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SCATTERING OF WATER WAVES BY A POROUS CIRCULAR ARC-SHAPED BARRIER SUBMERGED IN OCEAN

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ABSTRACT
In this paper, we study the problem of scattering of surface water waves by a thin circular arc shaped porous plate submerged in the deep ocean. The problem is formulated in terms of a hypersingular integral equation of the second kind in terms of an unknown function representing the difference of potential function across the curved barrier. The hypersingular integral equation is then solved by a collocation method after expanding the unknown function in terms of Chebyshev polynomials of the second kind. Using the solution of the hyper-singular integral equation, the reflection coefficient, transmission coefficient and energy dissipation coefficient are computed and depicted graphically against the wave number. Known results for the rigid curved barrier are recovered. It is observed that the porosity of the barrier reduces the reflection and transmission of the waves and enhances the dissipation of wave energy. The reflection coefficient and dissipation of wave energy decreases as the length of the porous curved barrier increases. Also the reflection coefficient is almost independent of the inertial force coefficient of the material of the porous barrier. However, the inertial force coefficient of the material of the porous barrier enhances transmission and reduces dissipation of wave energy.
Keywords: curved porous plate, dissipation of wave energy, reflection coefficient, transmission coefficient, water wave scattering.

1 INTRODUCTION

Interaction of water waves with thin plate assuming the linear theory has been a subject of considerable interest as this phenomenon serves as a model for a wide range of physical situations which include wave interaction with breakwaters, very large floating structures. Breakwaters are coastal structures which are widely constructed to attenuate the wave action in inshore water and thereby reduce the coastal erosion and provide safe harbourage. Usually these structures are rigid structures, which extend up to the full depth of ocean. However, these fixed structures are expensive and difficult to construct, particularly when the ocean is very deep. An useful alternative is to construct floating breakwaters.

Breakwaters are mathematically modeled as rigid impermeable thin vertical plate either partially immersed or submerged in the ocean. A study of interaction of waves with a thin, rigid vertical plate dates back mid twentieth century. A number of mathematical concepts have evolved to handle the boundary value problem associated with the study of water wave scattering by a thin rigid vertical plate present in ocean with free surface. In this context the works of Dean [1], Ursell [2] and Evans [3] may be mentioned. It may be noted here that exact solution of the aforesaid boundary value problem exists when the barrier is in the form of a rigid vertical plate present in the deep ocean and for normal incidence of the incoming wave train. In all other cases only approximate analytical or numerical methods are used to obtain approximate solution.

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Porous coastal structures like rubble mound breakwaters are important in coastal engineering as the structural voids in the porous breakwaters can dissipate wave energy efficiently. Mathematical modeling of porous structure as thin, porous vertical wave maker was pioneered by Chwang [4] in 1983, although Solitt and Cross [5] studied the problem of wave propagation through porous media in 1972. Scattering problems involving porous breakwater were studied by many researchers using various sophisticated mathematical techniques. Among them the works of Yu [6], Mclver [7], Evans and Peter [8], Tsai and Young [9] may be mentioned. Recently Gayen and Mandal [10] used second kind hypersingular integral equation formulation to study the problem of wave scattering by a submerged porous plate in ocean with free surface. From these works, it is observed that the porosity of the barrier induces energy dissipation due to which the amplitude of waves reflected or transmitted are reduced.

The problem of water waves scattering by a curved rigid thin plate form of an arc of a circle submerged in the deep ocean was studied by Parsons and Martin in [11] by using a first kind hypersingular integral equation formulation based on judicious application of Green's Integral theorem. They have shown that as the length of the curved barrier increases, the reflection coefficient decreases and for almost circular barrier, reflection coefficient becomes very small.

