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Maximum Active Power Transmission Efficiency and Apparent Power Definition with Arbitrary Waveforms

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ABSTRACT

A proof of the theorem giving the maximum available active power with constant transmission losses (i.e., maximum efficiency) valid for arbitrary waveforms, is proposed. This makes it possible to rigorously generalize the definitions of power factor based on the definition of apparent power as the maximum power available to apply them to systems with non-symmetric, non-sinusoidal, and eventually time-varying (i.e., non-periodic) waveforms or DC grids, thus extending its application to hybrid multi-wire systems with asymmetrical phases, with different voltages, frequencies or waveforms (i.e., non-sinusoidal waves, such as rectangular or PWM) and even DC, which can be combined eventually sharing the neutral conductor. Finally, an example of application to a hybrid system composed of DC and unbalanced load inverter-based non-sinusoidal AC subsystems is included.

1. INTRODUCTION

In polyphase systems, most of the current definitions of power factor proposed in the standards [1-18] are based on a theorem that gives the maximum power delivered for constant transmission losses [4, 6]. However, instead of naming this theorem as "Maximum Power Theorem" we prefer to name it as "Maximum Efficiency Power Transmission Theorem" to avoid confusions with the classical theorem of electrical circuits, which states that the power that a source could supply a load becomes maximum when the load impedance is the complex complement of the Thévenin impedance.

This maximum efficiency power transmission theorem is generally proven for sinusoidal waveforms operating in the time domain using trigonometric functions, or more simply using complex phasors [4, 6, 12, 15].

However, as it will be shown in the next section, the theorem is also valid for non-sinusoidal waveforms, even in time-varying systems.

This means that this theorem is valid for "polyphase" hybrid systems where some lines are DC supplies, other single phase sinusoidal, three phase unbalanced power subsystems and even high frequency rectangular waveform power sources (e.g., the typical 400 Hz power systems). This extends the application of the theorem to solve problems currently generated by the proliferation of power electronic loads [18-21].

2. THEOREM OF MAXIMUM EFFICIENCY

2.1 Maximum transmission efficiency with respect to the power source

This is the approach adopted to define the system apparent power used in standards like the IEEE1459 and DIN 40110.

In the circuit of Figure 1, the waveforms are not assumed sinusoidal, or even periodic (i.e., arbitrary waveforms).

It is assumed that both the resistances and the parasitic inductances of the lines can be different one from each other, although as will be seen in the demonstration that follows, it will not be necessary to know the value of the parasitic inductances to determine the maximum possible transmission efficiency (since these do not dissipate energy).

It is assumed that the functions of the waveforms of the voltages and currents are continuous, bounded, and satisfy the necessary conditions to permute the derivation and integration operations (practical waveforms have continuous derivatives bounded to values less than infinity).

The impedances of the load can be non-linear and eventually active (that is, they can contain voltage or current generators with arbitrary waveforms).

To calculate the energy transferred to the load, the instantaneous power is integrated during a time τ corresponding to the operating time of the system, or to the chosen analysis interval (which in the particular case of periodic waves can be one or multiple periods of the network).

Let m be the number of phases, the instantaneous power delivered by the source is:

$$p = \sum_{x=1}^{m} v_x\, i_x + v_N\, i_N \quad (1)$$

where, v_N and i_N are the neutral voltage and current.

Therefore, the energy consumed is:

$$W = \int_0^{\tau} p\, dt = \int_0^{\tau} \left(\sum_{x=1}^{m} v_x\, i_x + v_N\, i_N\right) dt \quad (2)$$

and the average active power is defined as:

$$P_{av} = W/\tau \quad (3)$$

An instantaneous power loss will dissipate in power lines, expressed by:

$$p_S = \sum_{x=1}^{m} R_{S_x}\, {i_x}^2 + R_N\, {i_N}^2 \tag{4}$$

During the time τ the energy lost in the lines will be:

$$W_S = \int_0^{\tau} p_S\, dt = \int_0^{\tau}\left(\sum_{x=1}^{m} R_{S_x}\, {i_x}^2 + R_N\, {i_N}^2\right)dt \tag{5}$$

The root mean square value of each current is defined as:

$${I_x}^2 = \frac{1}{\tau}\int_0^{\tau} {i_x}^2\, dt\,; \forall x = 1,2,3,\dots,m,N \tag{6}$$

and that of each voltage as:

$${V_x}^2 = \frac{1}{\tau}\int_0^{\tau} {v_x}^2\, dt\,;\ \forall x = 1,2,3,\dots,m,N \tag{7}$$

Defining the transmission efficiency as (W-W_S)/(W) in order to obtain its maximum, the minimum of W_S will be found for a constant total transmitted energy W. For this, the Lagrange multipliers method will be used.

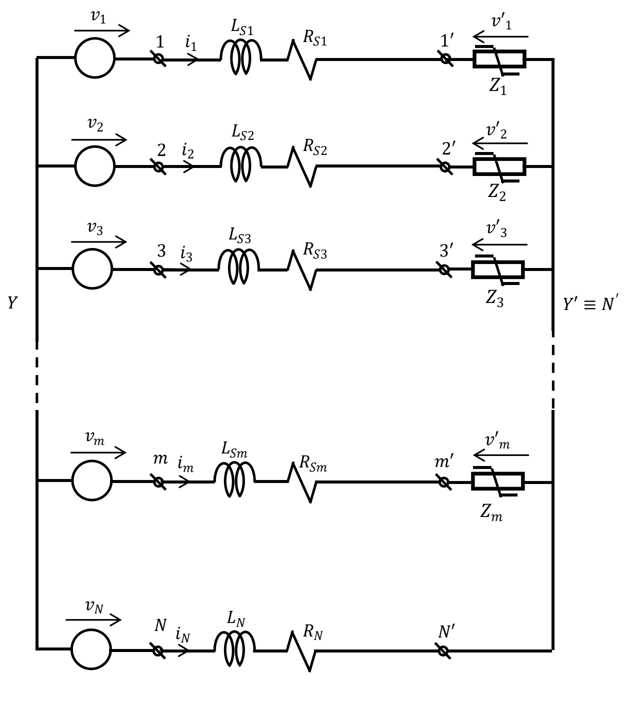


Figure 1. Circuit for the proof of the maximum efficiency theorem (m: number of phases)

It will be assumed that the functions corresponding to the waveforms of voltages and currents admit permutation between the operations of summation, integration and derivation (which happens for the waveforms used in practice, which are continuous and have continuous derivatives with bounded values less than infinity).

