USE OF EQUIVALENT CELERITY TO ESTIMATE MAXIMUM PRESSURE INCREASE IN SERIAL PIPES DURING WATER HAMMER - NUMERICAL SIMULATIONS IN MATLAB

AGNIESZKA MALESIŃSKA[1], MARIUSZ ROGULSKI[1], PIERFABRIZIO PUNTORIERI[2], GIUSEPPE BARBARO[2] & BEATA KOWALSKA[3]
[1]Warsaw University of Technology Faculty of Building Services, Hydro and Environmental Engineering, Warsaw, Poland.
[2]Department of Civil Engineering, Energy, Environmental and Materials, Mediterranea University, RC, Italy.
[3]Lublin University of Technology, Faculty of Environmental Engineering, Lublin, Poland.

ABSTRACT
Pipe lines are useful for transporting water for drinking, irrigation and for fire-ing over long distances, this pipe lines are called “Transmission line” and are used to carry conveying raw or treated water from a well field or remote storage (large lake, reservoir, etc.,) facility to a treatment plant and/or distribution storage tank. In water-carrying piping systems, dangerous phenomena may occur. One such phenomenon is water hammer.

The water hammer has always been an area of study, which has captivated the minds of researchers due to its complex and challenging phenomena. Modeling the phenomenon in real conditions is extremely difficult. Due to the dimensions of the piping systems, conducting research at real scales is impossible. However, thanks to the development of numerical methods, the study of water hammer and its effects can be performed using simulation programs. Unfortunately, the simulation results are not always consistent with the actual course of the phenomenon.

One of the parameters that describes the nature of the course of a water hammer is the velocity of propagation of the pressure wave, c, which is called celerity. The transient surge pressure, p, may be calculated from the pressure celerity c, and the sudden change in fluid flow velocity, Δv. In a piping system, the value of the pressure wave celerity is not equal to the individual celerity, c, for a single pipeline. Therefore for piping systems for Δp calculations the equivalent celerity shell be used.

This article presents value of the equivalent celerity calculated from equations derived using linear analysis of natural vibrations of the system. For implement of the equations, an algorithm in MATLAB has been developed that allows one to easily calculate the equivalent celerity, c_e, for N pipelines connected in series with varying diameter, length and material composition.
Keywords: equivalent celerity, long distance, numerical methods, water hammer.

1 INTRODUCTION

The effectiveness of the pipe lines for transporting water for drinking, irrigation and for fire-fighting is largely determined by its reliability. One of the elements that influences the water transport is the water supply. Water for drinking, irrigation and for fire-fighting [7] purposes is often transported over long distances, through piping systems of considerable geometrical dimensions. Such piping systems can be susceptible to the formation of fast, changeable phenomena. One of these phenomena is known as water hammer.

The water hammer has always been an area of study, which has captivated the minds of researchers due to its complex and challenging phenomena. It has been known to cause serious ruptures and losses in pipe systems. For these reasons, there are extensive studies on literature related to water hammer, for example [1–3, 6].

The water hammer phenomenon is usually modeled numerically. However, for the purpose of numerical simulation, the results still need to be verified in a physical model.

ISSN: 2046-0546 (paper format), ISSN: 2046-0554 (online), http://www.witpress.com/journals
DOI: 10.2495/CMEM-V7-N1-22-32

One of the parameters describing the phenomenon of water hammer is the velocity of propagation of the disturbance, c, which is called celerity. This value is closely connected with the pipe material. For steel pipes, it is assumed that this value reaches 1,280 m/s, and for plastic pipes, c = 390 m/s. The value of celerity c, in turn, determines the maximum pressure increase caused by the passing water hammer wave. As already mentioned, for piping systems with a constant internal diameter, the celerity of the pressure wave is related to the material of the pipe walls. However, for piping systems with different diameters, the value of celerity c cannot be determined based only on the wall material of the pipes.