In the present paper, we have considered the problem of scattering of water waves by a thin curved porous barrier in the form of an arc of a circle submerged in the deep ocean. Following Parsons and Martin [11] we use the Green's Integral theorem to reduce the corresponding boundary value problem to the second kind hypersingular integral equation. This hypersingular integral equation is solved using a collocation method by approximating the unknown function by Chebyshev polynomials. The reflection and transmission coefficients and amount of energy dissipation are evaluated and depicted graphically. Results for rigid curved barrier are recovered by making porosity parameter equal to zero. It is observed from the numerical results that porous curved barrier enhances the dissipation of wave energy and consequently the reflection and transmission of wave energy are much less than that of a rigid curved barrier. This shows a porous curved barrier serves as a better model for a breakwater than for a rigid curved barrier. Also, it is observed that transmission coefficient increases while reflection coefficient and energy dissipation decreases as the arc length of porous circular curved barrier increases. Also the inertial force coefficient of porous material of the barrier does not have a significant effect on reflection coefficient, but for a porous curved barrier with non zero inertial force coefficient the transmission coefficient of waves is enhanced and the dissipation of wave energy is reduced.

2 FORMULATION OF THE PROBLEM

We consider two dimensional, time harmonic and irrotational motion in water, which is assumed to be incompressible inviscid homogeneous fluid, due to scattering of a time harmonic incident wave on a porous curved barrier Γ submerged in the deep ocean. We choose a rectangular Cartesian coordinate system in which y axis is vertically downwards in to fluid region $y \geq 0$ and the plane $y = 0$ denotes the position of the undisturbed free surface. The geometry of the curve plate Γ is given in Fig. 1, where d is the depth of the mid-point of Γ from the free surface and the length of the plate is $2a = 2b\theta$. From Fig. 1, we see that any point $q \equiv (\xi, \eta)$ on Γ is given by

$$\xi(t) = b\sin t\theta, \eta(t) = d + b - b\cos t\theta, -1 \leq t \leq 1 \text{ and } -\pi < \theta \leq \pi \tag{1}$$

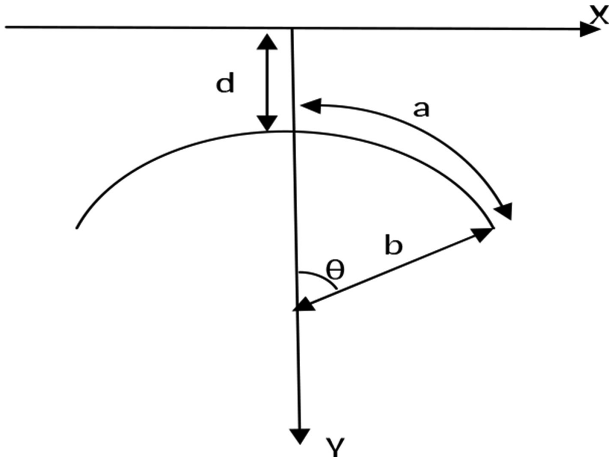

Figure 1: Geometry of the porous curved plate.

Assuming linear theory, the time harmonic train of waves represented by the potential function $Re\{\phi^{inc}(x, y)\exp(-i\sigma t)\}$ is incident on the barrier from the direction of $x = -\infty$, where σ is the circular frequency and $\phi^{inc}(x, y)$ is given by

$$\phi^{inc}(x, y) = \exp(-Ky + iKx), \tag{2}$$

where $K = \frac{\sigma^2}{g}$, g is the acceleration due to gravity. The resulting motion in the water is described by the velocity potential function $Re\{\phi(x, y)\exp(-i\sigma t)\}$, where $\phi(x, y)$ satisfies

$$\nabla^2\phi = 0 \text{ in the water region,} \tag{3}$$

the free surface condition

$$K\phi + \phi_y = 0 \text{ on } y = 0 \tag{4a}$$

bottom condition

$$\nabla\phi \to 0 \text{ as } y = \infty \tag{4b}$$

The boundary condition of the porous curved plate surface Γ is given by

$$\frac{\partial\phi_1}{\partial n} = \frac{\partial\phi_2}{\partial n} = -iK\beta[\phi] \text{ on } \Gamma \tag{5}$$

Here $[\phi] = \phi_2(x, y) - \phi_1(x, y)((x, y) \in \Gamma)$ is the difference of potential functions across the curved barrier Γ where the potential functions $\phi_2(x, y)$ is in the region $x^2 + (y - b - d)^2 > b^2$ and $\phi_1(x, y)$ is in the region $x^2 + (y - b - d)^2 < b^2$, $((x, y)\varepsilon\Gamma)$ and $\frac{\partial}{\partial n}$ denotes the normal derivative at a point on Γ.

