

# Exponentially-Weighted Based Dynamic Pilot Power Allocation in Massive MIMO Systems

Muhammet Pakyürek, Osman Dikmen\*, Selman Kulaç

Department of Electrical Electronics, Faculty of Engineering, Duzce University, Duzce 81620, Turkey

Corresponding Author Email: osmandikmen@duzce.edu.tr

https://doi.org/10.18280/ts.390225	ABSTRACT
<b>Received:</b> 31 March 2022	In this paper, we propose a distinctive pilot power allocation algorithm to maximize the sum

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#### Keywords:

dynamic programming, massive MIMO, pilot power allocation, pilot contamination, optimization In this paper, we propose a distinctive pilot power allocation algorithm to maximize the sum rate in a multi-cell multi-user massive multiple-input multiple-output (MIMO) system. The algorithm optimizes pilot powers by polarizing the corresponding SINR values. In order to polarize SINRs, the difference between average SINR per cell and individual SINR is calculated for each user of the whole cells. The exponential form of the difference is used in the calculations of the weights for power allocation. New power values are obtained in proportion to these weights. Therefore, the power budget is utilized more efficiently thanks to these optimized power values. The efficiency of the algorithm is measured using the cumulative average SINR of the simulation system. Furthermore, equal pilot power allocation (EPPA) and water-filling pilot power allocation (WF-PPA) schemes are also implemented to compare the performances under the same simulation environments. A vast number of simulations results prove that our proposed heuristic approach is more efficient than EPPA and WF-PPA methods.

### **1. INTRODUCTION**

Massive MIMO technology is one of the most significant next generation wireless communication architecture equipped with hundreds of antennas at the base station [1]. The massive MIMO's ability to increase spectrum and energy efficiency, as well as to improve capacity and reliability are some of the reasons why it is a powerful technology [2, 3]. There are a variety of research areas of the massive MIMO such as hardware impairment, channel estimation, low latency, achievable rate and power allocation.

The power allocation is an essential process in data transmission for the power efficiency. In the case of a small number of antennas in the base stations, the conventional systems are only able to provide a small number of power allocation possibilities to users. MIMO systems are a technology that emerged with the equipping of hundreds of antennas at the base station. Therefore, there are more antennas on the transmitting side compared to conventional methods. In the case of a single or several antennas in conventional systems, it was relatively easy to allocate the pilot power. Because generally the total power was transferred to these antennas. However, after the development of MIMO systems and the fact that 5G and beyond technologies play an important role in increasing their capacity, problems began to arise about how the total power should be used. The optimal allocation of the total power to the transmit antennas is an optimization problem that needs to be solved in order to increase the MIMO capacity. The optimal power allocation is a significant optimization problem that needs to be solved to increase the MIMO capacity [4]. Uniform pilot power allocation, in other words, equal pilot power allocation (EPPA), is also used in MIMO systems where the CSI is known at the receiver but not at the transmitter [5]. One of the most important reasons for choosing this method is that EPPA basically provides a solution with low complexity [6]. The EPPA had been mostly preferred in the early days of MIMO systems. On the other hand, complex system requires more sophisticated power assignment approach to utilize the limited power more efficiently. Therefore, it is important to develop different power allocation strategies regarding the users channel conditions [7].

Every user's SNRs are different from each other. Therefore, they also require different power values to maximize the sum rate. Water-filling pilot power allocation (WF-PPA) algorithm is such an algorithm to calculate powers to each users' regarding their channel states. The method used in solving WF problems is the Lagrangian method [8]. WF-PPA algorithm has been used in some MIMO studies to increase energy efficiency [9, 10]. In another study [8], the WF algorithm is also used to maximize sum achievable rate. WF algorithm, which is considered as the optimum solution in the literature, is an important power allocation method as base algorithm to compare. However, with the WF-PPA algorithm, only users with higher powers are improved while other users are counted as ignored giving the too small weights in some experimental scenarios. Although the WF-PPA method is an optimal algorithm, it is not a global solution. Because there are discontinuity points due to the jacobian of SINR. Therefore, it is possible to get better results from the WF-PPA algorithm.