Using natural vibration analysis to calculate the equivalent celerity c_e, it is possible to estimate the basic parameters of water hammer in a fast and easy way, and among other things, the expected maximum pressure increase. This, in turn, can translate into a fast verification of the proposed piping system for estimating the water hammer phenomenon as a function of connected pipes of different diameters, different lengths and different material compositions. To facilitate the use of the equations, an algorithm implemented in MATLAB has been presented that allows specifying the value of the equivalent celerity c_e for any number of connected pipes.

2 THE ESSENCE OF THE WATER HAMMER PHENOMENON

To simulate the water hammer phenomenon, linear analysis methods can be used, provided that the pulsing in the relevant system is a periodic motion. This is confirmed by harmonic analysis of the phenomenon, using the classical oscillation equations, and limiting it to the case of damped natural vibrations (Wrona in 1969, Potter 1982). Vibrations are periodic movements in which all points of the vibrating system repeatedly return to the initial state after a fixed time interval. The simplest vibrating motion is the harmonic motion. Any periodic vibration can be formed by the superposition of the basic harmonic vibrations of the same period and in a limited case from an infinite number of higher harmonic vibrations of appropriately selected amplitudes and phases. Vibrations triggered by the action of a single pulse are natural vibrations. An example of this type of vibration is the water hammer formed under the influence of a single pulse, such as, for example, the sudden closure of a valve.

In an attempt to predict certain parameters of the water hammer phenomenon, one can use a theory based on the natural vibrations of the system utilizing the equations of fluid mechanics.

Often, when using equations governing the variable flow of liquid in the pipeline system, the solution reduces to a function of time. Thus, the analysis is limited only to a temporary state or the method of triggering this state. According to harmonic analysis of the water hammer phenomenon, a variable flow may sometimes be referred to as periodically pulsing or showing its own vibration. With such a description of the phenomenon, the solution of the equations allows one to determine the frequency (i.e., one can search for solutions directly specifying a steady pulsation cycle). In this case, achieving a uniform rate of the pulsation cycle is important.

Frequency-dependent factors, such as friction or wave celerity, have a major impact on the dynamic behavior of liquids in pulsating conditions as a result of extraordinary energy dissipation or due to fluctuations of wave celerity.

Borrowing methods from the theory of linear vibrations requires reducing the equations describing the course of the phenomenon to a linear form (Pipes, 1958). To this end, the differential equation of motion and continuity is adopted in simplified form, i.e., it is assumed that the average flow parameters are “constant”. Thus, their derivatives are both zero, and the friction is reduced to a linear form.

The basic formula for further investigations were equations [5, 8]:

Motion equation $gH_x + VV_x + V_t + g\sin\alpha \dfrac{fV|V|}{2D} = 0$ (1)

Continuity equation (derived by T.P. Propson) $VH_x + H_t - V\sin\alpha + \dfrac{a^2}{g}V_x = 0$ (2)

After simplifying and making above equations Q and H dependent, we get respectively [5]:

$$\frac{\partial H}{\partial x} + \frac{1}{gQ}\frac{\partial Q}{\partial t} + \frac{\lambda Q^n}{2gDA^n} = 0 \tag{3}$$

$$\frac{\partial Q}{\partial x} + \frac{gA}{c^2}\frac{\partial H}{\partial t} = 0 \tag{4}$$

where: H – piezometric head [m] of the liquid column, Q – volumetric flow rate [m^3/s], λ – multiplication factor of the friction element [-], n – power exponent [-], D – pipeline diameter [m], A – pipeline cross-section area [m2], c – wave head speed in water hammer [m/s], g – acceleration due to gravity [m/s^2].

If in equation (3) the loss factor $\dfrac{\lambda Q^n}{2gDA^n}$ is omitted, equations (3) and (4) take the form of the formulas given by Allevi in 1913. However, in the presented discussions the equations (3) and (4) were used to analyse system's free vibrations, which allowed to partially take into account the influence of losses on the parameters of the water hammer (the calculated frequency of the equivalent celerity is influenced by the frequency of the ω oscillation of the system).