Also $\beta(= \beta_r + i\beta_i)$ is the porous-effect parameter (cf [10, 12]) given by

$$\beta = \frac{\gamma(f + iS)}{K\tau(f^2 + S^2)} \tag{6}$$

Here γ is the porosity, f is the resistance force coefficient, S is the inertial force coefficient and τ is the thickness of the porous medium. If the resistance is predominant in the porous medium, ie, $S << f$, then β is purely real. In this case, the barrier is considered to be densely packed. However, both resistance and inertial force coefficient of the porous material are equally important [cf [12]). The condition at infinity described by

$$\phi(x,y) \to \begin{cases} \phi^{inc}(x,y) + R\phi^{inc}(-x,y) \text{ as } x \to -\infty \\ T\phi^{inc}(x,y) \qquad\qquad\qquad \text{ as } x \to \infty, \end{cases} \tag{7}$$

where R and T be the complex reflection and transmission coefficients respectively. In the next section we proceed to determine R and T.

3 METHOD OF SOLUTION

Let $Re\left\{G(x,y;\xi,\eta)e^{-\iota\sigma t}\right\}$ denotes the potential function due to the presence of a line source at $(\xi,\eta)(\eta > 0)$. Then $G(x,y;\xi,\eta)$ satisfies the following boundary value problem

$$\nabla^2 G = 0 \text{ except at } (\xi, n),$$

$$KG + G_y = 0 \text{ on } y = 0,$$

$$G \sim \log \text{as} = \left((x-\xi)^2 + (y-\eta)^2\right)^{\frac{1}{2}} \to 0,$$

$$\nabla G \to 0 \text{ as } y \to \infty$$

G behaves as outgoing waves as. $|x-\xi| \to \infty$. The expression for G is given by (cf [10])

$$G(x,y,\xi,\eta) = \log\left(\frac{r}{r'}\right) - \int_0^\infty \frac{\exp(-k(y+\eta))}{(k-K)} \cos(k(x-\xi))\,dk \tag{8}$$

where $r, r' = \left((\text{x-}\xi)^2 + (y \mp \eta)^2\right)^{\frac{1}{2}}$ and denotes the integral along the positive real axis in the complex k-plane indented below the pole at k = K. Applying Green's theorem to the functions $\phi(P) - \phi^{inc}(P)$ and $G(x,y;\xi,\eta)(\equiv G(P,Q))$ suitably, we have the integral representation of $\phi(Q)$ as

$$\phi(Q) = \exp(-K\eta + iK\xi) - \frac{1}{2\pi}\int_\Gamma [\phi](p)\frac{\partial}{\partial n_p} G(p,Q)\,ds_p \tag{9}$$

where $P(x,y), Q(\xi,\eta)$ be points in fluid and p, q denote points on Γ and $\dfrac{\partial}{\partial n_p}$ denote the normal derivative at p on Γ. Using the boundary condition (5), we have the following hypersingular integral equation for the unknown discontinuity in potential across Γ, $[\phi]$ as

$$\frac{1}{2\pi}\int_\Gamma [\phi]\frac{\partial^2}{\partial n_q \partial n_p} G(p,q)\,ds_p - iK\beta[\phi](\eta) = \frac{\partial \phi^{inc}}{\partial n_q}(\xi,\eta), q \in \Gamma, \tag{10}$$

Now for curved plate Γ, the length of the plate is $2a = 2b\theta$ and any point $q(\xi,\eta), p(x,y)$ on Γ are given by $\xi(t) = b\sin t\theta,\ \eta(t) = d+b-b\cos t\theta, x(s) = b\sin s\theta, y(s) = d+b-b\cos s\theta,$ where $-1 \le t,s \le 1$ and $-\pi < \theta < \pi$. The unit normal at $p,q \in \Gamma$ are given by, $n(p) \equiv (n_1^p, n_2^p) \equiv (-\sin s\theta, \cos s\theta), n(q) \equiv (n_1^q, n_2^q) \equiv (-\sin t\theta, \cos t\theta)$.

