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Reliability Analysis for Multistate Consecutive k-out-of-n: F System Using Bayesian Network

Ranganathan Sakthivel[1,2], Gundala Vijayalakshmi[1*]

[1] Department of Mathematics, College of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur 603203, Tamil Nadu, India
[2] Department of Career Development Center, College of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur 603203, Tamil Nadu, India

Corresponding Author Email: vijayalg@srmist.edu.in

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ABSTRACT

This paper aims to determine the reliability of a complex system using a Bayesian network. A Bayesian network (BN) is a probabilistic graphical model that represents knowledge about an uncertain domain where each component corresponds to a random variable and each edge represents the corresponding conditional probability. Bayesian network is used to estimate the multistate consecutive k-out-of-n: F system reliability. This paper presents the Bayesian network construction and the reliability of the proposed system. The reliability of linear and circular multistate consecutive k-out-of-n: F systems based on the Bayesian network are compared. Furthermore, the reliability of proposed system is shown to be significantly greater than the exact reliability obtained by Amirian, Khodadadi, and Chatrabgoun.

1. INTRODUCTION

Both the system and its components are permitted to take one of two states in classic reliability theory: functioning or failed. In a multistate system, the system and its components might be in more than one state, such as fully functional, partially functional, or entirely failed. The modelling of equipment situations is more flexible using a multi-state system reliability model.

The concept of the successive k-out-of-n: F system was first proposed by Chiang and Niu [1] and has been greatly considered in the idea of reliability. If a system with n components fails whenever k successive components fail, it is called a successive k-out-of-n: F system. The system is linear if the n components are arranged in a line. The system is circular if the n components are arranged in a circle.

The essential ideas of multistate system dependability were largely defined in these studies. The multistate system (MSS) qualities were first explored when the system structure function, minimal cut set and minimal path set, coherency, and element relevance were specified. Ross [2] developed and proved the analogue of the closure theorem for a rising failure rate average stochastic process, and later did the same for new better than used stochastic processes. The idea of binary coherent systems, proposed by Barlow and Wu [3], has been expanded for multistate components. With the binary structure and reliability function principles, many of the conclusions for the binary case can be obtained for multistate systems. El-Neveihi et al. [4] introduced the first MSS reliability optimization issue, which included distributing multistate elements into k-series systems with the goal of increasing the predicted number of systems operating at that level or above. The reliability evaluation of monotone multistate systems with s-independent multistate components, Aven [5] presented two efficient methods employing minimum pathways and minimal cuts. Levitin [6] utilized the universal generating function approach to determine that the dependability of multistate systems had two failure mechanisms.

Zaitseva et al. [7] and Zaitseva and Levashenko [8] used the theory of multi-valued logic to analyse the reliability of multistate parallel systems, series systems, series-parallel systems, and k/n systems, as well as the consequence of component state variations on system reliability, in order to avoid the tedious calculation of minimal path set and cut set. Hudson and Kapur [9, 10] discuss simulations and their uses in reliability investigation for scenarios where the structure can have a wide range of states and all of its modules can have a wide range of many states. They also discuss inclusion-exclusion constraints and how they compare to disjoint subset bounds. The later bounds are based on Abraham's recursive disjoint products, which have been generalized. Since 1996, Lisnianski and Levitin [11] have used the universal generating function approach and genetic algorithm to solve the redundancy optimization problem of a series-parallel power system and a bridge structure. For computing the dependability of a multistate system, Hwang and Yao [12] presented an O (Kn) approach, where K (K<n) is the maximum number of states for a component. Aggarwal et al. [13] developed utilizing minimal path vectors an effective heuristic solution to addressing the restricted redundancy optimization issue in multistate systems. Zhai and Lin [14] discussed how to set up and build a Bayesian network-based multi-state system model, as well as how to use prior and posterior likelihood to perform bidirectional implication investigation and directly estimate the system's dependability measures using prior probability and Conditional Probability Tables (CPT).

Langseth and Portinale [15] emphasized the characteristics of the modelling framework that make BNs particularly suitable for reliability applications, as well as on-going research important to reliability practitioners. Wilson and Huzurbazar [16] discussed how to conduct joint inference about all of the nodes in a network using multilevel discrete data. When system architectures are too complicated to be represented using fault trees, several approaches can be used. According to Khakzad et al. [17], BN is a superior safety analysis model because of its flexible structure, which allows it to match a wide range of accident situations. The dependability of subsea Blowout Preventer control systems is assessed at any given moment using Cai et al. [18] suggested Bayesian network models, and the difference between posterior and prior probability of any single component given system failure is calculated. Martins and Maturana [19] present a BN-based technique for assessing human dependability and applies it to the operation of an oil tanker, with an emphasis on collision risk. The model was used to estimate the most likely sequence of hazardous occurrences and therefore isolate essential activities in the ship's operation in order to investigate Internal Factors, Organizational Factors, Management and Skills that should be given greater attention in order to reduce risk.