The use of these equations in the linear vibration analysis after the introduction of the hyperbolic functions allows the derivation of equations describing the amount of pressure and flow as a function of position in the pipeline:

$$H(x) = H_U \cosh\gamma x - Z_C Q_U \sinh\gamma x \tag{5}$$

$$Q(x) = -\frac{H_U}{Z_C}\sinh\gamma x + Q_U \cosh\gamma \tag{6}$$

where: index U – means the beginning of the pipeline, Q and H as in eqns (3) and (4), Z_C – resistivity determined for the liquid in a specific pipeline. Characteristic impedance Z_C can also be found in other methods used to analyze the water hammer phenomenon such as: Impedance Method or Transfer Matrix Method.

Eqns (5) and (6) are equations of pressure and flow transfer. Using the above equations allows computing (after appropriate substitutions and transformations) the equivalent celerity thanks to the linking of pipe properties (A_i, c_i, L_i) with the natural frequency of the system. As will be shown, the use of the method of natural vibration provides a simple solution to this problem. Further discussion will focus on the serial connection of two and three pipes made of different materials and for different ratios of diameters and lengths of connected sections.

For the correct solution of the issue, a correct definition of the boundary conditions of system operation is important.

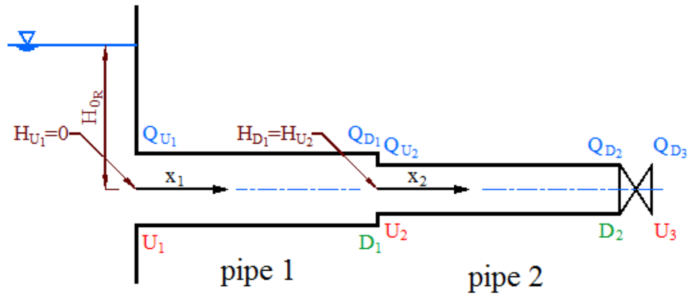


Figure 1: Scheme of a serial connection of two pipes.

Figure 1 shows an example of a serial connection of two pipes. According to the presented scheme, the boundary conditions are as follows:

$$H_{D1} = H_{U1} \text{ and } Q_{D1} = Q_{U2}, \tag{7}$$

where index U indicates the beginning of the pipeline and index D indicates the end of the pipeline.

Where: U – upstream, D – downstream, H – pressure value in the given section, Q – flow value in the given section, H_{0R} – static pressure in the tank.

For any point of the pipe, an equation for the transfer of pressure and flow can be written as given in eqns (5) and (6).

Using the boundary conditions and the fact that the pipes are serially connected, the following equation can be obtained:

$$-\frac{Z_{c_1}}{Z_{c_2}} \tanh \gamma_1 l_1 \tanh \gamma_2 l_2 = 1 \tag{8}$$

where Z_C – resistivity $Z_C = \frac{\gamma}{Cs}$, C – constant, passive capacitive reactance [m], s – permanent, complex frequency (Laplace transform) [-] s = σ +iω, σ – real part, iω – imaginary part, γ – propagation constant of disturbance $\gamma^2 = Cs\left(L_b s + R\right)$, L_b – passive inertial resistance [s^2/m^3], R – resistivity of the pipeline per unit of length. In further consideration, component R was omitted. This is not, however, equivalent to completely excluding the influence of losses on the water hammer parameters, but merely reducing the impact.

To obtain only the dependencies between geometric parameters of the connected pipes and the vibration frequency of the entire system from the equation above, adequate substitutions for Z_C and γ must be made.

For this purpose, it should be noted that if the vibration pulsation has a regular amplitude at each point x of the hydraulic system, the size of the real part of s must be zero.