In eqn (10) we substitute $X = x - \xi = b(\sin s\theta - \sin t\theta)$, $Y = y + \eta = 2d + 2b - b(\cos s\theta + \cos t\theta)$, $y - \eta = -b(\cos s\theta + \cos t\theta)$and

$$[\phi](p) = f(s) \tag{11}$$

to obtain

$$\int_{-1}^{1} \frac{f(s)}{(s-t)^2} ds + \int_{-1}^{1} f(s) L(t,s) ds + 2\pi b\theta i K\beta f(t) = 2\pi b\theta K e^{-K\eta + i(K\xi + t\theta)} \tag{12}$$

where

$$\begin{aligned} L(t,s) = {} & \frac{\theta^2}{4}\left\{ \frac{1}{\sin^2 \frac{(s-t)\theta}{2}} - \frac{4}{\theta^2 (s-t)^2} \right\} \\ & + b^2\theta^2 \cos(s-t)\theta \left\{ \frac{Y^2 - X^2}{(X^2+Y^2)} + \frac{2KY}{X^2+Y^2} + 2K^2 \Phi_0(X,Y) \right\} \\ & + 2b^2\theta^2 \sin(s-t)\theta \left\{ K\frac{\partial \Phi_0}{\partial X} - \frac{XY}{(X^2+Y^2)^2} \right\}, \end{aligned} \tag{13}$$

$$\Phi_0(X,Y) = i\pi e^{-KY + iK|X|} + \int_0^\infty \frac{e^{-k|X|}}{k^2+K^2} [k\cos kY - K\sin kY] dk,$$

with $f(\pm) = 0$. The hypersingular integral in eqn (12) is in the sense of Hadamard finite part integral. Now to solve the second kind hypersinguler integral eqn (12) we approximate f (s) as (cf [11])

$$f(s) \approx \sqrt{1-s^2} \sum_{n=0}^{N} a_n U_n(s) \tag{14}$$

where U_n(s) are the Chebyshev polynomial of the second kind and a_n (n = 0, 1, 2, ...N) are the unknown constant to be determined.

Now using the relation (14) in eqn (12) we obtain the following system of linear equations in a_n

$$\sum_{n=0}^{N} a_n C_n(t) = \upsilon(t). \tag{15}$$

where $\upsilon(t) = 2\pi Kb\theta e^{-K\eta+i(k\xi+t\theta)}$

$$C_n(t) = \left(-(n+1) + 2i\beta\theta Kb\sqrt{1-t^2}\right)\pi U_n(t) + \int_{s=-1}^{1}\sqrt{1-s^2}U_n(s)L(t,s)ds \tag{16}$$

Now substituting $t = t_j$ $(j = 0,1,2, ..., N)$ in (15), we obtain (N + 1) system of linear equation with $(N + 1)$ unknown a_0, ai, a_2, ..., a_N as

$$\sum_{n=0}^{N} a_n C_n(t_j) = \upsilon(t_j) \text{ for } j = 0,1,2,...,N \tag{18}$$

where t_j's are the collocation points chosen as

$$t_j = \cos\frac{2j+1}{2N+2}\pi, \quad j = 0,1,2,...,N \tag{19}$$

The linear system (18) is solved by standard method to obtain the constants $a_0, a_i, a_2, ..., a_N$ and hence have the approximate solution of the hypersin-gular integral eqn (12)

3.1 Reflection and transmission coefficients:

The reflection and transmission coefficients R and T can be obtained approximately in terms of a series involving the constants a_n $(n = 0,1,2,\ldots,N)$ defined in (18). This is achieved by making $\xi \to \pm\infty$ in $\phi(\xi,\eta)$ given in (9) and using the condition (7), (11) and (14) we can write R, T as

$$R = -iKb\theta\sum_{n=0}^{N} a_n \int_{t=-1}^{1}\sqrt{1-t^2}U_n(t)e^{-K\eta(t)+i(K\xi(t)t\theta)}dt \tag{20}$$

$$T = 1 - iKb\theta\sum_{n=0}^{N} a_n \int_{t=-1}^{1}\sqrt{1-t^2}U_n(t)e^{-K\eta(t)-i(K\xi(t)t\theta)}dt \tag{21}$$

Thus R and T can be obtained once a_n are known after solving the system of linear eqn (18). For impermeable $(\beta = 0)$ plate $|R|$ and $|T|$ must satisfy the identity $|R|^2 + |T|^2 = 1$.