Mi et al. [20] research centered on the dependability of complex multistate systems with epistemic uncertainty and common cause failures. The Dempster Shafer (DS) evidence theory is utilized to describe epistemic uncertainty in system through the state space reconstruction of MSS, and an uncertain state is obtained in the new state space based on the Bayesian network approach for reliability analysis of MSS. Li et al. [21] presented a way for embedding fuzzy probability and Bayesian networks into multistate systems using common cause failures. Amiran et al. [22] developed a novel technique that provides exact reliability for a large number of successive k-out-of-r-from-n: F systems. This work is completed in particular for equal and unequal component probabilities. Byun and Song [23] introduced the notion of composite state to extend the Matrix based Bayesian network (MBN) to multistate systems. The MBN definitions and inference methods are updated to accommodate the composite state, and parameter sensitivity formulas are also constructed for the MBN. In a complex multistate system, Jia et al. [24] focused on mixed uncertainty of state information in each unit caused by a lack of data, complicated structures, and inadequate comprehension, as well as common-cause failure between units.

Madhumitha and Vijayalakshmi [25] calculated the mean time to failure and confidence interval using Bayesian methods for the Cons.k/n:F systems. Bibartiu et al. [26] defined the memory growth for the k/n voting gate was lowered from exponential to polynomial in the range of input events due to a scalable Bayesian network model. Nashwan [27] provided formula to calculate the precise reliability and failure likelihood functions for the linear and circular r-gap successive k-out-of-m-from-n: F systems. Amirian and Khodadadi [28] developed a new algorithm, that can determine the exact reliability for a large class of sequential linear and circular systems. Jegatheesan and Gundala [29] produced an evaluation of the linear (circular) Cons.k/n:F system's fuzzy Bayesian reliability using the squared error loss function. Yin et al. [30] proposed a method for describing F systems with common components that combines the theoretical study of linear and circular k-out-of-n with the finite Markov chain imbedding approach. In addition, there are MATLAB programs that provide accurate reliability for sequential linear and circular systems. Our contribution is obtaining the Bayesian network reliability estimate of the multistate consecutive k-out-of-n: F system and comparing it with the exact reliability given by Amirian, Khodadadi, and Chatrabgoun.

In this paper, the reliability of the proposed system is derived by the Bayesian network. The remaining paper is organized as follows: Section 2 gives the background details and the brief introduction of multistate system and Bayesian network is explained in section 3. In section 4 the system reliability evaluation using BN is given. Section 5 gives the results and comparison and section 6 concludes the paper.

2. BRIEF INTRODUCTION OF MULTISTATE SYSTEM AND BAYESIAN NETWORK

2.1 Multistate system

In a particular context, all systems are designed to perform their intended functions. Some systems are capable of doing tasks at varied levels of efficiency, which are referred to as performance rates. A MSS is a system with a finite number of possible performance rates. A MSS is usually made up of components that can be multistate itself. In fact, the simplest example of an MSS with two unique states is a binary system (perfect functioning and complete failure).

A MSS is any system made up of various binary state units that have a cumulative influence on the overall system performance. Indeed, the availability of a system's units determines its performance rate, since various numbers of accessible units might give different levels of task performance. The well-known k-out-of-n systems are a basic illustration of such a situation. These systems are made up of n identical binary units, and depending on the number of units available, they can have an n+1 state. The performance rate of the system is expected to be proportional to the number of units available. Performance rates corresponding to greater than k-1 available units are assumed to be acceptable. Because different combinations of k available units might give different performance rates for the entire system when their contributions to the cumulative system performance rate differ, the number of potential MSS states increases significantly.

In general, every element's performance rate can range from flawless to utter failure. Partial failures are failures that result in a reduction in element performance. Elements continue to operate at decreased performance rates after partial failure, and after complete failure, they are completely incapable of performing their functions. Consider a transmission station-based wireless communication system. The number of consecutive stations covered in a station's range determines its status. This quantity is dependent not only on the availability of station amplifiers, but also on signal propagation circumstances, which are affected by weather, solar activity, and other factors.

2.2 Multi state Bayesian network

A Bayesian network is a directed acyclic graph that consists of variables and directed edges that are all connected by a table of conditional probabilities for each variable on all of its parents. As a result, it's a graphical depiction of unknown quantities that shows the model's information flow as well as the probabilistic causal dependency between variables. Professor Pearl of the University of California proposed the BN in 1986, which is a directed acyclic graph (DAG) with nodes and directed acyclic arcs. Nodes represent components, while arcs connecting pairs of nodes reflect the connections between the components. Root nodes are nodes that have no parents and have prior probability. Conditional probability tables (CPTs) are present in all other nodes, with leaf nodes being those without descendants. A node's conditional probability table provides the likelihood of each state of the node based on all possible combinations of its parents' states.

The BN is built on the well-known Baye's Rule. Assume that M and N are random components. The conditional probability of M given N can be written as:

$$P\left({}^{M}/_{N}\right) = \frac{P(M)P(N/M)}{P(N)} \tag{1}$$

Here $P(M)$ is prior likelihood, $P(M|N)$ is posterior likelihood, and $P(N|M)$ is likelihood ratio.

$$\mathrm{P(N)} = \sum_{i=1}^{l} P(N/M = a_i)P(M = a_i) \tag{2}$$

Suppose a Bayesian network possesses numerous components which are stated as $U=\{X_1,X_2,\dots X_n\}$, X_i ($i=1,2,\dots,n$) is a failure occurrence of a specific component or system that has to be investigated. The conditioned independency theory is an important concept for simplifying the joint probability distribution. Assume that $Par(X_i)$ is X_i's the parent set and $Nd(X_i)$ is X_i's non-descendant set. X_i is independent of $Nd(X_i)$ when conditioned on $Par(X_i)$.