Throughout the literature survey, we have detected a number of delicate points to enhance our algorithm as follows:

- The small number of cells in simulations as in studies [11, 12]
- Discontinuity of water-filling algorithm because of the Jacobian of SINR [8]

Therefore, the proposed method provides the capability of simulations where the number of cell is nineteen. Hence, the results of this study's simulations are more resembling to real life scenarios. Furthermore, exponentially-weighted based



dynamic pilot power allocation algorithm (EWD-PPA) algorithm as a nature of its name, it always updates the cell regarding the most recent SINR of users in power updates to ensure the enhancement in total achievable rate of the system. This also reduce the pilot contamination because SINR is inversely proportional to interference. In the power updates, EWD-PPA calculates weights using an exponential function of the difference of SINR. This weighting function is defined for every real number so it can always end up with a definite number. Furthermore, weighting function always require positive values to keep power values within predefined  $[\rho_{min}, \rho_{max}]$  range. The following parts of the paper are planned as follows: In Section 2, there are statements about the system model. The proposed new algorithm for power allocation is mentioned in Section 3. The simulation results of a proposed new power allocation algorithm in terms of spectrum efficiency are given in Section 4. An overview of the findings and discussions from the study, conclusions and recommendations for future work are presented in Section 5.

## **2. SYSTEM MODEL**

This paper considers a multi-cell multi-user uplink system with massive MIMO working scenarios have been designed for the simulations. In this L cells system, BSs have M antennas and serve to K users in each cell. The channel vector  $\mathbf{h}_{ijk}$  from the *k*th user in the *j*th cell to the BS in the *i*th cell is written as in [13].

$$\mathbf{h}_{ijk} = \mathbf{g}_{ijk} \sqrt{\beta_{ijk}} \tag{1}$$

where,  $\mathbf{g}_{ijk}$  is shown as a small scale fading coefficient vector. All the elements of this vector have complex Gaussian distribution with zero-mean and unit variance. The large scale fading coefficient  $\beta_{ijk}$ , which depends on shadow fading with path loss, can be written as:

$$\beta_{ijk} = \frac{z_{ijk}}{r_{ijk}^{\gamma}} \tag{2}$$

where,  $z_{ijk}$  is shadow fading and  $r_{ijk}$  is the distance between the *k*th user in the *j*th cell and the *i*th BS and  $\gamma$  represents path loss exponent.

In the uplink pilot transmission phase, the channel state information (CSI) is estimated by the BS after receiving uplink pilot sequences at coherence interval. All used uplink pilot sequences with  $\tau_p$  length for each cell are employed with the pilot set  $\boldsymbol{\psi} = [\mathbf{p}_1, \mathbf{p}_1, \dots, \mathbf{p}_K]$ . We assume that users in the same cell have different orthogonal pilot sequences. This is ensured using more pilot sequences than users. In a system where the pilot power is thought to be different for the users in each cell, the expression of the received signal in the BS *i*th can be shown as below similar to [12]:

$$\mathbf{Y}_{i} = \sum_{j=1}^{L} \sum_{k=1}^{K} \sqrt{\rho_{jk}} \mathbf{h}_{ijk} \mathbf{p}_{k}^{H} + \mathbf{N}_{i}$$
(3)

where,  $\mathbf{N}_i$  and  $\rho_{jk}$  corresponds to Gaussian noise and power of the pilot for *k*th user in *j*th cell, respectively. After obtaining Eq. (3), channel is estimated between BS in cell *i* and user *k* in cell *i* using  $\mathbf{p}_{K}$  of pilot sequence is as follows:

$$\hat{\mathbf{h}}_{iik} = \mathbf{Y}_i \mathbf{p}_K = \sqrt{\rho_{ik}} \mathbf{h}_{iik} + \sum_{j \neq i}^L \sqrt{\rho_{jk}} \mathbf{h}_{ijk} + \mathbf{N}_i \mathbf{p}_K$$
(4)

In uplink data transmission phase, the BSs employ CSI to detect the received signal when the channel estimate is completed. The detected signal of kth user in BS in ith cell can be calculated by method such as maximum-ratio combining (MRC) detector used in some studies [13]. The data received in BS in ith cell is given as:

$$\mathbf{y}_i = \sqrt{\rho_u} \sum_{j=1}^{L} \sum_{k=1}^{K} \mathbf{h}_{ijk} x_{jk} + \mathbf{n}_i$$
(5)

where, the data symbol from the *k*th user in the *j*th cell is represented by  $x_{jk}$  and uplink data transmission power is denoted by the  $\sqrt{\rho_u}$ . The detected signal of *k*th user in BS in *i*th cell can be calculated by methods such as maximum-ratio combining (MRC) detector used in some studies [13] as follows,

