ROBUST OPTIMIZATION OF FACILITY LOCATION MODELS AND FUNDAMENTAL RESOURCE ESTIMATIONS UNDER DEMAND UNCERTAINTY: A CASE STUDY OF RELIEF DISTRIBUTION

R. KASEMSRI, K. SANO, H. NISHIUCHI & A. JAYASINGHE
Department of Civil and Environmental Engineering, Nagaoka University of Technology, Japan.

ABSTRACT

Humanitarian logistics are recognized as significant issues of natural disaster operations and management. This study considers the vital item distribution network models to relieve the large number of surviving victims under their uncertainty by the reason that the post-disaster undergoes fluctuation of demand and imprecise prediction. The purpose of this study is to handle this demand uncertainty with the facility location model and to compare their sensitivity with the deterministic model. The expected results are to explore the location of facilities and optimize transportation link flows in order to minimize total delivery cost, which includes travel, facility and transhipment costs. We propose three distinct network models based on their hierarchy structures and truck sizes to determine the most efficient model with high robustness for both deterministic demand and uncertainty demand. We determine a single hierarchy and double hierarchies of the facility sites; each hierarchy is then distributed by the distinct truck sizes. The two hierarchies with the large truck's delivery offered preferable objectives; they are robust when demand becomes uncertain or unknown. We solve the problem by the ellipsoidal uncertainty set, which is a novel approach that has never been fully applied so far to solve the facility location. We also estimate the fundamental resource requirements, including the number of trucks and total working time of drivers. Therefore, this study can help the decision maker to plan for post-disaster distribution network and their systems when demand uncertainty occurs.

Keywords: facility locations, robust optimization, uncertainty demand.

1 INTRODUCTION

Post-disaster logistics functions are defined for two significant issues: providing essentials to surviving victims and rescuing the victims. This study focuses on vital item distribution to help surviving victims. There are three sub-problems in logistics activities: location, routing and location-routing, which are realized with cost efficiency, quick response, satisfied demand and an environment issue, for example the air pollution from vehicle's exhaust and the noise pollution of vehicle in urban area. Moreover, the efficiency of planning and coordinating logistic activities is necessary to treat the problems.

The problem of location is one of the most important aspects in logistic activities. Some research has been done on the appropriate location of medical centres where the evacuees can be quickly accessed (Mete and Zabinky [1]). The research is conducted not only on the medical centres but also on the location of shelters. Lin et al. [2] focused on the improvement of logistics efficiency. They said that the prioritized items for delivery and an extensive time period are important for humanitarian logistics. They presented the location of temporary depots around the disaster-affected area between the long travel distances of demand points and the central depots. This study intends to design the depot locations by considering cost

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efficiency and also the satisfaction with the demand. Furthermore, real situations usually meet with fluctuations of parameter uncertainty. The methodology to handle this demand fluctuation is *robust optimization* (RO), which is opposite to deterministic models. Snyder [3] surveyed the facility location under uncertainty. Snyder illustrated the surveys in several articles, which were categorized by his approach to uncertainty. Those are stochastic location problems and robust location problems. However, there are no location problems solved using a robust counterpart. Therefore, this study also stresses the importance of uncertainty of parameters; here is the demand uncertainty.

There have been enormous impacts and a humanitarian crisis following the 2011 Tohoku earthquake and tsunami [4]. Japan's central bank said that the economic losses of Kobe quake in 1995 were 10 trillion yen for both immediate problems with industrial production suspended in many factories and the longer-term issue of the cost of rebuilding. However, the Japanese government and BOJ Governor Masaaki Shirakawa had estimated that this cost is much higher than the cost of just the direct material damage and could exceed 25 trillion yen. Moreover, there are several costs generated to recover the situations during disaster and post-disaster periods, for example reconstruction cost, rescue cost and logistics cost. The logistic cost was presented by Nagurney [5] as approximately 80% from the overall operation responding cost. Therefore, cost efficiency should be one of the many aspects that must be considered. For this reason, this study focuses on the logistic cost efficiency. An improved supply distribution cost can reduce the expenditure of the whole operation cost during the amelioration period. A bottle of water is considered to be a requisite item for a preliminary succour. Even the total delivery cost minimization is one to consider in humanitarian logistics; however, it is a good criterion to compare the results of distinct network systems.