The minimum of W_S will be found with the restrictions that the total energy (W) transmitted from the source is constant:

$$f(i_x) = \int_0^{\tau} p\, dt - W = 0; \forall x = 1,2,3,\dots,m,N \tag{8}$$

and the restriction imposed by the node equation:

$$g(i_x, i_N) = \sum_{x=1}^{m} i_x + i_N = 0 \tag{9}$$

which is equivalent to:

$$\int_0^{\tau} g\, dt = \int_0^{\tau}\left(\sum_{x=1}^{m} i_x + i_N\right)dt = 0 \tag{10}$$

With which, from the Lagrange method, the system of equations in partial derivatives to solve is:

$$\frac{\partial W_S}{\partial i_x} - \lambda_1 \frac{\partial f}{\partial i_x} - \lambda_2 \frac{\partial \int_0^{\tau} g\, dt}{\partial i_x} = 0 \tag{11}$$

By exchanging derivation and integration, the system to solve becomes:

$$\frac{\partial p_S}{\partial i_x} - \lambda_1 \frac{\partial p}{\partial i_x} - \lambda_2 \frac{\partial g}{\partial i_x} = 0; \forall x = 1,2,3,\dots,m,N \tag{12}$$

Since p_S is always greater than zero, after finding its minimum instantaneous value, it will be integrated to obtain the minimum total energy lost.

Notice that if a minimum instantaneous function p_S is found for each instant, its integral during time τ (which is W_S) must also be minimum.

Deriving (1), (4) and (10), and substituting in (12) one obtains:

$$2\, i_x - \lambda_1\left({}^{v_x}/_{R_{S_x}}\right) - \left({}^{\lambda_2}/_{R_{S_x}}\right) = 0; \forall x = 1,2\dots m \tag{13a}$$

$$2\, i_N - \lambda_1\left({}^{v_N}/_{R_N}\right) - \left({}^{\lambda_2}/_{R_N}\right) = 0 \tag{13b}$$

Adding member to member (13a) and (13b) gives:

$$2\left(\sum_{x=1}^{m} i_x + i_N\right) - \lambda_1\left[\sum_{x=1}^{m}\left({}^{v_x}/_{R_{S_x}}\right) + \left({}^{v_N}/_{R_N}\right)\right] - \left({}^{\lambda_2}/_{R_{S_{//}}}\right) = 0 \tag{14}$$

where,

$$R_{S_{//}} = {}^{1}\Big/\left[\sum_{x=1}^{m}\left({}^{1}/_{R_{S_x}}\right) + \left({}^{1}/_{R_N}\right)\right] \tag{15}$$

is the result of associating in parallel all the parasitic resistances of the lines.

Applying the law of nodes (9), one gets:

$$\lambda_2 = -\lambda_1 v_O \tag{16}$$

where,

$$v_O = R_{S_{//}}\left[\sum_{x=1}^{m}\left({}^{v_x}/_{R_{S_x}}\right) + \left({}^{v_N}/_{R_N}\right)\right] \tag{17}$$

is a reference voltage that will be zero when the phase voltages have no homopolar component and the neutral voltage is negligible.

Substituting (16) and (17) in (13a) and (13b), it results:

$$i_x = (\lambda_1/2)\left[(v_x - v_O)/R_{S_x}\right] \tag{18a}$$

$$i_N = (\lambda_1/2)\left[(v_N - v_O)/R_N\right] \tag{18b}$$

Substituting in the equation of energy losses (W_S) (5), yields:

$$W_S = (\lambda_1/2)^2 \int_0^\tau \{[\sum_{x=1}^{m} (v_x - v_O)^2/R_{S_x}] + \cdots + [(v_N - v_O)^2/R_N]\} dt \quad (19)$$

Defining:

$$V_{xO}{}^2 = \frac{1}{\tau}\int_0^\tau (v_x - v_O)^2 dt\,; \forall x = 1, 2, 3, \ldots m, N \quad (20)$$

and substituting in (19), exchanging sum and integral, one obtains:

$$W_S = (\lambda_1/2)^2\, \tau\left[\left(\sum_{x=1}^{m} V_{xO}{}^2/R_{S_x}\right) + \left(V_{NO}{}^2/R_N\right)\right] \quad (21)$$

from where it follows:

$$2/_{\lambda_1} = \sqrt{\tau/_{W_S}}\sqrt{\left(\sum_{x=1}^{m} V_{xO}{}^2/R_{S_x}\right) + \left(V_{NO}{}^2/R_N\right)} \quad (22)$$

Using definitions (5) and (6):

$$W_S = \tau\left[\left(\sum_{x=1}^{m} R_{S_x}\, I_x{}^2\right) + R_N\, I_N{}^2\right] \quad (23)$$

Substituting (23) in (22) it results:

$$2/_{\lambda_1} = \sqrt{\left(\sum_{x=1}^{m} V_{xO}{}^2/R_{S_x}\right) + \left(V_{NO}{}^2/R_N\right)} \Bigg/ \sqrt{\left(\sum_{x=1}^{m} R_{S_x}\, I_x{}^2\right) + R_N\, I_N{}^2} \quad (24)$$

On the other hand, from (18a) and (18b):

$$v_x = \left(2/_{\lambda_1}\right) R_{S_x}\, i_x + v_O; \forall x = 1, 2, 3, \ldots, m \quad (25a)$$

$$v_N = \left(2/_{\lambda_1}\right) R_N\, i_N + v_O \quad (25b)$$

Substituting (25a) and (25b) in (2) yields:

$$W = \tau\, P_{av} = \int_0^\tau \left\{\left(2/_{\lambda_1}\right)\left[\left(\sum_{x=1}^{m} R_{S_x}\, i_x{}^2\right) + R_N\, i_N{}^2\right] + v_O\left(\sum_{x=1}^{m} i_x + \; + i_N\right)\right\} dt \quad (26)$$

Applying the law of nodes $\sum_{x=1}^{m} i_x + i_N = 0$, it results:

$$W = \tau\, P_{av} = \left(2/_{\lambda_1}\right)\int_0^\tau \left[\left(\sum_{x=1}^{m} R_{S_x}\, i_x{}^2\right) + R_N\, i_N{}^2\right] dt \quad (27)$$

With the definitions given by (6), exchanging summation and integration, one obtains:

$$W = \tau\, P_{av} = \tau\left(2/_{\lambda_1}\right)\left[\left(\sum_{x=1}^{m} R_{S_x}\, I_x{}^2\right) + R_N\, I_N{}^2\right] \quad (28)$$

Substituting (24) in (28), gives:

$$P_{av} = \sqrt{\left(\sum_{x=1}^{m} V_{xO}{}^2/R_{S_x}\right) + \left(V_{NO}{}^2/R_N\right)}\;\sqrt{\left(\sum_{x=1}^{m} R_{S_x}\, I_x{}^2\right) + R_N\, I_N{}^2} \quad (29)$$