$$\begin{aligned} \hat{x}_{ik} &= \mathbf{h}_{iik}^{H} \mathbf{y}_{i} \\ &= \left(\sum_{j=1}^{L} \sqrt{\rho_{jk}} \mathbf{h}_{ijk} + \mathbf{N}_{i} \mathbf{p}_{K}\right)^{H} \left(\sqrt{\rho_{u}} \sum_{j=1}^{L} \sum_{k=1}^{K} \mathbf{h}_{ijk} x_{jk} + \mathbf{n}_{i}\right) \\ &= \sqrt{\rho_{u}} \left( \underbrace{\sqrt{\rho_{ik}} \mathbf{h}_{iik}^{H} \mathbf{h}_{iik} x_{ik}}_{Desired signal} \right) \\ &+ \sqrt{\rho_{u}} \left( \underbrace{\sum_{\substack{j\neq i \\ Inter-cell interference}}^{L} \sqrt{\rho_{jk}} \mathbf{h}_{ijk}^{H} x_{jk}}_{Noise and interference}} \right) \end{aligned}$$
(6)

where, the combination of uncorrelated interference and noise is stated by  $\varepsilon_{ik}$ .

In this study, the uplink SINRs can be expressed for the MIMOs of this study as in the study [12],

$$SINR_{ik}^{u} = \frac{\rho_{ik} |\mathbf{h}_{iik}^{H} \mathbf{h}_{iik}|^{2}}{\sum_{j \neq i} \rho_{jk} |\mathbf{h}_{ijk}^{H} \mathbf{h}_{ijk}|^{2} + \frac{|\varepsilon_{ik}^{u}|^{2}}{\rho_{u}}}$$
(7)

where, uplink data transmission power is denoted by the  $\rho_u$ and the uncorrelated interference and noise is stated by  $\varepsilon_{ik}$ [12]. The representation of this equation, depending on  $\beta_{ijk}$ large scale fading coefficients, can be written as in the study [13], assuming number of M antennas in the base station goes to infinity

$$SINR_{ik}^{u} \to \frac{\rho_{ik}\beta_{iik}^{2}}{\sum_{j \neq i}\rho_{jk}\beta_{ijk}^{2}}$$
(8)

Taking into account the uplink SINR equations, the expression of the capacity can be written as

$$R_{ik}^{u} = \mathbb{E}\{\log_2(1 + SINR_{ik}^{u})\}\tag{9}$$

When the whole system is considered, the expression of uplink achievable sum rate for the whole cells is as follows:

$$R_{net}^{u} = \sum_{i=1}^{L} \sum_{k=1}^{K} R_{ik}^{u}$$
(10)

#### **3. PROPOSED METHOD**

In this section, proposed pilot power allocation algorithm based on the dynamic approach will be discussed. Different from the equal power allocation approach used in conventional massive MIMO systems, a novel pilot power allocation algorithm has been developed based on the difference of each user's SINR from the average SINR per cell. The main idea behind this approach is to polarize SINRs. In other words, it is to make the good channels conditions better and the bad ones worse. This is inspired from the most famous work of Polar Codes [14]. During these updates, the quality of service (QoS) is ensured staying within  $[\rho_{min}, \rho_{max}]$  range. Before elaborating the algorithm, the optimization problem of power allocation has to be revealed with critical constraints.

#### 3.1 Problem definition

In this paper, the maximization of total uplink achievable rate under certain constraints is the main object function to allocate powers efficiently. We express the power allocation optimization problem as P and its constraints as follows:

$$i \in [1, 2, \dots L], k \in [1, 2, \dots K]$$

$$\max \sum_{i=1}^{L} \sum_{k=1}^{K} R_{ik}^{u} \Longrightarrow \max \sum_{i=1}^{L} \sum_{k=1}^{K} \rho_{ik} \zeta_{ik}$$

$$C_{1} : \sum_{k=1}^{K} \rho_{ik} \le P_{t}$$

$$C_{2} : \rho_{min} \le \rho_{ik} \le \rho_{max}$$

$$(11)$$

When problem Eq. (11) is solved, it means that pilot power of user is assigned. So, the parameter to be optimized under the given constraints for solving this problem is  $\rho_{ik}$ . With the solution of the P problem, the system capacity will increase. As seen in Eq. (9), the expression of the capacity is directly proportional to the SINR. Therefore, the aim is also to increase the SINR explicitly. A simplified expression of the SINR can be written as follows:

$$SINR_{ik}^{u} = \rho_{ik}\zeta_{ik} \tag{12}$$

where,  $\zeta_{ik} = \frac{\beta_{iik}^2}{\sum_{j \neq i} \rho_{jk} \beta_{ijk}^2}$  from Eq. (8).  $C_1$  and  $C_2$  show the constraints. The  $C_1$  constraint states that the sum of new pilot powers cannot be greater than given total budget  $P_t$  for each cell. Last constraint  $C_2$  indicates that the pilot power must be included within the range of defined minimum and maximum power values. We determine the  $\rho_{min}$  and  $\rho_{max}$  powers obtained as in the study [13]. This optimization problem is solved satisfying the constraints by this proposed heuristic

power allocation algorithm. Simulation results clearly shows that this solution ends up with more efficient pilot power allocations.