This study considers robust counterpart in RO, which is provided by AIMMS software and has more recently been applied to handle under uncertainty of the parameters in the models. RO is designed to meet some major challenges associated with uncertainty-affected optimization problems as follows: to operate under lack of full information on the nature of uncertainty, to model the problem in a form that can be solved efficiently and to provide guarantees about the performance of the solutions. RO is an uncertainty modelling approach suitable for a situation where the uncertainty ranges are known and not necessarily the distribution. Typically some inputs take an uncertain value anywhere between a fixed minimum and a maximum. This demand uncertainty can show how the worst case presents itself when we consider the fluctuation of the demand, which was recommended by Holguín-Veras et al. [6, 7]; from the Tohoku experience, the disaster planners must design for worst-case scenarios from small disasters to large ones to improve future response efforts and Holguín-Veras et al. [8, 9] suggested the policy on the important and overlooked point of material convergence phenomenon. RO is suitable for our situations as only simple inputs are required from the user about the data uncertainty because there are no scenarios or distribution functions to be defined. The advantage of RO models is that their complexity grows only slightly when uncertainty is added. As the result, the model can be solved efficiently. Many fields of the academic study had discussed uncertainty parameter handling with RO approaches, for instance, the design and operations of chemical processes, an electrical capacity system, supply chain networks and transportation planning design.

2 OBJECTIVES

The case study was carried out on Miyagi prefecture, the most affected area in a severe Tohoku earthquake in 2011. The compositions of this study are summarized as major objectives of the study and expected results of the models. The major objectives are the following:

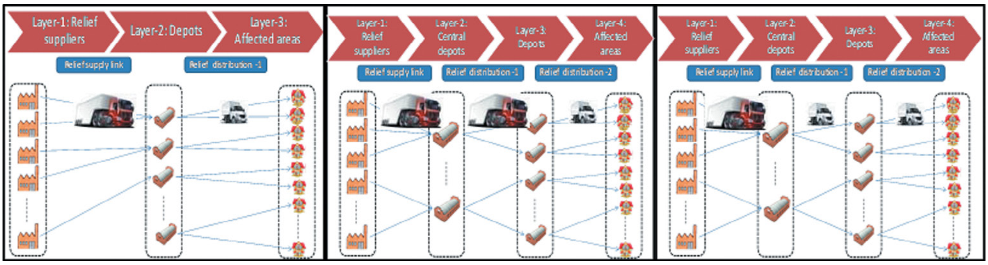


Figure 1: The three different network frameworks.

1. To propose the three different models by structure and truck sizes and then compare the total delivery cost efficiency and their sensitivity of those networks
2. To handle the facility location problem for both deterministic demand and uncertainty demand and compare their robustness

The expected results of facility location model are defined as follows:

1. To search the appropriate locations of depots to distribute the relief items in Miyagi prefectures
2. To allocate the transportation link flow at each network configuration
3. To minimize the total delivery cost which includes the transportation cost, the opening facility cost and the transhipment cost

3 MODEL STRUCTURE

The problem is designed for three different network frames. We categorize the distinct networks by the network configurations and the dispatched truck sizes. Two types of the network configurations are single hierarchy and double hierarchies of facility sites, defined as central depots and depots, respectively. Then, the problem becomes one of three network configurations with two echelons and four network configurations with three echelons. The first network element is the location where the serviceable supports are known as suppliers. The second network element is the central relief depot in case of double hierarchies. The third element is the relief depots for double hierarchies and the relief depot in case of a single hierarchy. These second and third network locations are unknown and need to be defined with the most efficiency. Finally, a possible area that was attacked by the natural disaster is called an affected area which can be defined based on known locations as demands. The transportation truck sizes are 10-ton trucks and 4-ton trucks. The configurations of the networks are illustrated in Fig. 1.