A total equivalent system voltage (or DC equivalent voltage) can be defined as:

$$V_{eq-tot} = \sqrt{R_{S_{//}}}\sqrt{\left(\sum_{x=1}^{m} V_{xO}{}^2/R_{S_x}\right) + \left(V_{NO}{}^2/R_N\right)} \quad (30)$$

and a total equivalent system current (or DC equivalent current):

$$I_{eq-tot} = \sqrt{\left(\sum_{x=1}^{m} R_{S_x}\, I_x{}^2\right) + R_N\, I_N{}^2} \Big/ \sqrt{R_{S_{//}}} \quad (31)$$

Using both definitions, one gets:

$$P_{av} = V_{eq-tot} \cdot I_{eq-tot} \quad (32)$$

To show that the found losses W_S correspond to a minimum, the second order derivatives of the auxiliary function of the Lagrange method must be considered:

$$h_{(i_x)} = p_S - \lambda_1\, f_{(i_x)} - \lambda_2\, g_{(i_x)}; \forall x = 1, 2, 3 \ldots m, N \quad (33)$$

where, the constants λ_1 and λ_2 can be calculated using (16), (17) and (24).

Using (16), the auxiliary function (33) can be written as:

$$h_{(i_x)} = \left(\sum_{x=1}^{m} R_{S_x}\, i_x{}^2\right) + R_N\, i_N{}^2 - \lambda_1\left[\sum_{x=1}^{m} (v_x - - v_O)\, i_x + (v_N - v_O)\, i_N\right] \quad (34)$$

and the second order derivatives are:

$$\partial^2 h/_{\partial i_x{}^2} = 2\, R_{S_X}; \forall x = 1, 2, 3, \ldots, m \quad (35a)$$

$$\partial^2 h/_{\partial i_N{}^2} = 2\, R_N \quad (35b)$$

$$\partial^2 h/_{\partial i_x\, \partial i_y} = 0; \forall\, x \neq y \quad (36)$$

The Hessian matrix that can be formed with the second-order partial derivatives of the Lagrange's method auxiliary function, is a diagonal matrix in which all the determinants of its principal submatrices (called *principal minors*) are greater than zero (*positive definite matrix* according to the Sylvester criterion). Therefore, the singular point found is a minimum.

For example, for $m = 3$ it is:

$$\begin{bmatrix} \partial^2 h/_{\partial i_1{}^2} & \partial^2 h/_{\partial i_1\, \partial i_2} & \partial^2 h/_{\partial i_1\, \partial i_3} & \partial^2 h/_{\partial i_1\, \partial i_N} \\ \partial^2 h/_{\partial i_2\, \partial i_1} & \partial^2 h/_{\partial i_2{}^2} & \partial^2 h/_{\partial i_2\, \partial i_3} & \partial^2 h/_{\partial i_2\, \partial i_N} \\ \partial^2 h/_{\partial i_3\, \partial i_1} & \partial^2 h/_{\partial i_3\, \partial i_2} & \partial^2 h/_{\partial i_3{}^2} & \partial^2 h/_{\partial i_3\, \partial i_N} \\ \partial^2 h/_{\partial i_N\, \partial i_1} & \partial^2 h/_{\partial i_N\, \partial i_2} & \partial^2 h/_{\partial i_N\, \partial i_3} & \partial^2 h/_{\partial i_N{}^2} \end{bmatrix} = 2\begin{bmatrix} R_{S_1} & 0 & 0 & 0 \\ 0 & R_{S_2} & 0 & 0 \\ 0 & 0 & R_{S_3} & 0 \\ 0 & 0 & 0 & R_N \end{bmatrix}$$

In this matrix it is evident that all the *principal minors*, that is, all the determinants of the *principal submatrices*, are greater than zero, so both the instantaneous power p_S and its integral, which is the lost energy, are minimal.

For the constant energy W, the losses found are minimal and the transmission efficiency is maximum, so the opposite is also valid, that is, for constant losses, the transferred power calculated through (29) will be the maximum possible for these effective values of current and voltage (*rms* values with periodical waveforms).

Consequently, this maximum possible power is the one that should be adopted as the denominator to calculate the power factor, and (29) can be adopted as the definition of apparent power for the more general case of a polyphase system discussed here.

Particular Cases of Special Interest.

There are two situations of particular interest:

(1) First, when the parasitic resistances of the lines are equal to each other, but different from the resistance of the neutral conductor. That is:

$$R_{S_x} = R_S \neq R_N; \forall x = 1, 2, 3, \ldots, m \quad (37)$$

With this condition one gets:

$$R_{S_{//}} = 1 \Big/ \left[\left(m/R_S \right) + \left(1/R_N \right) \right] \quad (38)$$

and defining:

$$\rho = R_N / R_S \quad (39)$$

it results:

$$R_{S_{//}} = R_S \Big/ \left[m + \left(1/\rho \right) \right] \quad (40)$$

Substituting (40) in (17):

$$v_O = \left(\sum_{x=1}^{m} v_x + v_N \right) / \left[m + \left(1/\rho \right) \right] \quad (41)$$

and then (29) yields:

$$P_{av} = \sqrt{\sum_{x=1}^{m} V_{xO}{}^2 + \left(V_{NO}{}^2/\rho \right)} \sqrt{\sum_{x=1}^{m} I_x{}^2 + \rho\, I_N{}^2} \quad (42)$$

(2) Case in which all the parasitic resistances of the lines are equal to each other. That is:

$$R_S = R_N \Longrightarrow \rho = 1 \quad (43a)$$

and

$$R_{S_{//}} = R_S / (m + 1) \quad (43b)$$

Therefore:

$$v_O = \left(\sum_{x=1}^{m} v_x + v_N \right) / (m + 1) \quad (44)$$

turns out to be the homopolar, or zero sequence, voltage of the polyphase system formed by v_1, v_2, v_3, ..., v_m, v_N.

Thus, it results:

$$P_{av} = \sqrt{\sum_{x=1}^{m} V_{xO}{}^2 + V_{NO}{}^2} \sqrt{\sum_{x=1}^{m} I_x{}^2 + I_N{}^2} \quad (45)$$

An equivalent system voltage per phase can be defined as:

$$V_{eq} = \sqrt{\left(\sum_{x=1}^{m} V_{xO}{}^2 + V_{NO}{}^2 \right) / m} \quad (46)$$

and an equivalent system current per phase:

$$I_{eq} = \sqrt{\left(\sum_{x=1}^{m} I_x{}^2 + I_N{}^2 \right) / m} \quad (47)$$

which is the equivalent current originally proposed by Buchholz. With these definitions it results:

$$P_{av} = m\, V_{eq}\, I_{eq} \quad (48)$$

which is the maximum active power obtainable in a balanced and symmetric polyphase system with *m* phases operating with the given currents and effective voltages.