After several appropriate substituting and transformations final equations can be derived:

$$tg\frac{\omega l_1}{c_1} tg\frac{\omega l_2}{c_2} = \frac{c_2 A_1}{c_1 A_2} \tag{9}$$

Solving the equation above with regard to ω, the equivalent celerity of the pressure wave c for the entire pipeline may be determined. Because the entire period of a complete cycle, 2T, amounts to hence:

$$2T = \frac{4L}{c} = \frac{2\pi}{\omega} \tag{10}$$

$$c = \frac{2}{\pi} L\omega \tag{11}$$

where $L = L_1 + L_2$, is length of the entire pipeline.

It should be noted that using the dependency (eqn. 9) substantially simplifies and speeds up the computation of the equivalent celerity of pressure oscillation changes upon reaching the natural vibrations of the system. Eqn (9) is only true for a system consisting of two pipes. The analysis of natural vibration oscillations for three pipes connected in series requires the derivation of a new dependency:

$$\frac{c_1 A_2}{c_2 A_1} \tan\frac{\omega l_1}{c_1} \tan\frac{\omega l_2}{c_2} + \frac{c_1 A_3}{c_3 A_1} \tan\frac{\omega l_1}{c_1} \tan\frac{\omega l_3}{c_3} + \frac{c_2 A_3}{c_3 A_2} \tan\frac{\omega l_2}{c_2} \tan\frac{\omega l_3}{c_3} = 1 \tag{12}$$

Similarly, as was the case with two pipes, the solution of the equation with respect to ω allows for the computation of the equivalent celerity c_e.

3 NUMERICAL VERIFICATION

For implement of the equations, an algorithm in MATLAB software package (version 2016) has been developed that allows one to easily calculate the equivalent celerity, c_e (for MATLAB calculation c_e = C), for N pipelines connected in series with varying diameter, length and material composition. A function was implemented that allows the determination of ω and the equivalent celerity of the pressure wave C for any number of pipes, based on pipe parameters. This function has the following form:

function [*Omega*, *C*] = pipes(c, l, A, start_fzero, script_name)

Function arguments:

- *c* – celerity in the given individual pipe (in the form of a line vector, with parameters for all pipes);
- *l* – length of individual pipe (in the form of a line vector, with parameters for all pipes);
- *A* – cross section of the given pipe (in the form of a line vector, with parameters for all pipes);
- *start_fzero* – starting point to find a solution by means of the MATLAB function *fzero()*;
- *script_name* – name of script, into which the sequence of executed commands might be saved.

The function returns two parameters:

- *Omega* – omega value for the system;
- *C* – equivalent celerity of the pressure wave for the entire system.

In the first step, the size of vectors *c, l* and *A* is verified in order to determine the number of pipes (ref. N) composing the system. If the number of elements in individual vectors is not equal, the function returns an error.

The essential transformation is performed using symbolic computation (syms package, Symbolic MATLAB Toolbox). First, a starting point in form of two equations is assumed:

$$H_{D1} = -Q_{U1} Z_{C1} \sinh \gamma_1 l1 \tag{13}$$

$$Q_{D1} = Q_{U1} \cosh \gamma_1 l_1 \tag{14}$$

with symbolic variables H_{D1} and Q_{D1}.

Then, iteratively for steps k=2 … N-1, new symbolic variables and substitutions are made:

$$\mathrm{H_{Uk}}=\mathrm{H_{Dk\text{-}1}} \tag{15}$$

$$\mathrm{Q_{Uk}} = \mathrm{Q_{Dk\text{-}1}} \tag{16}$$

$$H_{Dk} = H_{Uk} \cosh \gamma_k l_k - Q_{Uk} Z_{Ck} \sinh \gamma_k l_k \tag{17}$$

$$Q_{Dk} = -\frac{H_{Uk}}{Z_{Ck}} \sinh \gamma_k l_k + Q_{Uk} \cosh \gamma_k l_k \tag{18}$$

The last step (N) includes the creation of further symbolic variables and final substitutions:

$$\mathrm{H_{UN}}=\mathrm{H_{DN\text{-}1}} \tag{19}$$

$$\mathrm{Q_{UN}} = \mathrm{Q_{DN\text{-}1}} \tag{20}$$

$$Q_{DN} = -\frac{H_{UN}}{Z_{CN}} \sinh \gamma_N l_N + Q_{UN} \cosh \gamma_N l_N \tag{21}$$

In this manner, the expression Q_{DN} is obtained, containing one variable. Using *matlabFunction(),* the expression is converted into a handle, which is then used in function *fzero()* with the parameter *start_fzero*. The function *fzero()* finds the closest zero of a function relative to the starting point *start_fzero*.