Therefore,

$$P(X_i/Par(X_i), Nd(X_i)) = P(X_i/Par(X_i)) \tag{3}$$

The joint probability distribution may be calculated using the conditioned independency assumption as follows:

$$\mathrm{P(U)} = P(X_1, X_2, \dots X_n) = \prod_{i=1}^{n} P(X_i/Par(X_i)) \tag{4}$$

X_i 's probability distribution may be estimated as follows:

$$P(X_i) = \sum_{except\ X_i} P(U) \tag{5}$$

The posterior probability distribution of nodes may be determined given the evidence E.

$$P\left({}^{U}/_{E}\right) = \frac{P(U,E)}{P(E)} = \frac{P(U,E)}{\sum_U P(U,E)} \tag{6}$$

3. SYSTEM RELIABILITY EVALUATION BY BN

3.1 Linear multistate consecutive k-out-of-n: F system

All the basic components (nodes) $N_1,N_2,\dots N_n$ are placed in linear arrangement, and all components have multiple states i.e., more than two states. The purpose of intermediate components $M_1,M_2,\dots,M_{n-k+1}$ is to find system reliability (SR) component. In the beginning LMC (k\n:F) system will function normally but it will fail only when consecutive k components fail. The intermediate component M_1 connects the first k parent components with temporal link in the linear arrangement, and this will fail after all k parent components have failed. Similarly, k consecutive parent components are connected with temporal link to corresponding intermediate components $M_2,M_3,\dots,M_{n-k+1}$ respectively. All the intermediate components depend on the respective parent component. This arrangement of components is shown in Figure 1. If any one of the intermediate components fail, then LMC (k\n:F) system will fail.

Assumption of System reliability functions are as follows:

- Basic components $N_1,N_2,\dots N_n$ contains three states namely Working State (WS), Partial Working State (PWS), Failed State (FS).
- Intermediate components $M_1,M_2,\dots,M_{n-k+1}$ contains two states namely Working State (WS), Failed State (FS).
- System reliability component SR contains two states namely Working State (WS), Failed State (FS).

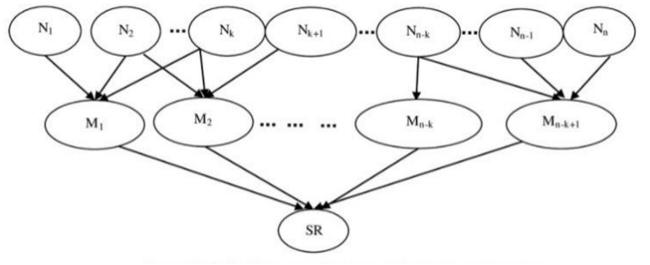

Figure 1. BN for linear multistate consecutive k-out-of-n: F system

LMC(k\n:F) system reliability calculation are given below. Joint Probability function is:

$$\begin{gathered} P(Z) = P(N_1, \dots N_n, M_1, \dots M_{n-k+1}, SR) = \\ P(SR/Par(SR)) \prod_{i=1}^{n-k+1} P(M_i/Par(M_i)) \prod_{i=1}^{n} P(N_i) \end{gathered} \tag{7}$$

$$\begin{gathered} P(Z) = P(SR/M_1, \dots, M_{n-k+1}) \\ P(M_1/N_1, .. N_k) \\ P(M_2/N_2, \dots, N_{k+1}) \dots \dots \\ P(M_{n-K-1}/N_{n-k}, \dots, N_{n-1})\ P(M_{n-k+1}/N_{n-k}, \dots, N_n) \\ P(N_1)P(N_2) \dots \dots\ P(N_n) \end{gathered} \tag{8}$$

Marginal Probability function is:

$$\begin{gathered} P(SR) = \\ \sum_{\substack{N_1=FS,\dots N_n=FS, \\ M_1=FS,\dots M_{n-k+1}=FS}}^{WS} P(SR/Par(SR)) \prod_{i=1}^{n-k+1} P(M_i/Par(M_i)) \prod_{i=1}^{n} P(N_i) \end{gathered} \tag{9}$$

$$\begin{aligned} P(SR) \\ = \sum_{\substack{N_1=FS,\dots N_n=FS, \\ M_1=FS,\dots M_{n-k+1}=FS}}^{WS} & P(SR/M_1, \dots, M_{n-k+1})P(M_1/N_1, \dots, N_k) \\ & P(M_2/N_2, \dots, N_{k+1}) \dots \dots \\ & P(M_{n-K-1}/N_{n-k}, \dots, N_{n-1}) \\ & P(M_{n-k+1}/N_{n-k}, \dots, N_n)\ P(N_1) \\ & P(N_2) \dots \dots\ P(N_n) \end{aligned} \tag{10}$$