2.2 Maximum transmission efficiency with respect to the power delivered to the load

Observing Figure 1, it can be concluded that it would be more interesting and appropriate to search the maximum efficiency by minimizing transmission losses for a constant power consumed in the load, instead of imposing that the power delivered by the source be kept constant. That is, finding the minimum of W_S given by (5), keeping constant the total energy consumed by the load:

$$W' = \tau\, P_{av}' = \int_0^{\tau} p'\, dt = \int_0^{\tau} \left(\sum_{x=1}^{m} v_x'\, i_x \right) dt \quad (49)$$

where,

$$p' = \sum_{x=1}^{m} v_x'\, i_x \quad (50)$$

is the instantaneous power consumed by the load and

$$P_{av}' = W'/\tau \quad (51)$$

is the average active power consumed by the load.

The instantaneous power losses p_S continues to be given by (4), and therefore the energy lost in transmission is also W_S given by (5).

Now, using the method of Lagrange multipliers, the minimum of W_S will be found keeping W' constant, with which the first restriction is:

$$f(i_x) = \int_0^{\tau} p'\, dt - W' = 0; \forall x = 1, 2, 3, \ldots, m \quad (52)$$

and the second restriction is the law of nodes, expressed by (9) or (10).

In an entirely similar way to what was done before, one must solve the system formed by:

$$\frac{\partial p_S}{\partial i_x} - \lambda_1 \frac{\partial p'}{\partial i_x} - \lambda_2 \frac{\partial g}{\partial i_x} = 0; \forall x = 1, 2, 3, \ldots, m, N \quad (53)$$

Differentiating (4), (9) and (50), one obtains:

$$2\, i_x - \lambda_1 \left(v_x' / R_{S_x} \right) - \left(\lambda_2 / R_{S_x} \right) = 0; \forall x = 1, 2, \ldots m \quad (54)$$

$$2\, i_N - \left(\lambda_2 / R_N \right) = 0 \quad (55)$$

Adding member to member (54) and (55) yields:

$$2(\sum_{x=1}^{m} i_x + i_N) - \lambda_1 \left[\sum_{x=1}^{m}\left({v_x}'/R_{S_x}\right)\right] - \left(\lambda_2/R_{S_{//}}\right) = 0 \quad (56)$$

where, $R_{S_{//}}$ is given by (15) and applying the law of nodes (9) it results:

$$\lambda_2 = -\lambda_1 v_O' \quad (57)$$

where,

$$v_O' = R_{S_{//}}\left[\sum_{x=1}^{m}\left({v_x}'/R_{S_x}\right)\right] \quad (58)$$

Substituting (57) in (54) gives:

$$i_x = (\lambda_1/2)\left[({v_x}' - v_O')/R_{S_x}\right] \quad (59)$$

and from (55) and (57) it results:

$$i_N = -(\lambda_1/2)\ v_O'/R_N \quad (60)$$

Substituting in the expression of energy losses (W_S), (5):

$$W_S = (\lambda_1/2)^2 \int_0^\tau \left\{\left[\sum_{x=1}^{m}({v_x}' - v_O')^2/R_{S_x}\right]\left({v_O'}^2/R_N\right)\right\} dt \quad (61)$$

Defining:

$${V_{xO}'}^2 = \frac{1}{\tau}\int_0^\tau ({v_x}' - v_O')^2\, dt\ ;\ \forall x = 1, 2, 3, \dots m \quad (62)$$

$${V_O'}^2 = \frac{1}{\tau}\int_0^\tau (v_O')^2 dt \quad (63)$$

and substituting in (61), exchanging summation and integral, gives:

$$W_S = (\lambda_1/2)^2\ \tau\left[\left(\sum_{x=1}^{m} {V_{xO}'}^2/R_{S_x}\right) + \left({V_O'}^2/R_N\right)\right] \quad (64)$$

from which it yields:

$$2/\lambda_1 = \sqrt{\tau/W_S}\ \sqrt{\left(\sum_{x=1}^{m} {V_{xO}'}^2/R_{S_x}\right) + \left({V_O'}^2/R_N\right)} \quad (65)$$

Substituting (23) in (65), yields:

$$2/\lambda_1 = \sqrt{\left(\sum_{x=1}^{m} {V_{xO}'}^2/R_{S_x}\right) + \left({V_O'}^2/R_N\right)} \Big/ \sqrt{\left(\sum_{x=1}^{m} R_{S_x} {I_x}^2\right) + R_N {I_N}^2} \quad (66)$$

Substituting (59) into (49):

$$W' = P_{av}'\ \tau = \int_0^\tau \left[\left(2/\lambda_1\right)\left(\sum_{x=1}^{m} R_{S_x}\ {i_x}^2\right) + v_O'\left(\sum_{x=1}^{m} i_x\right)\right] dt \quad (67)$$

Solving from the node equation, it is $\sum_{x=1}^{m} i_x = -i_N$ and using (60):

$$W' = P_{av}'\ \tau = \left(2/\lambda_1\right)\int_0^\tau \left[\left(\sum_{x=1}^{m} R_{S_x}\ {i_x}^2\right) + R_N\ {i_N}^2\right] dt \quad (68)$$

Permuting integral and summation, using the definitions given by (6), one obtains:

$$W' = P_{av}'\ \tau = \left(2/\lambda_1\right)\tau\left[\left(\sum_{x=1}^{m} R_{S_x}\ {I_x}^2\right) + R_N\ {I_N}^2\right] \quad (69)$$

Substituting (66) into (69) gives:

$$P_{av}' = \sqrt{\left(\sum_{x=1}^{m} {V_{xO}'}^2/R_{S_x}\right) + \left({V_O'}^2/R_N\right)}\ \sqrt{\left(\sum_{x=1}^{m} R_{S_x} {I_x}^2\right) + R_N {I_N}^2} \quad (70)$$

As in the previous case, a total equivalent system voltage (or DC equivalent voltage) and a total equivalent system current (or DC equivalent current) can be defined such that:

$$P_{av}' = {V'}_{eq-tot} \cdot {I'}_{eq-tot} \quad (71)$$

where,

$${V'}_{eq-tot} = \sqrt{R_{S_{//}}}\sqrt{\left(\sum_{x=1}^{m} {V_{xO}'}^2/R_{S_x}\right) + \left({V_O'}^2/R_N\right)} \quad (72)$$

$${I'}_{eq-tot} = I_{eq-tot} = \sqrt{\left(\sum_{x=1}^{m} R_{S_x}\ {I_x}^2\right) + R_N\ {I_N}^2} \Big/ \sqrt{R_{S_{//}}} \quad (73)$$

where, this current is the same as that of the preceding case.