After determining the zero of a function, the additional parameter *C* returned by the function as well as the determined value of *Omega* is calculated.

The flowchart of the resolving algorithm is shown in Fig. 2.

Example of calling the function:

```
c=[390 390 390 390];
L=[30 30 30 30];
A=[ 0.063*0.063*pi 0.050*0.050*pi 0.032*0.032*pi 0.020*0.020*pi];
[omega, C]=pipes(c,L,A,1,pipes4.m').
```

4 MAXIMUM PRESSURE INCREASE ESTIMATION IN WATER HAMMER

However not only equivalent celerity is important for designers, engineers or users of the pipe network, but first of all possible maximum pressure increase caused by water hammer. The pressure increase can be easily calculated by using equivalent celerity.

Joukowski equation can be use for pressure increase calculation [5]:

$$\Delta P = \rho\, c_e\, \Delta V \tag{22}$$

Dimensions: F=Force, L=Length, M=Mass, T=Time

c_e = equivalent celerity [L/T].

ΔP = Maximum pipe pressure increase in water hammer phenomenon [F/L^2].

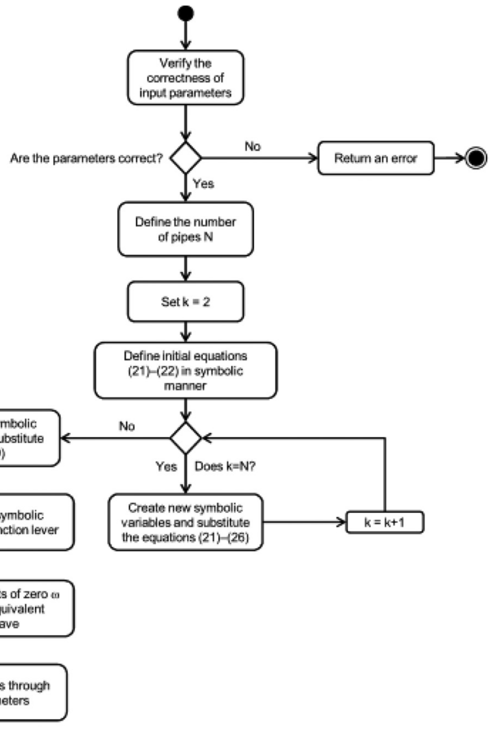


Figure 2: Algorithm flowchart.

ΔV = Change in velocity in water hammer [L/T].
ρ = Fluid density [M/L^3].

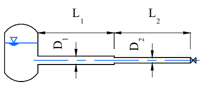
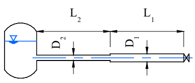
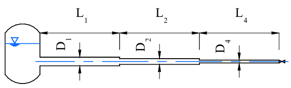
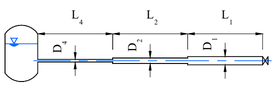
To confirm validity of numerical calculations of equivalent celerity, compliance of calculated pressure increases series of measurements was done. In the article few schemes was shown (Table 1).

The measurements was conducted for straight pipes of various lengths, diameters, and material configurations. Each time, the pipes were fixed to the base in a way that prevented any displacement.