3.2 The energy identity:

Porosity of the plate Γ causes the dissipation of wave energy, so in this case $|R|^2 + |T|^2 \leq 1$. To prove it mathematically, we use the Green's integral theorem to the functions ϕ and $\bar{\phi}$ in the region bounded by the lines $y = 0, -X \leq x \leq X; x = X, 0 \leq y \leq Y; y = Y, -X \leq x \leq X;$ $x = -X, 0 \leq y \leq Y$ and a contour enclosing the plate Γ. Finally making $X, Y \to \infty$, we obtained the energy identity as

$$|R|^2 + |T|^2 = 1 - 2K\beta_r b\theta\int_{\Gamma}\left|[\phi]\right|^2 dt \tag{22}$$

where β_r = Re part of β. After using (11), (14) and (22) we have

$$|R|^2 + |T|^2 = 1 - J. \tag{23}$$

Here J is the amount of dissipation of wave energy which is given by

$$J = 2K\beta_r b\theta\int_{t=-1}^{1}\left|\sqrt{1-t^2}\sum_{n=0}^{N} a_n U_n(t)\right|^2 dt \tag{24}$$

and J is positive so that $|R|^2 + |T|^2 < 1$. If $\beta = 0$ (i.e. for impermeable plate) then $J = 0$ and the energy identity in this case is $|R|^2 + |T|^2 = 1$.

4 NUMERICAL RESULTS

The reflection coefficient $|R|$, the transmission coefficient $|T|$ and the amount of wave energy dissipated J can be computed numerically from eqns (20), (21) and (24) respectively, once a_n is known by solving the system of linear eqn (18). For numerical computation the value of N in (18) is chosen N = 15.

In Figs 2–4, $|R|, |T|$ are plotted against Ka for $\theta \in \left\{\frac{\pi}{10}, \frac{5\pi}{10}, \frac{8\pi}{10}\right\}$ and for various values of porosity parameter $\beta \in \left\{0, 1, 1+\frac{i}{2}\right\}$ and for fixed $\frac{d}{a} = 0.1$. Here $\beta = \beta_r + i\beta_i$ where β_r is associated with resistance force coefficient while β_i is associated with the inertial force coefficient of the porous material of the curved barrier as given by eqn (6). In Fig. 5, the amount of wave energy dissipated J is plotted against Ka for various values of β and fixed $\frac{d}{a} = 0.1$. The energy identity (23) has been verified for various values of Ka, θ, β.

From Figs 2–4 the graph of $|R|$ and $|T|$ for rigid barrier$(\beta = 0)$ exactly coincides the result given by Parsons and Martin [11], for $\theta \in \left\{\frac{\pi}{10}, \frac{5\pi}{10}, \frac{8\pi}{10}\right\}$. This shows that the numerical results are fairly accurate. It is observed that for rigid curved barrier when $\theta = \frac{\pi}{10}$, $|R|$ and $|T|$ shows oscillatory behaviour. However as θ increases, the frequency of oscillation in $|R|$ and $|T|$ decreases. Also, $|R|$ decreases and $|T|$ increases as θ increases. It is also observed from eqn (24) that there is no dissipation of energy for rigid curved plate ie, $J = 0$ for $\beta = 0$.

From Fig. 2, it is observed that when $\theta = \frac{\pi}{10}$, $\beta \in \left\{1, 1+\frac{i}{2}\right\}$, $|R|$ shows a slow oscillatory behaviour for $Ka < 2.5$ and then gradually decreases almost to zero as $K a$ increases. Also $|R|$