4 MATHEMATICS

Indices

M : Set of the supplier nodes (i) (i = 1, 2, 3,…, M)
N : Set of the candidate central depots (j) (j = 1, 2, 3, … N)
L : Set of the candidate depots (k) (k = 1, 2, 3, …, L)
P : Set of the demand nodes or shelters (l) (l = 1, 2, 3, …, P)
TS : Set of the truck size (s)

Notations

x_{ij}^1, x_{jk}^2, x_{kl}^3 : The flow of items from *i* to *j*, *j* to *k* and *k* to *l*

C_j^1 : The capacity at the candidate central depots *j* ($j = 1, 2, 3, \ldots, N$)

C_k^2 : The capacity at the candidate depots *k* ($k = 1, 2, 3, \ldots, L$)

Sets of parameters

S_i : The amount of items at the supply nodes *i*

D_l : The demand at the affected area nodes *l*

S_i : The amount of items at the supply nodes *i*

D_l : The demand at the affected area nodes *l*

c_{ij}^1, c_{jk}^2, c_{kl}^3 : The travel cost from *i* to *j*, *j* to *k* and *k* to *l*

f_j^1, f_k^2 : The opening depot cost at *j* and *k*

tc_j^1, tc_k^2 : The transhipment cost at *j* and *k*

$v_{ij}^1, v_{jk}^2, v_{kl}^3$: The capacity of truck from *i* to *j*, *j* to *k* and *k* to *l*

$w_{ij}^1, w_{jk}^2, w_{kl}^3$: The maximum working time of drivers from *i* to *j*, *j* to *k* and *k* to *l*

$d_{ij}^1, d_{jk}^2, d_{kl}^3$: The distance from *i* to *j*, *j* to *k* and *k* to *l*

$t_{ij}^1, t_{jk}^2, t_{kl}^3$: The travel time from *i* to *j*, *j* to *k* and *k* to *l*

R_s : The energy consumption rate of truck size *s*

S_s : The driver salary of truck size *s*

T_s : The truck cost of truck size *s*

4.1 Objective function

$$\min\left\{\sum_{j=1}^{N}\sum_{i=1}^{M}c_{ij}^1x_{ij}^1+\sum_{k=1}^{L}\sum_{j=1}^{N}c_{jk}^2x_{jk}^2+\sum_{l=1}^{P}\sum_{k=1}^{L}c_{kl}^3x_{kl}^3+\sum_{j=1}^{N}f_j^1Y_j+\sum_{k=1}^{L}f_k^2Z_k+\sum_{j=1}^{N}tc_j^1Y_j+\sum_{k=1}^{L}tc_k^2Z_k\right\} \quad (1)$$

,
when

$$c_{ij}^1=\left(R_sd_{ij}^1\right)+\left(S_st_{ij}^1\right)+T_s \quad (2)$$

$$c_{jk}^2=\left(R_sd_{jk}^2\right)+\left(S_st_{jk}^2\right)+T_s \quad (3)$$

$$c_{kl}^3=\left(R_sd_{kl}^3\right)+\left(S_st_{kl}^3\right)+T_s \quad (4)$$

Decision variables

$$Y_j = \begin{cases} 1, & \text{if central depot is located at } j \\ 0, & \text{otherwise} \end{cases} \quad \text{for } j \in N, \tag{5}$$

$$Z_k = \begin{cases} 1, & \text{if depot is located at } k \\ 0, & \text{otherwise} \end{cases} \quad \text{for } k \in M, \tag{6}$$