Reliability for Final node (SR) is:

$$\begin{aligned} & R(SR) = P(SR/M_1 = FS, M_2 = WS, \dots, M_{n-k+1} = \\ & WS)\ P\ (M_1 = FS)\ P\ (M_2 = WS) \dots \dots P(M_{n-k+1} = \\ & WS) + P(SR/M_1 = WS, M_2 = FS, \dots, M_{n-k+1} = \\ & WS)P(M_1 = WS)P(M_2 = FS) \dots \dots P(M_{n-k+1} = \\ & WS) + \\ & \vdots \\ & P(SR/M_1 = FS, M_2 = FS, M_3 = WS, \dots, M_{n-k+1} = \\ & WS)P(M_1 = FS)P(M_2 = FS)P(M_3 = \\ & WS) \dots \dots P(M_{n-k+1} = WS) + \\ & \vdots \end{aligned} \tag{11}$$

$$
\begin{aligned}
&P(SR/M_1 = FS, M_2 = FS, M_3 = FS, \dots, M_{n-k+1} = WS)\, P(M_1 = FS)\, P(M_2 = FS)\, P(M_3 = FS) \dots \dots P(M_{n-k+1} = WS) + \\
&\vdots \\
&P(SR/M_1 = FS, M_2 = FS, M_3 = FS, \dots, M_{n-k+1} = FS) P(M_1 = FS) P(M_2 = FS) P(M_3 = FS) \dots \dots P(M_{n-k+1} = FS) + \\
&\vdots \\
&R(SR) \\
&= P(SR/M_1 = WS, M_2 = WS, M_3 = WS, \dots, M_{n-k+1} = WS) P(M_1 = WS) P(M_2 = WS) P(M_3 = WS) \dots \dots P(M_{n-k+1} = WS)
\end{aligned}
$$

Probability Calculation for Intermediate nodes,

$$
\begin{aligned}
&P(M_1 = WS) = P(M_1 = WS/N_1 = WS, N_2 = WS, N_3 = WS, \dots, N_k = WS) P(N_1 = WS) P(N_2 = WS) P(N_3 = WS) \dots \dots P(N_k = WS) + \\
&P(M_1 = WS/N_1 = PWS, N_2 = WS, \dots, N_k = WS)\ P(N_1 = PWS) P(N_2 = WS) \dots \dots P(N_k = WS) + \\
&P(M_1 = WS/N_1 = FS, N_2 = WS, \dots, N_k = WS)\ P(N_1 = FS) P(N_2 = WS) \dots \dots P(N_k = WS) + \\
&P(M_1 = WS/N_1 = WS, N_2 = PWS, \dots, N_k = WS)\ P(N_1 = WS) P(N_2 = PWS) \dots \dots P(N_k = WS) + \\
&P(M_1 = WS/N_1 = WS, N_2 = FS, \dots, N_k = WS)\ P(N_1 = WS) P(N_2 = FS) \dots \dots P(N_k = WS) + \\
&\vdots \\
&P(M_1 = WS/N_1 = PWS, N_2 = PWS, N_3 = WS, \dots, N_k = WS) \\
&P(N_1 = WS) P(N_2 = PWS) P(N_3 = WS) \dots \dots P(N_k = WS) + \\
&\vdots \\
&P(M_1 = WS/N_1 = FS, N_2 = FS, N_3 = WS, \dots, N_k = WS)\ P(N_1 = FS) P(N_2 = FS) P(N_3 = WS) \dots \dots P(N_k = WS) + \\
&\vdots \\
&P(M_1 = WS/N_1 = PWS, N_2 = PWS, N_3 = PF, \dots, N_k = WS) \\
&P(N_1 = PWS) P(N_2 = PWS) P(N_3 = PWS) \dots \dots P(N_k = WS) + \\
&\vdots \\
&P(M_1 = WS/N_1 = FS, N_2 = FS, N_3 = FS, \dots, N_k = WS) \\
&P(N_1 = FS) P(N_2 = FS) P(N_3 = FS) \dots \dots P(N_k = WS) + \\
&\vdots \\
&P(M_1 = WS/N_1 = FS, N_2 = FS, \dots, N_{k-1} = FS, N_k = WS) \\
&P(N_1 = FS)\, P(N_2 = FS)\, P(N_3 = FS) \dots \dots P(N_{k-1} = FS) P(N_k = WS)
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
P(M_1 = FS) = P(M_1 = FS/N_1 = FS, \dots, N_{k-1} = FS, N_k = FS) \\
P(N_1 = FS) \dots \dots P(N_{k-1} = FS)\, P(N_k = FS)
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
&P(M_{n-k+1} = WS) = \\
&P(M_{n-k+1} = WS/N_{n-k} = WS, \dots, N_n = WS) \\
&P(N_{n-k} = WS) \dots \dots P(N_n = WS) + \\
&P(M_{n-k+1} = WS/N_{n-k} = PWS, N_{n-k+1} = WS, \dots, N_n = WS) \\
&P(N_{n-k} = PWS) P(N_{n-k+1} = WS) \dots \dots P(N_n = WS) + \\
&P(M_{n-k+1} = WS/N_{n-k} = FS, N_{n-k+1} = WS, \dots, N_n = WS) \\
&P(N_{n-k} = FS) P(N_{n-k+1} = WS) \dots \dots P(N_n = WS) + \\
&P(M_{n-k+1} = WS/N_{n-k} = WS, N_{n-k+1} = PWS, \dots, N_n = WS) \\
&P(N_{n-k} = WS) P(N_{n-k+1} = PWS) \dots \dots P(N_n = WS) + \\
&P(M_{n-k+1} = WS/N_{n-k} = WS, N_{n-k+1} = FS, \dots, N_n = WS) \\
&P(N_{n-k} = WS) P(N_{n-k+1} = FS) \dots \dots P(N_n = WS) + \\
&\vdots \\
&P(M_{n-k+1} = WS/N_{n-k} = FS, N_{n-k+1} = PWS, \dots, N_n = WS) \\
&P(N_{n-k} = FS) P(N_{n-k+1} = PWS) \dots \dots P(N_n = WS) + \\
&\vdots \\
&P(M_{n-k+1} = WS/N_{n-k} = FS, \dots, N_{n-1} = FS, N_n = WS) \\
&P(N_{n-k} = FS) P(N_{n-k+1} = FS) \dots \dots P(N_{n-1} = FS) P(N_n = WS)
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
&P(M_{n-k+1} = FS) \\
&P(M_{n-k+1} = FS/N_{n-k} = FS, \dots, N_{n-1} = FS, N_n = FS) \\
&P(N_{n-k} = FS) P(N_{n-k+1} = FS) \dots \dots P(N_{n-1} = FS) P(N_n = FS)
\end{aligned} \tag{15}
$$