2.3 Comparison between both approaches

The expression of the maximum power obtained with the second approach is very similar to that obtained with the first one, which has been the preferred criterion by most international standards. The numerical results do not differ substantially between the two criteria when the neutral voltages are small and the transmission efficiencies are high. However, note that in the case of optimizing the efficiency with respect to the power transferred to the load, the reference voltage v_O', given by (58), does not depend on the neutral voltage, which, as can be deduced from observing Figure 1, is a voltage difficult to measure from the point of electrical energy consumption where the user is located.

For the case in which the parasitic resistances of the lines are equal to each other, but different from the resistance of the neutral conductor (condition 37), it is:

$$v_O' = \{m/[m + (1/\rho)]\} v_\Sigma' \quad (74a)$$

where, v_Σ' is the homopolar component of the load voltages:

$$v_\Sigma' = (1/m)\ \sum_{x=1}^{m} {v_x}' \quad (74b)$$

Thus, (72) and (73) become:

$${V'}_{eq-tot} = \sqrt{\left(\sum_{x=1}^{m} {V_{xO}'}^2\right) + \left({V_O'}^2/\rho\right)} \Big/ \sqrt{m + (1/\rho)} \quad (75)$$

$${I'}_{eq-tot} = \sqrt{\sum_{x=1}^{m} {I_x}^2 + \rho\ {I_N}^2}\sqrt{m + (1/\rho)} \quad (76)$$

This second approach based on (70) allows defining the apparent power without the need to know the neutral voltage drop as:

$$S' = \sqrt{\left(\sum_{x=1}^{m} {V'_{xO}}^2\right) + \left({V'_O}^2/\rho\right)}\sqrt{\sum_{x=1}^{m} {I_x}^2 + \rho\, {I_N}^2} \qquad (77)$$

which would be a simpler definition than those based on the first approach (which is used in IEEE Std. 1459).

When there is no homopolar component in the output voltages, (75) and (76) are simplified as:

$$V'_{eq-tot} = \sqrt{\left(\sum_{x=1}^{m} {V'_x}^2\right)/m} \qquad (78)$$

$$I'_{eq-tot} = \sqrt{\sum_{x=1}^{m} {I_x}^2 + \rho {I_N}^2}\,\sqrt{m} \qquad (79)$$

and (77) becomes:

$$S' = \sqrt{\left(\sum_{x=1}^{m} {V'_{xO}}^2\right)}\sqrt{\sum_{x=1}^{m} {I_x}^2} \qquad (80)$$

In both optimization approaches, losses can be expressed as:

$$P_S = \left(\sum_{x=1}^{m} R_{S_x} {I_x}^2\right) + R_N\, {I_N}^2 \qquad (81)$$

with which, (29) and (70) become:

$$P_{av} = V_{eq-tot}\big]_{crit\#1}\sqrt{P_S}\Big/\sqrt{R_{S//}} \qquad (82a)$$

$$P'_{av} = V'_{eq-tot}\big]_{crit\#2}\sqrt{P_S}\Big/\sqrt{R_{S//}} \qquad (82b)$$

where, $V_{eq-tot}\big]_{crit\#1}$ and $V'_{eq-tot}\big]_{crit\#2}$ are given by (30) and (72) respectively. From where the equivalent voltages per phase can be defined, according to:

$$V_{eq}\big]_{crit\#1} = V_{eq-tot}\big]_{crit\#1}/\sqrt{m} \qquad (83a)$$

$$V'_{eq}\big]_{crit\#2} = V'_{eq-tot}\big]_{crit\#2}/\sqrt{m} \qquad (83b)$$

Note that in the most general case it results:

$$V'_{eq}\big]_{crit\#1} = V_{eq}\big]_{crit\#1} - R_{S//} I_{eq-tot} \neq V'_{eq}\big]_{crit\#2} \qquad (84)$$

which should be taken into account when proposing the equivalent circuit models.

With either criteria, at least three equivalent circuit topologies can be proposed for a polyphase system with arbitrary waveforms (Figure 2).

(a) Polyphase equivalent model with sinusoidal alternating current, with symmetrical source and balanced load (Figure 2a)

The current of each phase is adopted as $I_{eq} = I_{eq-tot}/\sqrt{m}$ and the equivalent voltages per phase V_{eq} and V'_{eq} will have expressions that will depend on the optimization criteria adopted. The loss equivalent series resistors will be:

$R_{S_{eq}} = R_{S//}$, such that:

$$P_S = m\, {I_{eq}}^2 R_{S_{eq}} = {I_{eq-tot}}^2 R_{S//} = \left(\sum_{x=1}^{m} R_{S_x} {I_x}^2\right) + +R_N {I_N}^2 \qquad (85)$$

When it is:

$$R_{S_x} = R_S;\ \forall x = 1, 2, 3, \dots m \qquad (86a)$$

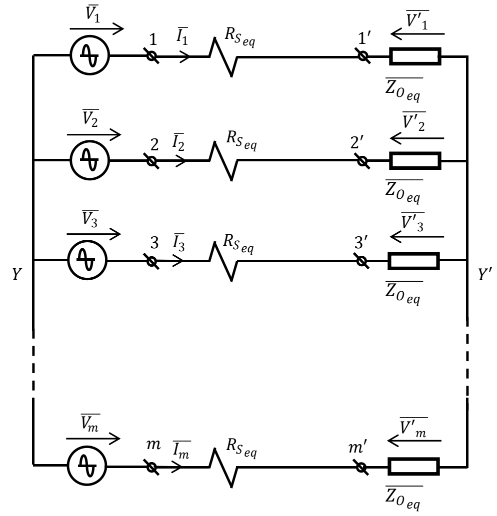
$$R_N = \rho\, R_S \qquad (86b)$$

based on (42), the model can be expressed by:

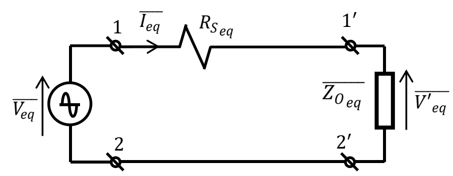
$$R_{S_{eq}} = m\, R_{S//} = R_S/[1 + (1/m\,\rho)] \qquad (87)$$

$$I_{eq} = \sqrt{\sum_{x=1}^{m} {I_x}^2 + \rho\, {I_N}^2}\,\sqrt{1 + (1/m\,\rho)} \qquad (88)$$

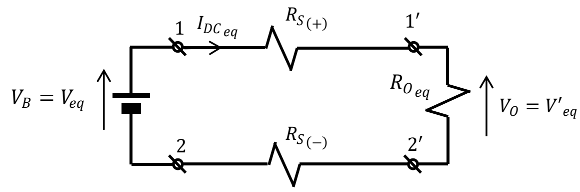
with V_{eq} given by (83a), for criterion # 1, or V'_{eq} given by (83b) for criterion #2.



