube flow with mixed convection," PhD Dissertation, Department of Chemical and Biological Engineering, Boulder, University of Colorado, 2005.

[9] Alam, M. S., Rahman, M. M. and Sattar, M. A., "Effects of variable suction and thermophoresis on steady MHD free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined flat plate in the presence of thermal radiation," *Int. J. Thermal Sci.*, vol. 47, no. 6, pp. 758-765, 2008.

[10] Alam, M. S., Rahman, M. M., "On the effectiveness of variable heat and mass fluxes on hydromagnetic free convection and mass transfer flow along an inclined permeable stretching surface with thermophoresis," *Int. J. Energy & Tech*., vol. 5, no. 1, pp. 1-10, 2013.

[11] Eckert, E. R. G.and Drake, R. M., *Analysis of Heat and Mass Transfer*, McGraw- Hill, New York, 1972.

[12] Kafoussias, N. G. and Williams, E. W., "Thermal-diffusion and diffusion-thermo effects on mixed free forced convective and mass transfer boundary layer flow with temperature dependent viscosity," *Int. J. Engng. Sci*., vol.33, pp. 1369-1384, 1995.

[13] Anghel, M., Takhar, H. S. and Pop, I., "Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium," Studia Universitatis Babes-Bolyai, *Mathematica*, vol. XLV, pp. 11-21, 2000.

[14] Postelnicu, A., "Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects," *Int. J. Heat Mass Transfer*, vol.47, pp. 1467-1472, 2004.

[15] Alam M. S. and Rahman M. M., "Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction," *Nonlinear Analysis; Modelling and Control*., vol. 11, no. 3, pp. 3-12, 2006.

[16] Acharya M. Singh L. P. and Dash G. C., "Heat and mass transfer over an accelerating surface with heat source in the presence of suction and blowing," *Int. J. Engng. Sci*., vol. 37, pp. 189-211, 1999.

[17] Nachtsheim, P. R. and Swigert, P., "Satisfaction of the asymptotic boundary conditions in numerical solution of the system of non-linear equations of boundary layer type," NASA TND-3004, 1965.

[18] Alam, M. S., Rahman, M. M. and Samad, M. A., "Numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion," *Non-linear Analysis; Modeling and Control*., vol. 11, no. 4, pp. 331-343, 2006.

[19] Andersson, H. I., Bech, K. H. and Dandapat, B. S., "Magnetohydrodynamic flow of a power-law fluid over a stretching sheet," *Int. J. Non-linear Mech*., vol. 27, pp. 929-936, 1992.

[20] Chen, C. H., "Effects of magnetic field and suction/injection on convection heat transfer of non-Newtonian power-law fluids past a power-law

stretched sheet with surface heat flux," *Int. J. Thermal Sci*., vol. 47, no. 7, pp. 954-961, 2008.
[21] Elbashbeshy, E. M. A., "Heat transfer over a stretching surface with variable surface heat flux," *J. Phys. D: Appl. Phys*., vol. 31, pp. 1951-1954, 1998.

NOMENCLATURE

A_1, A_2, b	Prescribed constants
B_0	Magnetic induction
C	Concentration
D	Mass diffusivity
Df	Dufour number
f	Dimensionless stream function
f_w	Dimensionless wall suction
g	Acceleration due to gravity
Gr_x	Local Grashof number
Gm_x	Local modified Grashof number
g_s	Temperature buoyancy parameter
g_c	Mass buoyancy parameter
M	Magnetic field parameter
Nu_x	Local Nusselt number
Pr	Prandtl number
Q	Heat generation/absorption parameter
Sc	Schmidt number
T	Temperature
u, v	Velocity components in the x- and y-direction respectively
x, y	Axis in direction along and normal to the plate

Greek symbols

η	Pseudo-similarity variable
λ	Thermal conductivity of fluid
τ	Thermophoretic parameter
θ	Dimensionless temperature
ϕ	Dimensionless concentration

Subscripts

w	Condition at wall
∞	Condition at infinity