The measurements was conducted in accordance with the following assumptions:

1. Measurements and analysis has been concerned a simple water hammer
2. The experience was performed at 281°K
3. The pressure characteristics were measured in four measuring sections on the pipeline: at the downstream near the ball valve, at the change of the cross-sections area of the connected in series pipes; as a control point, pressure was also measured in the middle of the pipeline just behind the tank
4. Pressure was measured by using a system consisting of a strain gauge set, extensometers amplifier (ZEP-101) and a computer with AD/DA.
5. Pressure values were measured with a time step of 5μs
6. Pressure gauges have linear operating characteristics with a correlation coefficient of more than 0.999

Table 1: Schema examples (*S – steel pipes; P – PVC)*

<table>
<tr><th>Schemas</th><th>Seria</th><th>L1 [m]</th><th>L2 [m]</th><th colspan="2">D1 [mm]</th><th>D2 [mm]</th></tr>
<tr><td rowspan="3"></td><td>S1S2</td><td>26.45</td><td>21.20</td><td colspan="2">42.0</td><td>35.0</td></tr>
<tr><td>P1P2</td><td>24.00</td><td>24.00</td><td colspan="2">40.8</td><td>32.6</td></tr>
<tr><td>P1P4</td><td>24.25</td><td>25.00</td><td colspan="2">40.8</td><td>20.4</td></tr>
<tr><td rowspan="3"></td><td>S2S1</td><td>26.45</td><td>21.20</td><td colspan="2">35.0</td><td>42.0</td></tr>
<tr><td>P2P1</td><td>24.00</td><td>24.00</td><td colspan="2">40.8</td><td>32.6</td></tr>
<tr><td>P4P1</td><td>24.25</td><td>25.00</td><td colspan="2">40.8</td><td>20.4</td></tr>
<tr><td></td><td>P1P2P3</td><td colspan="2" rowspan="2">All pipes have the same length L = 12.00 m</td><td>D1 [mm]</td><td>D2 [mm]</td><td>D4 [mm]</td></tr>
<tr><td></td><td>P3P2P1</td><td>42</td><td>35</td><td>21</td></tr>
</table>

7. Measurement range of gauges 1.2 MPa and 2 MPa
8. Steady motion parameters before the water hammer appear were also measured
9. The initial pressure was chosen to prevent the occurrence of cavitations during a water hammer
10. Each measurement had calculated measurement error

As a result of the experiments, the characteristics of the pressure change caused by water hammer wave propagation were obtained and recorded. Only two example characteristics for two configuration of the same pipes are shown below (Figs. 3 & 4).

On the presented charts, the following symbols were adopted:

Δ p - first pressure increase caused by water hammer;
pż - pressure increase calculated based on Joukowski formula;
Δ pmax - maximum pressure increase observed during water hammer phenomenon.

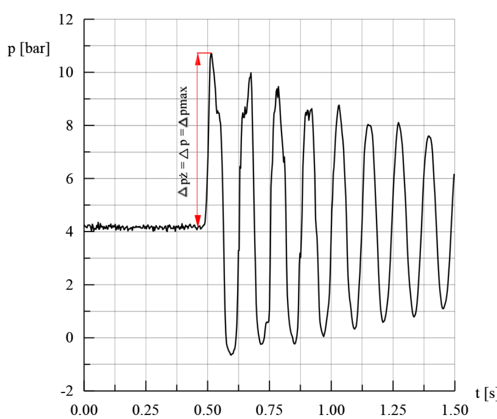


Figure 3: Pressure characteristics for three measuring sections for a steel pipe S1S2.

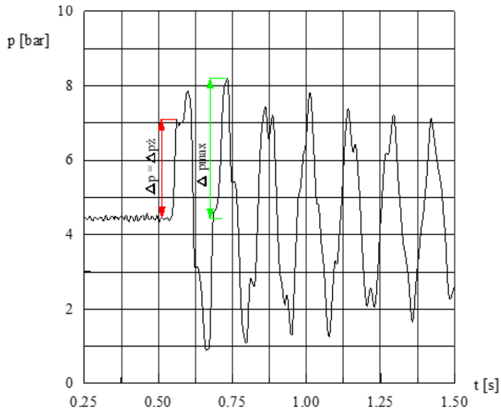


Figure 4: Pressure characteristics for three measuring sections for a steel pipe S2S1.