# **3.2** Exponentially-weighted based dynamic pilot power allocation algorithm (EWD-PPA)

Alg. 1 shows the fundamental steps of the proposed EWD-PPA approach. First of all, SINRs are calculated with equal power allocation for all users. Then, pilot power updates are performed starting from the outermost cell with the least interference possibility. New superior pilot powers are assigned after calculating optimal weight values. In the revise of weights, the difference between the SINR of each user and the average cell SINR are calculated regarding the all updates in previously modified cells. The corresponding weights are calculated using the difference of SINRs. This weight calculation is an exponential function of SINR difference as defined 2<sup>sinr\_dif</sup>. It is quickly able to response to changes in SINR difference value. Then, users' powers are allocated regarding these updated weights. During the power allocations, each individual power is ensured in the range of  $[\rho_{min}, \rho_{max}]$ values by thresholding. Furthermore, it is asserted that the total budget is not exceeded by normalizing the summation of weights to the total budget. As the algorithm iterates over the cells, it takes into account the updated SINRs of the previous cells. Therefore, the proposed method is dynamically adapted to instant changes in SINRs. Since it can work in real time, it can be practically employed in real-life scenarios. SINR has been increased with more superior power allocation to users regarding the channel states. It is obviously concluded that interference (pilot contamination in uplink) is also compensated more efficiently thanks to the SINR based exponential weighting approach.

#### Algorithm 1. EWD-PPA algorithm

Input: SINRs of equally power allocation Output: Optimized power values Initialization:  $1: \rho_{ik} = \frac{P_t}{K}$ SINR<sub>ik</sub> = EPA\_SINR<sub>ik</sub>  $globalAvgSINR = mean(sum (EPA_SINR_{ik}))$ 2: **for** i = L to 1 **do** 3:  $avgCell_SINR = mean (sum (SINR (i, :)))$  $W_{ik} = 1$ 4: **for** k = K to 1 **do** 5:  $sinr_dif = SINR(i, k) - avgCell_SINR$  $W_{ik} = 2^{sinr\_dif}$  $\rho'_{ik}^{ik} = W_{ik} * \rho_{ik}$ 6: end for  $7: \rho'(\rho' < \rho_{min}) = \rho_{min}$  $\rho'(\rho' > \rho_{max}) = \rho_{max}$ 8:  $\rho'(i, :) = \frac{\rho'(i, :)}{sum(\rho'(i, :))} * P_t$ 9: (SINR (:, :))= SINR\_calculation( $\rho'$ , H) avg sinr' = mean (sum (SINR')) 10: if avg *sinr'* > globalAvg*SINR* then 11:  $\rho(i,:) = \rho'(i,:)$ globalAvgSINR= avg sinr' SINR = SINR'12: end if 13: end for 14: **return** ( $\rho(L, K:)$ )

Before the algorithm steps, it is essential to define some mathematical expressions and parameters which are used in the Alg. 1. Let  $P_t$  be the total budget per cell,  $\rho_{ik}$  is individual power of user k of cell i,  $EPA\_SINR_{ik}$  is equally power allocated SINRs of the system,  $SINR_{ik}$  is updated SINR after new power allocation executed indexes for cell and users are as follows:

Table 1. Essential simulation parameters

Name	Value
Number of users for each cell	10
Number of pilots for each cell	15
Number of base stations	19
Number of antennas in BS	512
Radius of cells	1000 m
Minimum distance to base stations	100 m
Sigma shadowing fading	8 dBm
Uplink transmission power	10^(15/10) for 15 dBm
Uplink pilot power	10^(15/10) for 15 dBm

### 4. SIMULATION RESULTS

In this section, the performance of our proposed PS-PPA method is exhibited through comparison of other algorithms. 10K simulations is realized on MATLAB (The whole scripts this study found written for can be at https://gitlab.com/xxxx/pilot power allocation of the study's GitLab page.). During calculations of users' positions and their corresponding channel states, Monte Carlo Methodology is employed to compare the algorithms objectively by obtaining statistically significant results. Numerical results are obtained from the massive MIMO system simulations. For a massive MIMO system model with nineteen cells, one base station per cell is considered. The number of users in each cell is planned as ten. In addition, the allowed pilot sequence for each cell was determined as fifteen to ensure orthogonality. Other parameters in the system are as shown in Table 1. In these systems, there are K users in each cell and L cells having a base station with M antennas. System parameters are as shown in Table 1.