subject to

$$\sum_{j=1}^{N} x_{ij} \le S_i, \tag{7}$$

$$\sum_{i=1}^{M} x_{ij}^{1} \le \sum_{k=1}^{L} x_{jk}^{2} Y_j, \tag{8}$$

$$\sum_{k=1}^{L} x_{jk}^{2} \le C_j^{1} Y_j, \tag{9}$$

$$\sum_{j=1}^{N} x_{jk}^{2} \le \sum_{l=1}^{P} x_{kl}^{3} Z_k, \tag{10}$$

$$\sum_{l=1}^{P} x_{kl}^{3} \le C_k^{2} Z_k, \tag{11}$$

$$\sum_{k=1}^{L} x_{kl}^{3} \ge D_l, \tag{12}$$

$$\sum_{j=1}^{N} x_{ij}^{1} \le v_{ij}^{1}, \tag{13}$$

$$\sum_{k=1}^{L} x_{jk}^{2} \le v_{jk}^{2}, \tag{14}$$

$$\sum_{l=1}^{P} x_{kl}^{3} \le v_{kl}^{3}, \tag{15}$$

$$\sum_{j=1}^{N} t_{ij}^{1} \le w_{ij}^{1}, \tag{16}$$

$$\sum_{k=1}^{L} t_{jk}^{2} \leq w_{jk}^{2}, \tag{17}$$

$$\sum_{l=1}^{P} t_{kl}^{3} \leq w_{kl}^{3}, \tag{18}$$

$$x_{ij}, y_{jk}, z_{kl} \geq 0, \tag{19}$$

$$Y_j,\ Z_k \in \{0,1\} \quad \text{for all } j \text{ and } k. \tag{20}$$

Constraint (7) guarantees that the total amount flow from suppliers i to central depots j is not more than the amount of serving goods at suppliers i. Constraint (8) restricts for the summation of link flow from i to j not exceeding the capacity of opening the central depots j. Constraint (9) limits for the total amount of link flows from j to k not exceeding the total availability of goods at opening central depots j. Constraint (10) restricts that the summation amount of link flow from j to k must not be more than the capacity of next network configuration or depots k. Constraint (11) ensures that the total amount from depots k to demand l is not more than the availability of goods at depots k. Constraint (12) is confirmed that the total amount serving from depots k is satisfied with the demand l. Constraints (13)–(15) are determined to prohibit that the amount of a commodity cannot exceed the maximum truck volume restriction. Constraints (16)–(18) are restricted for the total driving hours of a driver which are not more than the maximum working time. Constraint (19) is confirmed that each link flow from site i to j, j to k and k to l needs to be defined with some amount of goods. Constraint (20) is generated to specify that both the decision variables Y_j and Z_k are binary variable 0 and 1; 1 is represented if the facility is located at site j and k and 0 otherwise.

4.2 Mathematical robust formulation using robust counterpart

This study focuses on the multi-source and multi-layer facility location problem with uncertainty demand by considering the ellipsoidal uncertainty set in the RO approach. Ben-Tal and Nemirovski [10] consider ellipsoidal uncertainty set with linear programming. Kouvelis and Yu [11] discussed the robust discrete optimization and its applications. They proposed an approach to find a solution that minimizes the worst case performance under a set of scenarios for the data. Bertsimas and Brown [12] proposed a methodology for constructing uncertainty sets for robust liner optimization based on decision maker risk preferences. Josef [13] gave an overview on the state-of-the-art and recent advances in mixed integer optimization to solve planning and design problems in the process industry. Stochastic programming for continuous linear programming problems is now part of most of the optimization packages, and there is encouraging progress in the field of stochastic mixed-integer linear programming (MILP) and robust MILP. Ben-Tal et al. [14] proposed a soft robust model for optimization under ambiguity. Whenever the uncertainty set of a mixed-integer robust problem is an ellipsoidal, the robust counterpart can be reformulated as a mixed-integer second-order cone programme.

This study focuses on the demand uncertainty parameter which deviates from the nominal value of the uncertain parameters. The demand uncertainty is expanded followed by the region of the ellipsoidal uncertainty set. The demand is defined as parameter D and ($\bar{D}$)

is the demand that deviates from historical or nominal values. The uncertainty demand is $\bar{D} \in R^d$, we consider the sets around the nominal value $D \in R^d$. Then, we use ρ^2 to restrict the region around the nominal value, which is equal to 1. We determine the interval range of demand $(D - \bar{D})$ as equivalent to maximum truck capacity.