For the investigated schemes, the equivalent celerity c_e was calculated using the MATLAB algorithm and then the pressure increase was calculated according to the eqn. (22). Calculated pressure values were compared with values obtained with the real characteristics. In addition, the maximum recorded pressure increase was read from the real characteristics. The results are summarized in Table 2.

As can be seen, a satisfactory correspondence between the values of Δp calculated from the eqn (22) by using the equivalent celerity and the pressure increase obtained from the real pressure characteristic (Figs. 3 & 4). Noteworthy is the case, for which the diameters are set in ascending order. The calculated value Δp is equal to the measured value but this is not the maximum recorded pressure increase (Fig. 4).

The use of the algorithm for the real water supply system in the capital city Warsaw Poland is presented below (Fig. 5). Calculations were made for one case.

Calculations for real pipeline, Warsaw, Poland.

Table 2: Summary of calculated and measured parameters

Series	c_e [m/s]	t_Z [s]	V_o [m/s]	Δp [bar]	Δp_{obl} [bar]	Δp_{max} [bar]
S1S2	1320	0.020	0.520	6.80	6.86	6.80
P1P2	435	0.034	1.160	5.06	5.04	5.06
P1P4	523	0.031	1.390	7.25	7.27	7.25
S2S1	1155	0.021	0.203	2.27	2.34	3.94
P2P1	350	0.033	0.950	3.30	3.32	4.35
P4P1	225	0.035	0.810	1.80	1.83	5.60
P1P2P3	458	0.028	0.917	4.20	4.19	4.20
P3P2P1	288	0.033	1.200	3.50	3.47	6.50

where: c_e – equivalent celerity; t_z – closing time of the ball valve; V_o – average velocity for steady flow; Δp – pressure obtained from the real characteristics; Δp_{obl} – Pressure calculated for the equivalent celerity from the eqn (22); Δp_{max} – maximum pressure obtained from the real characteristics.

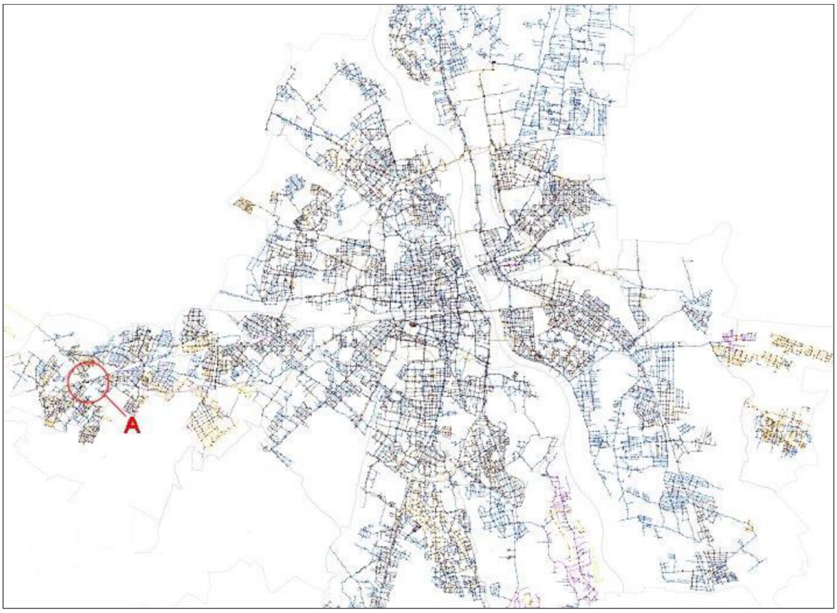


Figure 5: Warsaw water distribution network.