Failure of system reliability (SR) is:

$$
R(SR = FS) = \sum_{\substack{No.\ of\ failure \\ components\ in\ Par(SR) \geq 1}} P(SR = FS/Par(SR))\, P(Par(SR)) \tag{16}
$$

where,

$$
\begin{aligned}
&Par(SR) = M_1, M_2, M_3 \dots, M_{n-k+1} \\
&P(M_i = FS) = P(M_i = FS/Par(M_i = FS))\, P(Par(M_i = FS)) \\
&Par(M_i) = Consecutive\ k\ components\ from\ N_1, N_2, N_3 \dots, N_n\ (Start\ with\ i^{th}\ component, i = 1,2,3, \dots, n)
\end{aligned}
$$

3.2 Circular multistate consecutive k-out-of-n: F system

All the basic components (nodes) $N_1, N_2, \dots N_n$ are placed in circular arrangement, and all components have multiple states i.e., more than two states. The purpose of intermediate components $M_1, M_2, \dots, M_n$ is to find system reliability (SR) component. In the beginning CMC(k\n:F) system will function normally but it will fail only when consecutive k components fail. The intermediate component M_1 connects the first k parent components with temporal link in the linear arrangement, and this will fail after all k parent components have failed. Similarly, k consecutive parent components are connected with temporal link to corresponding intermediate components $M_2, M_3, \dots, M_{n-k+1}$ respectively. All the intermediate components depend on the respective parent component. This arrangement of components is shown in Figure 2. If any one of the intermediate components fail, then CMC(k\n:F) system will fail.

Assumption of System reliability functions are as follows:

- Basic components $N_1, N_2, \dots N_n$ contains three states namely Working State (WS), Partial Working State (PWS), Failed State (FS).
- Intermediate components $M_1, M_2, \dots, M_{n-k+1}$ contains two states namely Working State (WS), Failed State (FS).
- System reliability component SR contains two states namely Working State (WS), Failed State (FS).

CMC (k\n:F) system reliability calculation are given below,

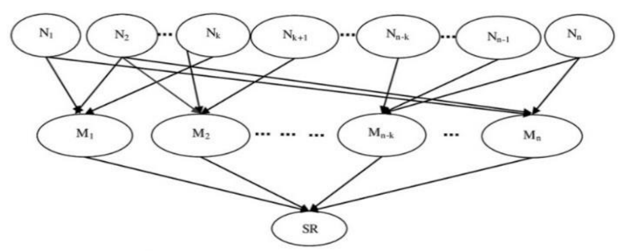

Figure 2. BN for circular multistate consecutive k-out-of-n: F system

Joint Probability function is:

$$
P(Z) = P(N_1, \dots N_n, M_1, \dots M_n, SR) = P(SR/Par(SR)) \prod_{i=1}^{n} P(M_i/Par(M_i)) \prod_{i=1}^{n} P(N_i) \tag{17}
$$

$$
\begin{aligned}
P(Z) = P(SR/M_1, \dots, M_{n-k+1}) P(M_1/N_1, .. N_k) \\
P(M_2/N_2, \dots, N_{k+1}) \dots \dots P(M_{n-k}/N_{n-k}, \dots, N_n) \\
P(M_n/N_n, N_1 \dots, N_{k-1})\, P(N_n) P(N_1) \dots \dots P(N_{k-1})
\end{aligned} \tag{18}
$$

Marginal Probability function is:

$$
P(SR) = \sum_{\substack{N_1 = FS, \dots N_n = FS, \\ M_1 = FS, \dots M_n = FS}}^{WS} P(SR/Par(SR)) \prod_{i=1}^{n} P(M_i/Par(M_i)) \prod_{i=1}^{n} P(N_i) \tag{19}
$$

$$P(SR) = \sum_{\substack{N_1=FS,\dots N_n=FS,\\ M_1=FS,\dots M_n=FS}}^{WS} \begin{matrix} P(SR/M_1,\dots,M_{n-k+1}) \\ P(M_1/N_1,\dots,N_k)P(M_2/N_2,\dots,N_{k+1})\dots \\ P(M_n/N_n,N_1\dots,N_{k-1}) \\ P(N_1)\dots\dots P(N_{k-1})P(N_n) \end{matrix} \quad (20)$$