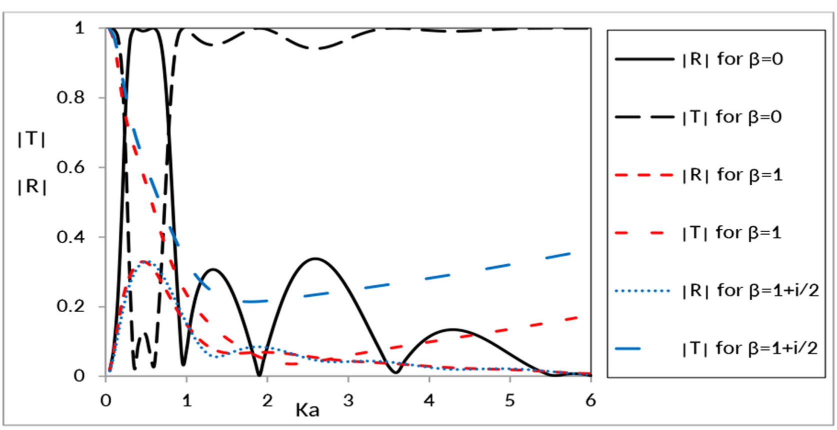

Figure 2: |R|, |T| against Ka, $\frac{d}{a} = 0.1$, for $\theta = \frac{\pi}{10}$

coincides almost for $\beta = 1$ and $\beta = 1 + \frac{i}{2}$• This shows that the inertial force coefficient of the porous material of the barrier has very little effect on the reflection coefficient |R|. Also, |R| for a porous barrier is much less than that of a rigid barrier. Similar behaviour in |R| is observed from Figs 3 and 4 for $\theta \in \left\{\frac{5\pi}{10}, \frac{8\pi}{10}\right\}$.

It is observed from Fig. 2 that $|T|$ for $\beta \in \left\{1, 1 + \frac{i}{2}\right\}$, decreases sharply for small value of Ka and then increases very slowly as Ka increases. For a particular value of wave number K a, $|T|$ for $\beta = 1 + \frac{i}{2}$ is more than |T| for $\beta = 1$. This shows that the inertial force coefficient of the porous material of the barrier increases the transmission coefficient. Also, |T| for a porous barrier is much less than that of a rigid barrier. Similar behaviour in $|T|$ is observed from Figs 3 and 4 for $\theta \in \left\{\frac{5\pi}{10}, \frac{8\pi}{10}\right\}$.

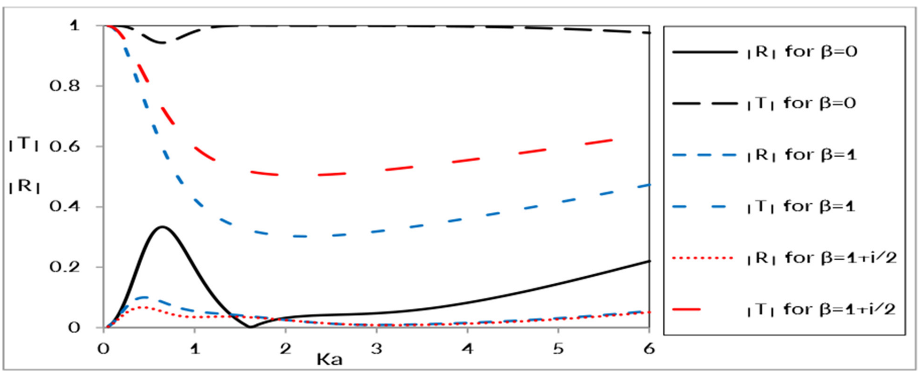

Figure 3: |R|, |T| against Ka, $\frac{d}{a} = 0.1$, for $\theta = \frac{\pi}{2}$

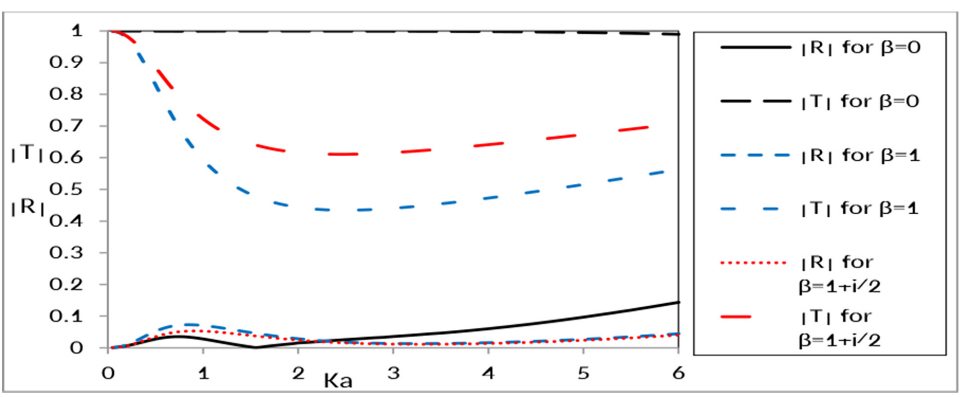

Figure 4: | R|, | T| against Ka, $\frac{d}{a} = 0.1$, for $\theta = \frac{8\pi}{10}$