(a)



(b)



(c)

Figure 2. Equivalent circuit models, (a) Sinusoidal polyphase equivalent system with symmetric source and balanced load, (b) Sinusoidal single-phase equivalent system, (c) Direct current equivalent system

Using (87) and (88) one may verify that P_S results the same given by (85).

It is clear that since all quantities are sinusoidal, phasors can be used.

b) Single-phase sinusoidal equivalent model (Figure 2b)

In this case, the sinusoidal voltage source will have an effective voltage equal to V_{eq-tot} (with the corresponding expression according to the optimization criterion adopted) and the sinusoidal current will have an *rms* value $V_{eq\text{-}tot}$, given by (31) and the equivalent series loss resistance will be $R_{S//}$.

c) Bifilar equivalent model of direct current (Figure 2c)

From Figure 2c it follows that:

$$R_{S_{tot}} = R_{S_{(+)}} + R_{S_{(-)}} = R_{S//} \quad (89)$$

The DC voltage V_B will be $V_B = V_{eq-tot}$ (with the expression corresponding to the optimization criteria adopted) and the direct current will be $I_{DC} = I_{eq-tot}$, given by (31).

3. APPARENT POWER AND POWER FACTOR DEFINITION IN HYBRID SYSTEMS

A hybrid system is generally composed of several subsystems of different nature, for example, polyphase, single-phase, two-wire direct current, multi-wire direct current and others with rectangular, trapezoidal or pulse width modulated waveforms, with separate or shared neutral wires.

Adopting the criterion of defining apparent power as the maximum active power that could be obtained from the system with the given effective current and voltage conditions, it is concluded that this maximum power will be:

$$S'_{tot} = \sum_{k=1}^{n} P'_{k_{max}} = \sum_{k=1}^{n} S'_k \quad (90)$$

where, *n* is the number of subsystems that make up the hybrid system and S'_{tot} is the total apparent power of the system that turns out to be the sum of the apparent powers of each subsystem (arithmetic apparent power) and:

$$S'_k = V'_{eq-tot_k} I'_{eq-tot_k} \quad (91)$$

where, V'_{eq-tot_k} is the total equivalent system voltage of subsystem *k* given by (72) and I'_{eq-tot_k} is the total equivalent system current of the same subsystem given by (73).

The power factor is the relation of the active power divided by the apparent power.

Thus, the power factor of each subsystem "k" will be:

$$PF_k = P'_k / S'_k \quad (92)$$

and the total power factor of the entire hybrid system is:

$$PF_{tot} = (\sum_{k=1}^{n} P'_k)/S'_{tot} \quad (93)$$

which can be also expressed as the weighted summation:

$$PF_{tot} = \sum_{k=1}^{n} FP_k \ (S'_k / S'_{tot}) \quad (94)$$

4. APPLICATION EXAMPLE: POWER FACTOR CALCULATIONS IN A HYBRID SYSTEM

In Figure 3, a hybrid system composed of three subsystems (numbered from # 1 to #3), with different nature and topologies of direct current and alternating current, is presented. It will be shown here how to define the apparent powers of each subsystem applying a definition based on the second optimization criterion proposed in Section 2, using (72) and (73) to define by (91) the apparent power of each subsystem.

It is assumed that the effective values of the voltages and currents indicated in the figures were experimentally measured, as well as the active powers of each load phase. With these data, the apparent powers and the power factors of each subsystem can be obtained and then through (93) or (94), the total apparent power and the total power factor.

4.1 Subsystem #1 (Figure 3a)

The subsystem shown in Figure 3a is a two-wire direct current power supply system with a number of phases *m*=1. Therefore, to apply definition (78), or also (91), one first gets from (58):

$$V_O{}'_{\#1} = \frac{1}{m+1} V'_{\#1} = \frac{1}{2} V'_{\#1} \quad \text{(E.1)}$$

from which the total equivalent system effective voltage (75) results:

$$V'_{eq-tot_{\#1}} = \sqrt{\left(V'_{\#1} - V_O{}'_{\#1}\right)^2 + V_O{}'_{\#1}{}^2} = \frac{1}{\sqrt{2}} V'_{\#1} \quad \text{(E.2)}$$

and the total equivalent system effective current is obtained by applying (76):

$$I_{eq-tot_{\#1}} = \sqrt{I_{\#1}{}^2 + I_N{}^2} = \sqrt{2}\, I_{\#1} \quad \text{(E.3)}$$

Therefore:

$$S'_{\#1} = V'_{eq-tot_{\#1}} I'_{eq-tot_{\#1}} = V'_{\#1}\, I_{\#1} \quad \text{(E.4a)}$$

Using the first optimization criterion, the following would have been obtained:

$$S_{\#1} = V_{eq-tot_{\#1}}\, I_{eq-tot_{\#1}} = V_{\#1}\, I_{\#1} \quad \text{(E.4b)}$$

which was naturally expected according to definition (93).

Assuming negligible voltage drops in the lines, it results: $V_{\#1} \cong V'_{\#1}$.

As $V_{\#1} \cong V'_{\#1} = 500\, V$ and $I_{\#1}$=50 A, it is: $S_{\#1} \cong S'_{\#1} = 25\, kVA$.

Since the load is pure resistive, it results: $FP_{\#1} = 1$.

4.2 Subsystem #2 (Figure 3b)

In Figure 3b a multi-wire direct current subsystem is shown, with 4 wires whose parasitic resistances are all the same. As in the previous case it will be assumed that voltage drops in the lines can be neglected compared to the voltages in the loads.

The measured voltages and currents are: $V_{1_{\#2}} = 400\, V$, $V_{2_{\#2}} = 300\, V$, $V_{3_{\#2}} = 260\, V$, $I_{1_{\#2}} = 10\, A$, $I_{2_{\#2}} = 20\, A$, $I_{3_{\#2}} = 30\, A$.