Reliability for Final node (SR) is:

$$\begin{aligned} R(SR) &= P(SR/M_1 = FS, M_2 = WS, \dots, M_n = WS) \\ &P(M_1 = FS)P(M_2 = WS)\dots\dots P(M_n = WS) + \\ &P(SR/M_1 = WS, M_2 = FS, \dots, M_n = WS) \\ &P(M_1 = WS)\ P(M_2 = FS)\dots\dots P(M_n = WS) + \\ &\vdots \\ &P(SR/M_1 = FS, M_2 = FS, M_3 = WS, \dots, M_{n-k+1} = WS) \\ &P(M_1 = FS)P(M_2 = FS)P(M_3 = WS)\dots\dots P(M_n = WS) + \\ &\vdots \\ &P(SR/M_1 = FS, M_2 = FS, M_3 = FS, \dots, M_n = WS) \\ &P(M_1 = FS)P(M_2 = FS)P(M_3 = FS)\dots\dots P(M_n = WS) + \\ &\vdots \\ &P(SR/M_1 = FS, M_2 = FS, M_3 = FS, \dots, M_n = FS) \\ &P(M_1 = FS)P(M_2 = FS)P(M_3 = FS)\dots\dots P(M_n = FS) + \\ &\vdots \\ R(SR) &= \\ &P(SR = WS/M_1 = WS, M_2 = WS, M_3 = WS, \dots, M_n = WS) \\ &P(M_1 = WS)P(M_2 = WS)P(M_3 = WS)\dots\dots P(M_n = WS) \end{aligned} \quad (21)$$

Probability Calculation for Intermediate nodes:

$$\begin{aligned} &P(M_1 = WS) = \\ &P(M_1 = WS/N_1 = WS, N_2 = WS, N_3 = WS, \dots, N_k = WS) \\ &P(N_1 = WS)P(N_2 = WS)P(N_3 = WS)\dots\dots P(N_k = WS) + \\ &P(M_1 = WS/N_1 = PWS, N_2 = WS, \dots, N_k = WS) \\ &P(N_1 = PWS)P(N_2 = WS)\dots\dots P(N_k = WS) + \\ &P(M_1 = WS/N_1 = FS, N_2 = WS, \dots, N_k = WS) \\ &P(N_1 = FS)P(N_2 = WS)\dots\dots P(N_k = WS) + \\ &P(M_1 = WS/N_1 = WS, N_2 = PWS, \dots, N_k = WS) \\ &P(N_1 = WS)P(N_2 = PWS)\dots\dots P(N_k = WS) + \\ &P(M_1 = WS/N_1 = WS, N_2 = FS, \dots, N_k = WS) \\ &P(N_1 = WS)P(N_2 = FS)\dots\dots P(N_k = WS) + \\ &\vdots \\ &P(M_1 = WS/N_1 = PWS, N_2 = PWS, N_3 = WS, \dots, N_k = WS) \\ &P(N_1 = PWS)P(N_2 = PWS)P(N_3 = WS)\dots\dots P(N_k = WS) + \\ &\vdots \\ &P(M_1 = WS/N_1 = FS, N_2 = FS, N_3 = WS, \dots, N_k = WS) \\ &P(N_1 = FS)P(N_2 = FS)P(N_3 = WS)\dots\dots P(N_k = WS) + \\ &\vdots \\ &P(M_1 = WS/N_1 = PWS, N_2 = PWS, N_3 = PWS, \dots, N_k = WS) \\ &P(N_1 = PWS)P(N_2 = PWS)P(N_3 = PWS)\dots\dots P(N_k = WS) + \\ &\vdots \\ &P(M_1 = WS/N_1 = FS, N_2 = FS, N_3 = FS, \dots, N_k = WS) \\ &P(N_1 = FS)P(N_2 = FS)P(N_3 = FS)\dots\dots P(N_k = WS) + \\ &\vdots \\ &P(M_1 = WS/N_1 = FS, N_2 = FS, \dots, N_{k-1} = FS, N_k = WS) \\ &P(N_1 = FS)\ P(N_2 = FS)\dots\dots P(N_{k-1} = FS)P(N_k = WS) \end{aligned} \quad (22)$$

$$\begin{aligned} &P(M_1 = FS) = P(M_1 = FS/N_1 = FS, \dots, N_{k-1} = FS, N_k = FS) \\ &P(N_1 = FS)P(N_2 = FS)\dots P(N_{k-1} = FS)P(N_k = FS) \end{aligned} \quad (23)$$