Figure 1 shows a cumulative distribution function (CDF) of the uplink achievable sum rate per cell from our proposed system along with two alternative methods: EPPA and WF-PPA. It clearly shows that an important sum rate enhancement can be obtained by performing the power allocation optimization suggested in Alg. 1. As seen in Figure 1, the proposed methodology is superior to the other algorithms. The order of algorithms SINRs' for the same CDF value is EPPA< WF-PPA<EWD-PPA. In other words, EWD-PPA provides much more channel capacity for a corresponding CDF value. With the EPPA method, it provides equal pilot power allocation to all users. Thus, the same power is allocated to the user with bad channel conditions or to the user with good channel conditions in the system. Thus, this affects the total channel capacity. With the WF-PPA method, all power is allocated equally for users above a certain threshold value, while users below the threshold value are ignored. Thus, the total capacity is increased. Unlike WF-PPA, in EWD-PPA that we proposed, users are not ignored. With the SINR polarization, more power factor is allocated to above-average users and less power factor is allocated to below-average users. Thus, since no user is ignored, the total capacity is much better than WF-PPA.

This enhancement is also valid in average per cell uplink achievable rate for a variety of changing number of antennas. Figure 2 and Figure 3 clearly proves that the proposed scheme shows better performance by providing a higher achievable rate than other schemes for the same number of antennas. The gap between the performances of algorithms is becoming wider when the number of antennas is getting higher.



**Figure 1.** Comparison of algorithms' CDF of the average per cell uplink achievable bit rate (bps) vs. SINR



Figure 2. Comparison of algorithms' average per cell uplink achievable rate against number of antennas M for each BS (zoomed out)



Figure 3. Comparison of algorithms' average per cell uplink achievable rate against number of antennas M for each BS (zoomed out)

The basis of the algorithm called sequentially gradual water-filling (SGWF) or sequentially extended gradual water-filling (SEGWF) in the study [12], which is considered for a seven-cell structure, is a WF-PPA algorithm. Our proposed algorithm is carried out for a model that can be called a multi-cellular system with nineteen cells. At the same time, EPPA and WF-PPA algorithms for comparisons are also implemented on this model. Therefore, it can be deduced that

the proposed algorithm is superior to the WF-PPA-based SEGWF algorithm [12], since it has better results than the WF-PPA algorithm as can be seen from the figures. In addition, Dao and Kim [12] stated that the SEGWF algorithm has higher performance than the joint user-cell grouping (JUCG) method [13]. Based on this explanation, it is also possible to assume that the our EWD-PPA algorithm is better than the JUCG algorithm considering our proposed algorithm is can be better than from the SEGWF algorithm.

Numerically, the average capacity per cell of all simulations also confirms that EWD-PPA has superior performance. EWD-PPA leads to 36% of capacity increase in average per cell when compared with EPPA, while WF-PPA 8% capacity increase.

#### **5. CONCLUSION AND FUTURE WORK**

This study provides a novel, highly effective optimization approach for pilot power allocation to increase the total uplink achievable rate. The comparison of the proposed algorithm with the other two algorithms and the total capacity rates obtained as the simulation suggests are as follows: Numerically, the average capacity of EWD-PPA is 0.527 while WF-PPA's is 0.427 and EPPA's is 0.398 for the whole simulations. The capacity optimization algorithm finds optimal weights in the power allocations using the exponential function of the SINR difference. During the update of the weights, each cell is optimized regarding the updated users' pilot powers of previous cells. The simulation results basically prove that this approach outperforms the two most popular methods of EPPA and WF-PPA.

As future works, pilot contamination can be solved by pilot assignment and power allocation. A joint version of these two approaches will be exhibited. Furthermore, Reinforcement Learning (RL) as machine learning technique is suitable for such a dynamically changing optimization problem. It's also aimed to solve this problem using RL to get better results.

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#### NOMENCLATURE

- $\mathbf{h}_{iik}$  channel vector
- $\mathbf{g}_{ijk}$  small scale fading coefficient vector
- $\beta_{iik}$  large scale fading coefficient
- $z_{ijk}$  shadow fading
- $\rho_{jk}$  power of the pilot for *k*th user in *j*th cell
- M Total number of base station antenna

## Greek symbols

 $\gamma$  path loss exponent

#### Subscripts

m	meter
dBm	decibel milliwats