In this subsystem it is *m* = 3 and from (58):

$V'_{O\#2} = \frac{1}{4}\left(V_{1\#2} + V_{2\#2} + V_{3\#2}\right) = 240\, V.$

According to (75) the total equivalent system voltage is:

$$V'_{eq-tot_{\#2}} = \sqrt{\left(V'_{1\#2} - V'_{O\#2}\right)^2 + \left(V'_{2\#2} - V'_{O\#2}\right)^2 + \left(V'_{3\#2} - V'_{O\#2}\right)^2 + V'_{O\#2}{}^2} = 295.3\, V \quad \text{(E.5)}$$

and the total equivalent system current according to (76) is:

$$I_{eq-tot_{\#2}} = I'_{eq-tot_{\#2}} = \sqrt{{I_{1_{\#2}}}^2 + {I_{2_{\#2}}}^2 + {I_{3_{\#2}}}^2 + {I_{N_{\#2}}}^2} \tag{E.6}$$

where according to the law of nodes is:

$${I_{N_{\#2}}}^2 = \left(I_{1_{\#2}} + I_{2_{\#2}} + I_{3_{\#2}}\right)^2 \tag{E.7}$$

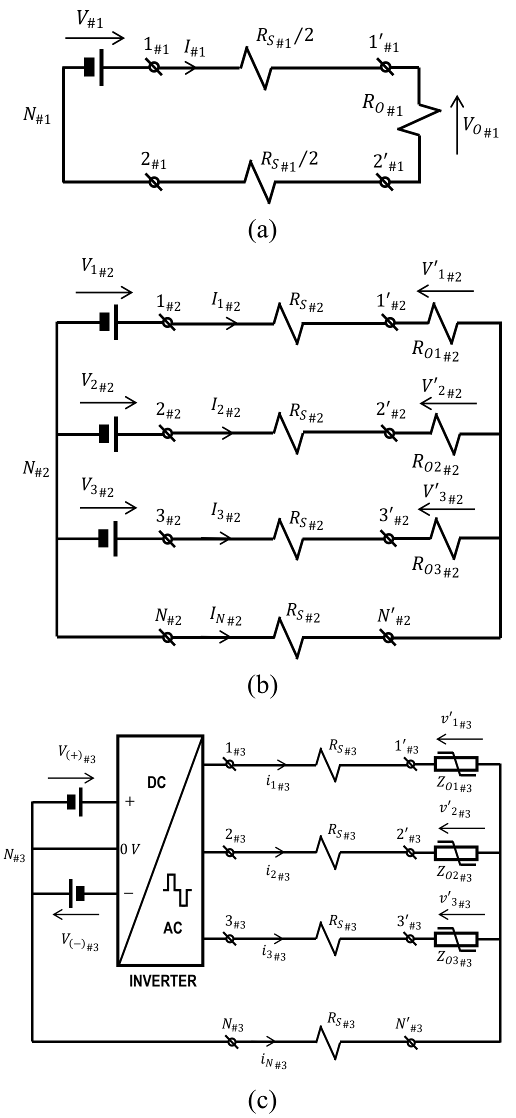


Figure 3. Hybrid system, (a) Two-wire direct-current subsystem, (b) Multiwire DC subsystem, (c) Three phase symmetric subsystem with rectangular waves

Substituting (E.7) in (E.6) and replacing by the numerical values given as data, yields:

$$I_{eq-tot_{\#2}} = I'_{eq-tot_{\#2}} = 70.7\ A \tag{E.8}$$

With the values given by (E.5) and (E.8) one obtains:

$$S'_{\#2} = V'_{eq-tot_{\#2}}\, I_{eq-tot_{\#2}} = 20.88\ kVA \tag{E.9}$$

According to data, the total active power of subsystem #2 is:

$$P_{tot_{\#2}} = V_{1_{\#2}}\, I_{1_{\#2}} + V_{2_{\#2}}\, I_{2_{\#2}} + V_{3_{\#2}}\, I_{3_{\#2}} = 17.8\ kW \tag{E.10}$$

and consequently, with (E.9) and (E.10), the power factor of subsystem #2 is:

$$PF_{\#2} = P_{tot_{\#2}}/S'_{\#2} = 0.852 \tag{E.11}$$

Note that the sum of the powers calculated as the product of each voltage and each current would also be the total apparent power considering each source and its respective load as separate subsystem from the others grouped as subsystem #2 (according to what was proposed in Section 3) which would be in disagreement with the result obtained through (E.9). What this equation shows is that more power could be transferred to the load with the same level of transmission losses, if the multi-wire system of Figure 3b were replaced by the equivalent two-wire system formed by a direct voltage source $V'_{eq-tot_{\#2}}$ delivering a continuous power to the load, equal to $S'_{\#2}$.

Obviously, when establishing a standard, it should be decided the criteria to address this issue regarding DC multi-wire systems.

When the source voltages are different and are intended to supply loads independent of each other, it should be more appropriate to consider each source with its load as a separate subsystem (even if they share the neutral).

In contrast, in symmetrical sources (delivering opposite voltages: $+V_B$ and $-V_B$), either single-pair or multi-pair (such as those used in some HVDC power transmission links) it is recommended to adopt a definition of apparent power (and consequently of power factor) based on the application of the maximum transmission efficiency theorem shown here (with either of the two optimization criteria exposed) because in this way, a figure of merit would be available to show the wasted transmission capacity of the link as a consequence of eventual load asymmetries.

4.3 Subsystem #3 (Figure 3c)

Figure 3c shows an inverter capable of delivering load phase voltages with rectangular waveforms, with a pulse width *DT/2* where *T* is the period and *D* is the duty cycle, between 0<D≤2/3 (see Figure 4). The *rms* currents are: $I_{1_{\#3}} = 11\ A$, $I_{2_{\#3}} = 20\ A$, $I_{3_{\#3}} = 14\ A$, $I_{N_{\#3}} = 5\ A$.

The active powers consumed by the non-linear impedances: $P_{Z\_O1_{\#3}} = 1100\ W$, $P_{Z\_O2_{\#3}} = 2500\ W$, $P_{Z\_O3_{\#3}} = 1400\ W$.

The symmetrical DC supply voltages are:

$V_{(+)_{\#3}} = V_{(-)_{\#3}} = 300\ V$.

There are two possible values for *D* that are of particular interest:

a) For $D = 2/3$ the pulse width is $2\pi/3$ and the waveform of the phase voltage obtained does not contain harmonic components of order 3 or their multiples, nor does it contain even components, so the first harmonic component that appears is of order 5 (which would facilitate filtering if it were decided to do so).

b) For $D = 1/2$ the crest factor (or peak factor) of the resulting rectangular wave is the same that would correspond to a sine wave ($F_{CR_{D=1/2}} = F_{CR_{sin}} = \hat{V}/V = \sqrt{2}$). This can be advantageous when powering equipment that must receive a peak voltage similar to that of a sinusoidal power line, for example, equipment powered by sources that include a rectifier with a direct capacitive filter and simultaneously, equipment containing transformers, filament lamps or heaters, which require an *rms* voltage similar to that of the public main.