$$\begin{aligned} &P(M_n = WS) = P(M_n = WS/N_n = WS, N_1 = WS, \dots, N_{k-1} = WS) \\ &P(N_n = WS)\dots\dots P(N_{k-1} = WS) + \\ &P(M_n = WS/N_n = PWS, N_1 = WS, \dots, N_{k-1} = WS) \\ &P(N_n = PWS)P(N_1 = WS)\dots\dots P(N_{k-1} = WS) + \\ &P(M_n = WS/N_n = FS, N_1 = WS, \dots, N_{k-1} = WS) \\ &P(N_{n-k} = FS)P(N_{n-k+1} = WS)\dots\dots P(N_n = WS) + \\ &P(M_n = WS/N_n = WS, N_1 = PWS, \dots, N_{k-1} = WS) \\ &P(N_n = WS)P(N_1 = PWS)\dots\dots P(N_{k-1} = WS) + \\ &P(M_n = WS/N_n = WS, N_1 = FS, \dots, N_{k-1} = WS) \\ &P(N_n = WS)P(N_1 = FS)\dots\dots P(N_{k-1} = WS) + \\ &\vdots \\ &P(M_n = WS/N_n = PWS, N_1 = PWS, \dots, N_{k-1} = WS) \\ &P(N_n = PWS)P(N_1 = PWS)\dots\dots P(N_{k-1} = WS) + \\ &\vdots \\ &P(M_n = WS/N_n = FS, N_1 = FS, \dots, N_{k-1} = WS) \\ &P(N_n = FS)P(N_1 = FS)\dots\dots P(N_{k-1} = WS) + \\ &\vdots \\ &P(M_n = WS/N_n = FS, N_1 = PWS, \dots, N_{k-1} = WS) \\ &P(N_n = FS)P(N_1 = PWS)\dots\dots P(N_{k-1} = WS) + \\ &\vdots \\ &P(M_n = WS/N_n = FS, N_1 = FS, \dots, N_{k-2} = FS, N_{k-1} = WS)\ P(N_n = FS)P(N_1 = FS)\dots\dots P(N_{k-2} = FS)P(N_{k-1} = WS) \end{aligned} \quad (24)$$

$$\begin{aligned} &P(M_n = FS) \\ &= P(M_n = FS/N_n = FS, N_1 = FS, \dots, N_{k-2} = FS, N_{k-1} = FS)\ P(N_n = FS)P(N_1 = FS)\dots\ P(N_{k-2} = FS)P(N_{k-1} = FS) \end{aligned} \quad (25)$$

Failure of system reliability (SR) is:

$$R(SR = FS) = \sum_{\substack{No.of\ failure \\ components\ in\ Par(SR)\geq 1}} P(SR = FS/Par(SR))\ P(Par(SR)) \quad (26)$$

where,

$$Par(SR) = M_1, M_2, M_3 \dots., M_n$$
$$P(M_i = FS) = P(M_i = FS/Par(M_i = FS))\ P(Par(M_i = FS))$$
$$Par(M_i) = Consecutive\ k\ components\ from\ N_1, N_2, N_3 \dots., N_n\ (Start\ with\ i^{th}\ component, i = 1,2,3, \dots., n)$$

3.3 Particular cases

Case (i)

When k=1, the system behaves like a series system, if any one of system component fails the entire system fails. In other words, if all of the components are operational, the system will function; otherwise, it will fail. Each component has failure probability $P(X_i)$, and then reliability formula is:

$$R_s(S) = \prod_i^n P(X_i)$$

Case (ii)

When k=n, the system requires all components to failure; so the system behaves like a parallel system. Each component has failure probability $P(X_i)$, and then reliability formula is:

$$R_P(S) = 1 - \prod_i^n (1 - P(X_i))$$

4. RESULTS AND COMPARISON

The BN reliability graph of the linear consecutive k-out-of-7: F system and circular consecutive k-out-of-7:F systems are shown in Figure 3 and Figure 4 respectively for the values given in Tables 1 and 2. The BN reliability of the system has been plotted for various probability values. It is observed that when the failure probability of component increases, reliability of the system decreases in both linear and circular systems. Table 3 shows the comparison of reliability Amirian et al. (2019) and BN reliability for both linear and circular systems. Figure 5 shows that the reliability evaluation for the proposed system using BN is approximately 10% and 8%

higher than the reliability (R) by Amirian, Khodadadi, and Chatrabgoun (2019). Since BN is a robust model giving accurate results. The reliability value for the various k values is shown in Table 4. Figure 6 reveals that the LMC(k\7:F) system structure has nearly 0.8% higher reliability in comparison to CMC(k\7:F) system structure. This is because circular system has more failures compared to linear system. The Figures 3-6 are drawn using MATLAB.

Table 1. BN reliability for linear consecutive k-out-of-7:F system

Q		**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
	k=3	0.995	0.966	0.898	0.785	0.633	0.457	0.281	0.131	0.033
BN R	k=4	0.999	0.995	0.975	0.928	0.844	0.715	0.544	0.345	0.147
	k=5	0.999	0.998	0.994	0.977	0.938	0.86	0.731	0.541	0.291

Table 2. BN reliability for circular consecutive k-out-of-7:F system

Q		**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
	k=3	0.994	0.955	0.867	0.73	0.555	0.367	0.197	0.073	0.012
BN R	k=4	0.999	0.991	0.96	0.891	0.773	0.609	0.413	0.217	0.062
	k=5	0.999	0.998	0.988	0.955	0.883	0.754	0.565	0.332	0.108

Table 3. Reliability versus BN Reliability for linear and circular at n=11 and k=2