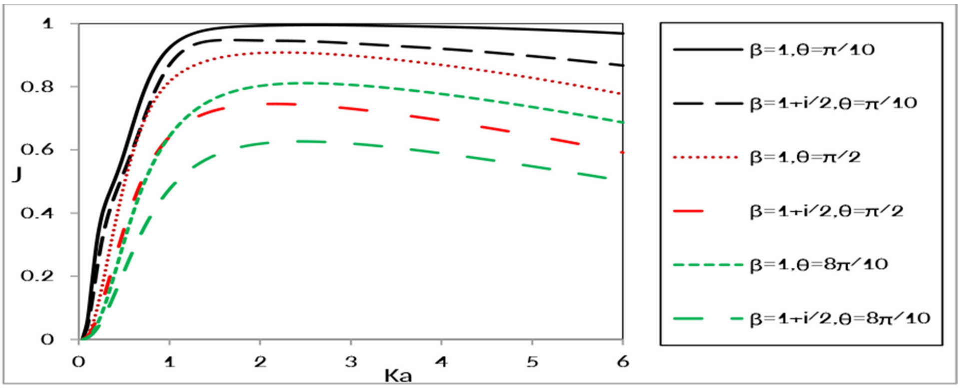

Figure 5: J against *Ka,* where $\frac{d}{a} = 0.1$, for various θ and β

From Fig. 5 it is observed that for $\theta = \frac{\pi}{10}$, J is less for $\beta = 1 + \frac{i}{2}$ than for $\beta = 1$ for a particular value of Ka. This shows that the inertial force coefficient of the porous material of the curved barrier reduces the dissipation of wave energy. Similar behaviour is observed in J for $\theta \in \left\{\frac{5\pi}{10}, \frac{8\pi}{10}\right\}$ and different value of β. Thus, it is observed that the inertial force coefficient of the porous material of the curved barrier has no significant effect on the reflection of wave energy, but it reduces the dissipation of wave energy and enhances the transmission of wave energy.

It is also observed for Figs 2–5 that $|R|$ and J decreases while $|T|$ increases as θ increases from $\frac{\pi}{10}$ to $\frac{8\pi}{10}$. This shows that as the length of the curved barrier increases, the transmission of wave energy increases while the reflection of wave energy and the dissipation of wave energy decreases. Thus, it is observed that there occurs dissipation of wave energy for porous barrier, which is in contrast with a rigid barrier where there is no dissipation of energy. Due to this reason the reflection and transmission coefficients of surface waves by a porous barrier is much less than that of a rigid barrier. Moreover the reflection coefficient decreases and transmission coefficient increases as θ increases, i.e. the length of the curved barrier increases. Also, it is observed that the dissipation of energy decreases as θ increases.

The reflection coefficient is almost independent of the inertial force coefficient of the material of the porous barrier while the inertial force coefficient of the material of the porous barrier reduces the dissipation of wave energy and increases the transmission coefficient.

5 CONCLUSION

In the present paper, the problem of scattering of water waves by a porous curved barrier is studied by using a second kind hypersingular integral equation based on judicious application of Green's integral theorem. The reflection coefficient, transmission coefficient and energy dissipation coefficient are evaluated and studied graphically. It is observed that (i) The porosity of the curved barrier enhances the dissipation of wave energy and thereby reduces the reflection and transmission of wave energy. So the porous curved barrier serves as the best model for a breakwater than a rigid barrier. (ii) An increase in the arc length of the porous barrier, increases the transmission of wave energy and decreases the reflection and dissipation of wave energy. (iii) The inertial force coefficient of the porous material of the barrier has no significant effect on reflection coefficient. But it enhances the transmission coefficient and reduces the dissipation of wave energy.

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