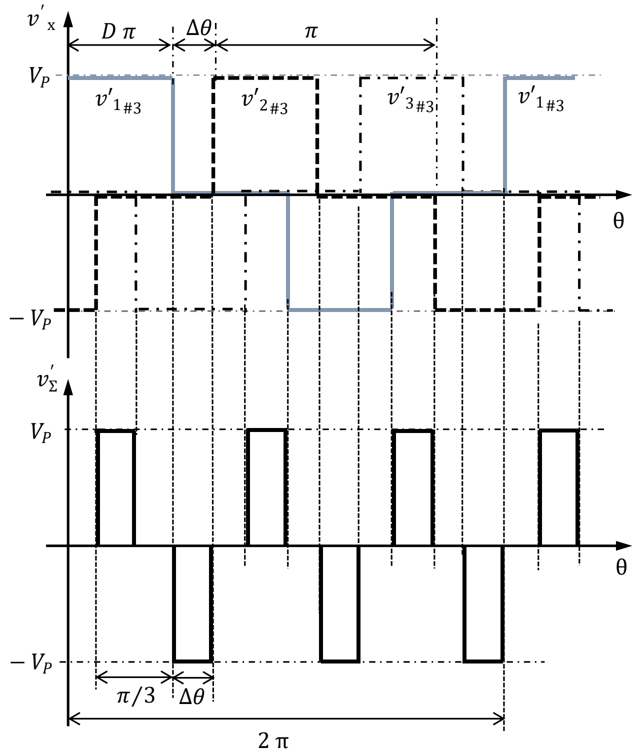


Figure 4. Waveforms of Subsystem # 3, which includes a rectangular wave inverter with non-linear and unbalanced load

In Figure 4 the phase voltages are shown for an arbitrary value of the duty cycle comprised between $0 < D \leq 2/3$.

The peak voltage of the rectangular wave is:

$$V_P \cong V_{(+)\#3} = V_{(-)\#3} \tag{E.12}$$

According to Figure 4 the pulse width is $D\pi$ and $\Delta\theta$ is:

$$\Delta\theta = \pi\left(\frac{2}{3} - D\right) \tag{E.13}$$

and also:

$$v'_{O\#3} = \frac{1}{4} v'_\Sigma = \frac{1}{4} \sum_{x=1}^{3} v'_{x\#3} \tag{E.14}$$

Notice that the period of $v'_{O\#3}$ is 3 times less than that of the phase voltages. From Figure 4 it is concluded that:

$$\frac{1}{2\pi} \int_0^{2\pi} \left(v'_{x\#3} - v'_{O\#3}\right)^2 d\theta = \frac{1}{2\pi} \int_0^{2\pi} \left({v'_{x\#3}}^2 - 2\, v'_{x\#3}\, v'_{O\#3} + {v'_{O\#3}}^2\right) d\theta = {V'_{x\#3}}^2 + {V'_{O\#3}}^2 \tag{E.15}$$

because,

$$\int_0^{2\pi} v'_{x\#3}\, v'_{O\#3}\, d\theta = 0 \tag{E.16}$$

being:

$${V'_{x\#3}}^2 = \frac{1}{2\pi} \int_0^{2\pi} {v'_{x\#3}}^2\, d\theta \tag{E.17a}$$

$${V'_{O\#3}}^2 = \frac{1}{2\pi} \int_0^{2\pi} {v'_{O\#3}}^2\, d\theta \tag{E.17b}$$

which gives:

$$V'_{eq-tot\#3} = \sqrt{3\, {V'_{\#3}}^2 + 4\, {V'_{O\#3}}^2} \tag{E.18}$$

From Figure 4 one obtains:

$${V'_{\#3}}^2 = D\, {V_P}^2 \tag{E.19}$$

$${V'_{O\#3}}^2 = \frac{\Delta\theta}{\pi/3} (V_P/4)^2 \tag{E.20}$$

Substituting (E.13), (E.19) and (E.20) in (E.18) gives:

$$V'_{eq-tot\#3} = \frac{V_P}{2} \sqrt{9\,D + 2} \tag{E.21}$$

Being $V_P = 300$ V, for $D = 2/3$ it results:

$$V'_{eq-tot\#3}\Big|_{D=2/3} = \sqrt{2}\, V_P = 424.3\, V \tag{E.22}$$

and for $D = 1/2$ it is:

$$V'_{eq-tot\#3}\Big|_{D=1/2} = 1.275\, V_P = 382.4\, V \tag{E.23}$$

The total equivalent system current (76) is:

$$I_{eq-tot\#3} = I'_{eq-tot\#3} = \sqrt{{I_{1\#3}}^2 + {I_{2\#3}}^2 + {I_{3\#3}}^2 + {I_{N\#3}}^2} = 27.24\, A \tag{E.24}$$

and the apparent power, neglecting the losses in the lines, can be expressed as:

$$S_{\#3} \cong S'_{\#3} = V'_{eq-tot\#3}\, I_{eq-tot\#3} \tag{E.25}$$

If it were $D = 2/3$ it would be:

$$S_{\#3} = 11.56\, kVA \tag{E.26a}$$

If instead it were $D = 1/2$ it would result:

$$S_{\#3} = 10.42\, kVA \tag{E.26b}$$

The total active power of subsystem #3 is:

$$P_{tot\#3} = P_{Z\_01\#3} + P_{Z\_02\#3} + P_{Z\_03\#3} = 5\, kW \tag{E.27}$$

and with these values the power factors may be easily obtained:

$$FP_{\#3}\rfloor_{D=2/3} = P_{tot\#3}/S_{\#3} = 0.43 \tag{E.28a}$$

$$FP_{\#3}\rfloor_{D=1/2} = P_{tot\#3}/S_{\#3} = 0.48 \tag{E.28b}$$

REMARK: In a real case, when modifying the duty cycle, the currents would be modified (depending on the converter topology and load features) and a different $I_{eq\text{-}tot}$ would have to be considered.

5. CONCLUSIONS AND PERSPECTIVES

The maximum efficiency theorem can be formulated following two optimization approaches, depending on the power supplied by the source or the power consumed by the load. The first approach has been preferred in the standards based on this type of definition of the apparent power obtained through demonstrations based mostly on the phasor representation (thus limiting its validity to the case of sine waves). Using phasors the first optimization approach implies a simpler demonstration that the one required if the second criterion here presented is adopted. The proof presented here is not based on the use of phasors and is valid for any practical waveform (continuous functions).

A second optimization criterion is proposed, optimizing with respect to the power consumed in the load, which leads to a simpler definition of the apparent power that does not depend explicitly on the neutral voltage of the consumer referred to the source one (sometimes cumbersome to measure). Perhaps, it would be better to adopt this second criterion to define the equivalent apparent power of the system, also simplifying its expression so as not to need to know the proportion of power consumed in star and delta (parameter ξ of the IEEE 1459 standard) that is normally unknown (which leads to adopt $\xi=1$ in such very common situations) [1, 4, 15].

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