P	**0.25**	**0.5**	**0.75**	**0.8**	**0.9**	**0.95**	**0.99**
R	0.00006	0.0166	0.29721	0.43621	0.77915	0.93128	0.99673
BN LR	0.00026	0.0563	0.524	0.665	0.904	0.975	0.999
BN CR	0.00011	0.0422	0.492	0.638	0.895	0.973	0.998

Table 4. Reliability of linear and circular system for various k values

k	**1**	**2**	**3**	**4**	**5**	**6**
BN LR	0.21	0.807	0.966	0.995	0.999	0.999
BN CR	0.21	0.783	0.955	0.991	0.998	0.999

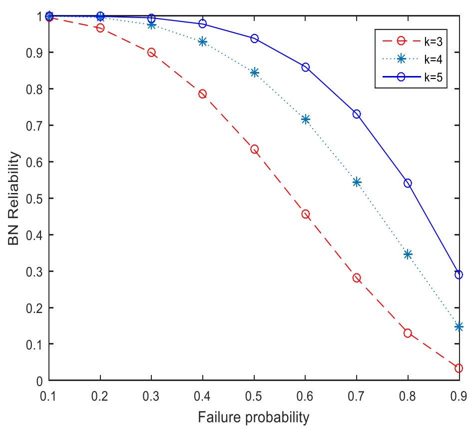

Figure 3. BN reliability for linear system

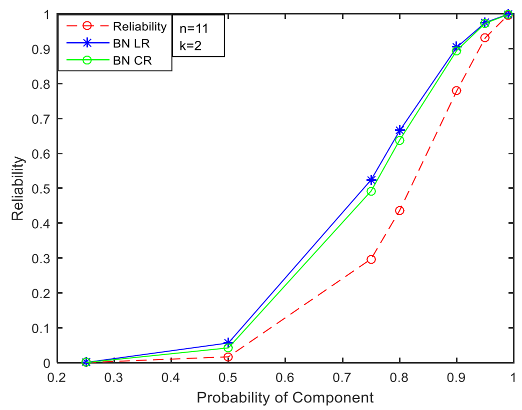

Figure 5. Reliability versus BN Reliability for linear and circular

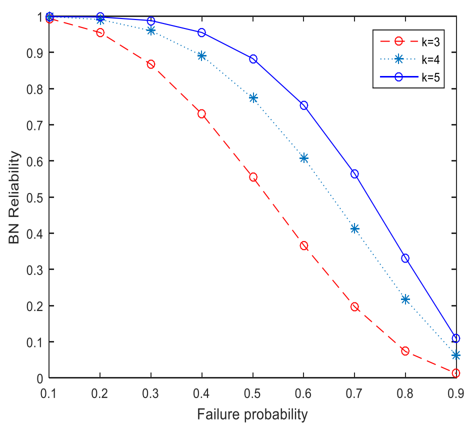

Figure 4. BN Reliability for circular system

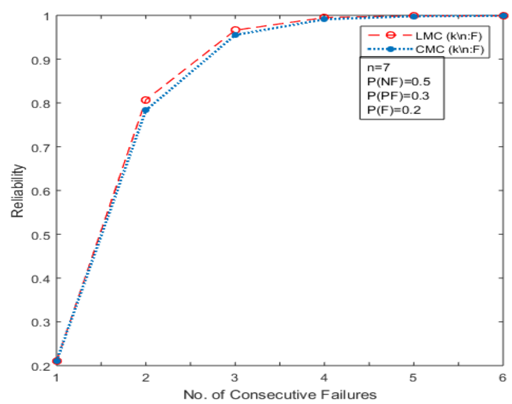

Figure 6. Reliability comparison for linear and circular system

5. CONCLUSION

We derived the reliability formula for the linear and circular multistate consecutive k-out-of-n: F system using Bayesian network. We derived particular cases series and parallel from the consecutive k-out-of-n: F using Bayesian network. The reliability of proposed system is shown to be 10% and 8% greater than the exact reliability obtained by Amirian, Khodadadi, and Chatrabgoun. LMC(k\n:F) system is shown to be 0.8% higher in comparison to the CMC(k\n:F) system. In future, the reliability analysis of LMC(k\n:F) and CMC(k\n:F) systems can be performed by constructing a continuous time Bayesian network and Dynamic Bayesian network. We can derive reliability formula for the linear and circular multistate consecutive k-out-of-n: F system using Bayesian network with non-identical components.

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NOMENCLATURE

LMC(k\n:F)	Linear Multistate Consecutive k-out-of-n: F system
CMC(k\n:F)	Circular Multistate Consecutive k-out-of-n: F system
$N_1, N_2, \dots N_n$	Independent and identically distributed components (Nodes) of the LMC(k\n:F) system
$M_1, M_2, \dots, M_{n-k+1}$	Intermediate Nodes of the LMC(k\n: F) system
$Par(X_i)$	Parents of node X_i
R	Reliability
SR	System Reliability
P	Success Probability
Q	Failure Probability
K	Number of Consecutive Failure
BN	Bayesian Network
BN R	Bayesian Network Reliability
BN LR	Bayesian Network Linear system Reliability
BN CR	Bayesian Network Circular system Reliability
MSS	Multistate System
DAG	Directed Acyclic Graph
CPTs	Conditional Probability Tables
CCFs	Common Cause Failures