Ellipsoidal:
$$U = \left(\frac{\left(D - \bar{D}\right)^t}{\delta \times \left(D - \bar{D}\right)} \right)^2 \leq \rho^2, \tag{21}$$

$$\min \left\{ \sum_{j=1}^{N} \sum_{i=1}^{M} c_{ij}^1 \overline{x_{ij}^1} + \sum_{k=1}^{L} \sum_{j=1}^{N} c_{jk}^2 \overline{x_{jk}^2} + \sum_{l=1}^{P} \sum_{k=1}^{L} c_{kl}^3 \overline{x_{kl}^3} + \sum_{j=1}^{N} f_j^1 \overline{Y_j} + \sum_{k=1}^{L} f_k^2 \overline{Z_k} + \sum_{j=1}^{N} tc_j^1 \overline{Y_j} + \sum_{k=1}^{L} tc_k^2 \overline{Z_k} \right\}, \tag{22}$$

$$\sum_{k=1}^{L} \overline{x_{kl}^3} \geq \overline{D_l} \tag{23}$$

5 RESULTS

We would like to illustrate circumstantial outcomes of both deterministic and uncertainty models. The expectation results for both circumstances are the total delivery cost of the three different network frames. As mentioned before, each network frame includes five demand scenarios; thus we prefer to report five expectation results for each. In order to identify the network efficiency by total delivery cost minimization and network robustness, we compare the total delivery cost of the three networks and indicate the best network structure. Then, we present the sensitivity analysis and compare the robustness of the three networks. Therefore, this study can help the decision maker to plan for post-disaster distribution network and their systems when the circumstance of demand uncertainty occurs.

5.1 Total delivery cost and their sensitivity

From the results, we found that the network configurations and their systems are affected with the total delivery cost of both deterministic demand and uncertainty demand as shown in Fig. 2. It can be seen that network 2 and network 3, as defined for two hierarchies of

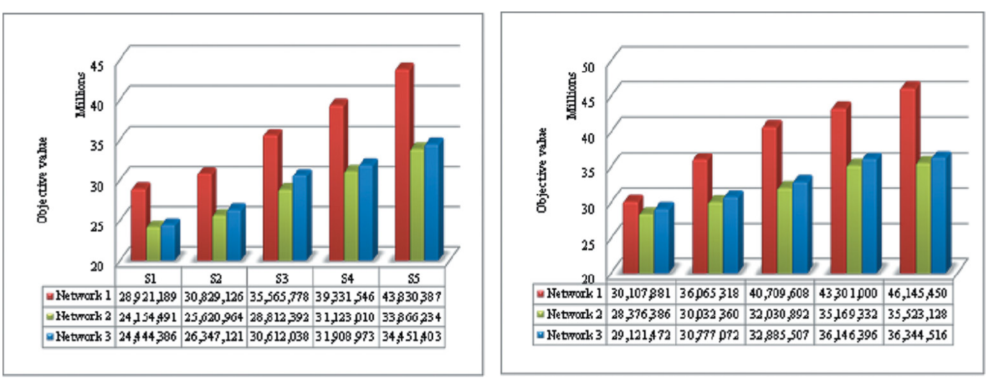


Figure 2: The total delivery cost.

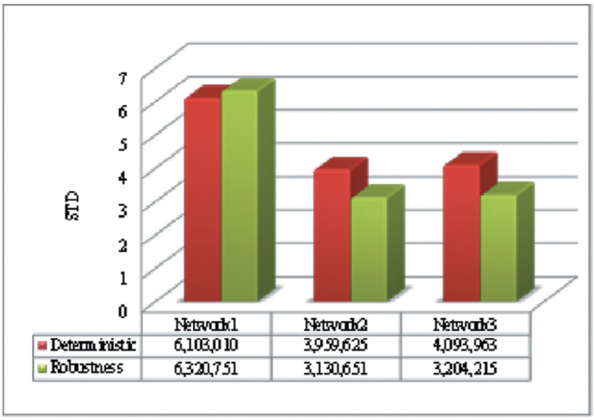


Figure 3: The standard deviation.

facility, have obviously preferable cost performance when compared with network 1 which is single hierarchy. The total delivery cost of network 2 and network 3 lessened by 17.96% and 16.78%, respectively. The total delivery cost is mostly generated by travel, which is more than about 90% and its rapid increase depends on the amount of transportation.

When comparing network 2 and network 3, all demand scenarios in network 2 can be reduced by 1.19%, 2.79%, 6.06%, 2.49% and 1.71%, respectively. These results demonstrate that not only network configurations but together with truck size, operations are significant with total delivery cost function. By using 10-ton truck to deliver from suppliers to central depots and from central depots to depots, we can have a benefit of cost reduction.