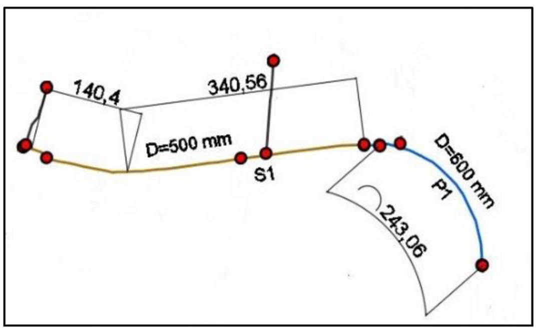


Figure 6: Detail A.

Table 3: Input pipe parameters A.

Pipe symbols	D_0 [mm]	Theoretic c [m/s]	L [m]
P1	600	390	243.06
S1	500	1280	480.96

Results for configuration S1P1: ω = 2.1667, C = 998.670 m/s, Q = 620 m^3/h, Δ p = 8.763 bar.

The case (Detail A, Fig. 6) shows a pipe system with two different materials (PVC, Steel) and two different diameters, whose calculation are shown in the Table 3.

5 SUMMARY

This paper shows an algorithm in MATLAB allows for an easy calculation of the equivalent celerity of the water hammer wave for any configuration of the series of connected pipes: diameter, length, material. The natural vibration analysis equations was used in MATLAB algorithm for a quick estimation of the equivalent celerity for a piping system made of different materials. The equations for manual calculation presented in this article include two and three pipe systems connected in series. For the calculations of any number of serially connected pipes of varying diameter, length, and material composition, a MATLAB algorithm has been presented. To calculate the equivalent celerity in any mixed system of serially connected pipes, it is enough to know the individual celerity for a given material, diameter, length and alignment sequence of the connected pipes. The accuracy of the obtained computations falls within common accuracy limits of engineering computation. Using the analysis of natural vibrations, one can create in an analogous manner an algorithm for a series of connected pipes with disbursements along the way. Using the algorithm will speed up the design process and facilitate the supervision of safety control water supply systems, such as those for fire prevention purposes (and far more).

Calculations of maximum pressure increase for decreasing diameter configurations give satisfactory compliance of numerical values and measured from real pressure characteristics.

For other diameter configurations, compatibility is also satisfactory, but this is not the maximum increase recorded during model measurements [4]. Further research should be made to find an algorithm that calculates the maximum pressure increase.

REFERENCES

[1] Shamloo, H. & Mousavifard, M., Numerical simulation of turbulent pipe flow for water hammer. *ASME Journal of Fluids Engineering*, **137**(11), 2015. https://doi.org/10.1115/1.4030806

[2] Libraga, J., Ribeiro, R., Covas, D. & Ramos, H., *Experimental and Numerical Analysis of Water Hammer in a Metal Pipe Rig*. 11th International Conference on Computing and Control for the Water Industry, Exeter, UK, 2011.

[3] Bruce, S. & David, S., *Water Hammer: Practical Solutions*, Butterworth-Heinemann, London, 1995.

[4] Meniconi, S., Brunone, B. & Ferrante, M., Water-hammer pressure waves interaction at cross-section changes in series in viscoelastic pipes. *Journal of Fluids and Structures,* **33**, pp. 44–58, 2012. https://doi.org/10.1016/j.jfluidstructs.2012.05.007

[5] Wylie, E.B. & Streeter, V.L., *Fluid Transient in Systems*, Prential Hall. Inc., Englewood Cliffs, New York, 1993.

[6] Puntorieri, P., Barbaro, G., Martins, N., Covas, D. & Vincenzo, F., *Hydraulic Transient Experimental Study in a Copper Pipe*. 9th International Conference on Computational & Experimental Methods in Multiphase & Complex Flow, (In Press), 2017.

[7] Malesińska, A., Experimental study of water hammer-induced forces and deformations in dry pipe fire protections sytems. *Fire Safety Journal,* **72**, pp. 16–24, 2015. https://doi.org/10.1016/j.firesaf.2015.02.014

[8] Wylie, E.B. & Streeter, V.L., *Fluid Transient In Systems.* Prential Hall. Inc., Englewood Cliffs, New York, 1993.