Figure 3 illustrates the standard deviation of objective function for each network. The standard deviation network 1 is higher than that of the others, which means that there are much fluctuations. The standard deviation of deterministic demand of network 1 is approximately 6 million, while there is around 4 million for network 2 and network 3.

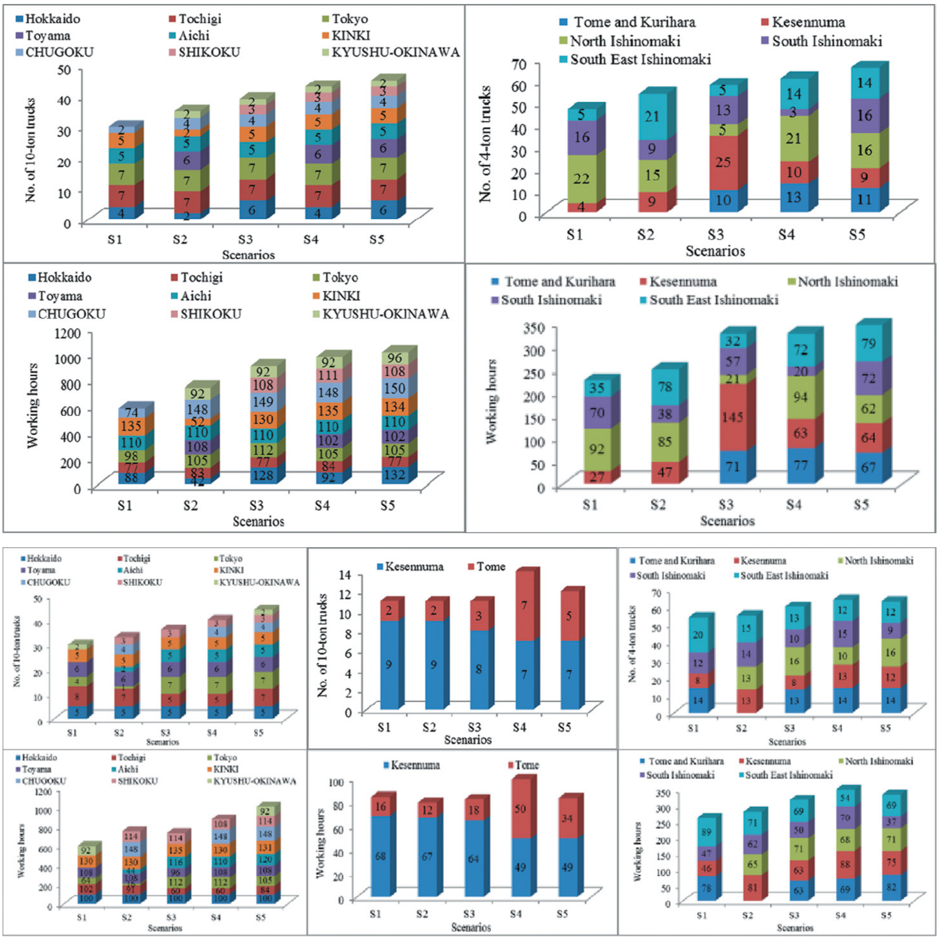
Comparing the deterministic demand and uncertainty demand, network 2 and network 3 are similar, which by using RO to handle the uncertainty demand illustrates more robustness than deterministic demand. In addition, the fluctuation between deterministic demand and uncertainty demand of network 2 is less than that of network 3, which means that network 2 is robust than the other networks.

5.2 Fundamental resource requirements

Figure 4 shows the maximum number of trucks that need to be used for relief distribution at each network configuration for uncertainty demand. From the results, we found that by dispatching a big lot size at central depots of network 2, the truck requirements can be reduced by approximately 50%. These trucks were not more than six vehicles at each central depot (echelon 2). This was an advantage on the transportation cost by reducing working time of drivers and their hiring cost. Thus, the whole system cost was reduced.

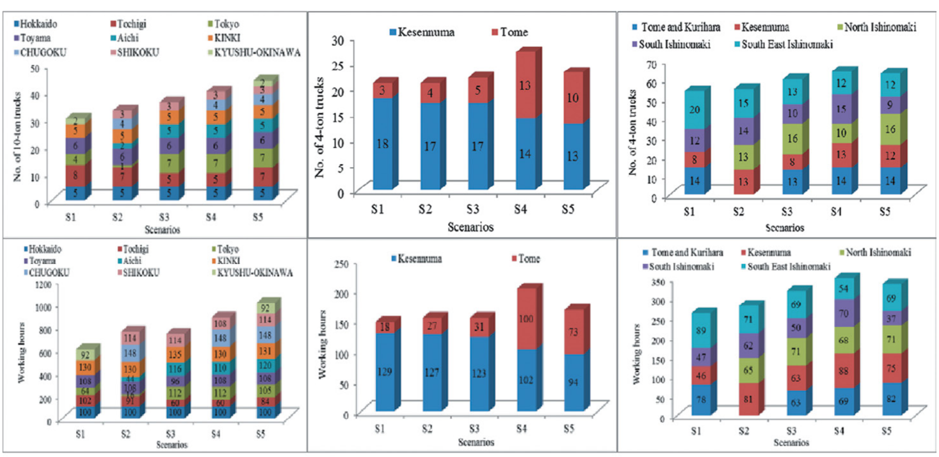
6 CONCLUSION AND FUTURE WORK

This study principally analysed the multi-facility location problems under both deterministic demand and uncertainty demand issues. We diagnose the uncertainty demand by the reason that it is quite difficult to predict the post-disaster demand. So, we determine the region of



Echelon 1 Echelon 2 Echelon 3

Network 2



Echelon 1 Echelon 2 Echelon 3

Figure 4: Fundamental resource requirements of uncertainty demand.

uncertainty demand as an ellipsoid uncertainty set that is suitable for our situation where only the uncertainty ranges are known and not necessarily the distribution. Moreover, an ellipsoid uncertainty set is a novel approach that has never been fully applied so far to solving facility location. We consider a whole distribution network starting from the beginning, suppliers, until the end, demands. We propose the three network structures which are the one network of single hierarchy facility and two networks of two hierarchies with distinct truck sizes (large trucks and small trucks). We determine the region of uncertainty demand as ellipsoid uncertainty set. Therefore, this study can help the decision makers to prepare the appropriate network with robustness for relief distribution.

First, the calculation results of both deterministic demand and uncertainty demand demonstrate that the network configurations are significant with total delivery cost. It can be seen clearly that the total delivery cost of network 2 and network 3 can reduce by about 18% because the travel cost is much reduced even though it requires more facility cost and transhipment cost. The results show that the travel cost has more significance than the opening facility cost. Moreover, the truck size operation is significant when the demand is high enough. This study found that large truck is appropriate to deliver both inbound and outbound supplies at the central depots. To apply the model, we suggest establishing the central depots and using large trucks to deliver both inbound and outbound supplies.

Furthermore, we would also prove that the networks are robust when the demand becomes uncertain or unknown. Here, we assume five different demand scenarios in each network based on the actual number of evacuees post-disaster. After solving the uncertainty demand by using RO, the results prove that the structural networks have an effect on the model robustness. The two hierarchies of facility provide an extra robustness than the single hierarchy of facility. Moreover, the uncertainty demand model is robust than deterministic demand model.

Finally, we discuss the interrelated aspects to improve the future work as follows: (1) we have not considered the other parameters that could possibly fluctuate during humanitarian logistics, for example the supply amount, the unit transportation cost, the opening facility cost and so on. Therefore, the uncertainty demand as well as the cost parameters should be considered simultaneously. (2) A new research can be improved with more efficiency by considering the vehicle routing problem together with our facility location problem. This model can be referred as location routing problem. The model might give more interesting results because the travel cost would be reduced by route detour. (3) The future work is considered the two-objective facility location problem. The model should be more reasonable by investigating both cost and time indicators simultaneously. After that the uncertainty of the demand is assigned to use with the model and then evaluates the robustness of the model.

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