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Numerical Study of Laminar Bingham Fluid in Axisymmetric Sudden Expansion

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ABSTRACT

A numerical study was carried out to investigate the laminar flow of Bingham fluid through an axisymmetric sudden expansion of four aspect ratios and various values of Reynolds number between [50~200] and Bingham number [0~2]. By using the commercial code Ansys-Fluent, this paper focuses on presenting Bingham's flow through an axisymmetric sudden expansion to determine the length and intensity of recirculation zones and shed light on the local loss coefficient. The results show an increase in the reattachment length and the eddy intensity of the recirculation zones by increasing the Reynolds number and the aspect ratio and decreasing with increasing the Bingham number and vice versa, the local loss coefficient increases as the aspect ratio increases for the Newtonian fluid this effect is reflected in the Bingham fluid, the increase of the Bingham number also increases the local loss coefficient, dimensionless equations has built to predict all the reattachment lengths, the eddy intensity and the local loss coefficient.

1. INTRODUCTION

The design of the industrial installation in the oil and gas processes requires knowledge of the pressure loss through the installation, the existence of fittings, valves and the sudden or gradual changes in the cross section of the duct that produces pressure drop.

Evaluation of pressure loss through the sudden expansion requires the determination of the friction coefficient *K*, which is calculated from the experimental measurement. In the literature, two equations are frequently common to elbows of (45°, 90°, and 180°), tee and valves. The first is suggested by Hooper [1].

$$K = (\frac{K_1}{\mathrm{Re}}) + K_\infty (1 + \frac{1}{D})$$

Further, Darby [2] improves the accuracy of the pressure loss calculation by taking size changes into account and developing the three *Ks* method;

$$K = (\frac{K_m}{\mathrm{Re}}) + K_i (1 + \frac{K_d}{D^{0.3}})$$

Another approach bases on a numerical result proposed by Oliveira and Pinho [3] relating the coefficient *K* with the Reynolds number in the sudden expansion, in their works, for a Newtonian fluid [4] they're varying the aspect ratio and developed an expression to predict the friction coefficient through the sudden expansion.

The *mi* coefficients in the following expression were given as a function of the aspect ratio.

$$K = \frac{m_1}{\mathrm{Re}^{m_2}} + m_3 + m_4 \log(\mathrm{Re}) + m_5 \left[\log(\mathrm{Re})\right]^2$$

The yield stress fluid was investigated by Kfuri et al. [5] through 1:2.6 and 1:4 in abrupt expansions and contractions, they give similar equations of the friction coefficient to these of Oliveira et al. [4] but as a function of the power low index for power low fluid and others for yield stress fluid as a function of the dimensionless yield stress.

Numerically, Rosa and Pinho [6] investigate Newtonian fluid through axisymmetric diffusers for $2<Re<200$, diffusion angle $0<\theta< 90°$ and aspect ratio 1:1.5 and 1:2. They developed two expressions of the friction coefficient as a function of Reynolds number and diffusion angle.

In the literature many studies are interested in the flows over the abrupt expansions, starting from Macagno and Hung [7], who studied experimentally a viscous Newtonian fluid over axisymmetric expansion, the calculation and the experiments reported that the streamlines and vortices presented as a function of Reynolds number. According to Alipour [8] the recirculation zone was found even at low Reynolds numbers and grew in size with increasing Reynolds number in step of the sudden expansion.

In order to scale the recirculation zone, Scott et al. [9] have created a model of dimensionless equations for the reattachment length L_r and the eddy intensity ψ^* as a function of Reynolds number for each aspect ratio then Badekas and Knight [10] developed the equation as functions of aspect ratio also. While Pak et al. [11] investigated a Newtonian and Non-Newtonian fluid flow over circular sudden expansion, they announced that the reattachment length is a function of the concentration of non-Newtonian fluid in which the reattachment length decreases with increasing concentration of fluid, and it is shorter than those of Newtonian fluid for a laminar flow. However, in the turbulent flow, the reattachment length doubles twice or three times that for Newtonian flow and gradually increases with increasing concentration.

Through the sudden expansion Scott et al. [12] studied numerically a viscoplastic fluid flow Casson and Bingham, they found a reduction in the length and the strength of recirculation zone for the viscoplastic fluid flow compared with a Newtonian fluid, the same results presented by Vradis and Otugen [13] in which higher yield stress produces small recirculation zones, generally, the yield stress number has the opposite effect to the Reynolds number effect, this result has been also provided by Hammad et al. [14], and Hegaj and Borzenko [15] for Herschel-Bulkley fluid. The aspect ratio has an effect according to Hammad [16] in which the results showed intensive and large recirculation zones for $\delta = 5$ than thus of $\delta = 2$.

Another phenomenon related to the yielded and un-yielded zones was studied by Jay et al. [17] for a yield-stress fluid through a 1:4 sudden axisymmetric expansion, they announced that the yield stress generates an un-yielded zone, the inertia and yield stress act in opposite ways, they proved also that the pressure loss increases with the yield stress fluid.

Regardless of the Reynolds number Mitsoulis and Huilgol [18] confirmed when the Bingham number goes to an infinite value, a lack of size and intensity of the recirculation zones, and the un-yielded zone is enlarged.

From the aforementioned discussion, it is clear that the analysis of viscoplastic fluid flows through the sudden expansion remained limited. Due to the important role of the yield stress fluid in the oil industry, the aim of this study is to formulate mathematical model equations for the reattachment length, the eddy intensity, and the friction coefficient for Bingham fluid through an axisymmetric sudden expansion of a variable aspect ratio.

2. MATHEMATICAL FORMULATION

2.1 Problem description

The geometry studied is an axisymmetric sudden expansion, different expansion ratios δ were considered 1:1.5, 1:2, 1:3, and 1:4 as depicted in Figure 1.

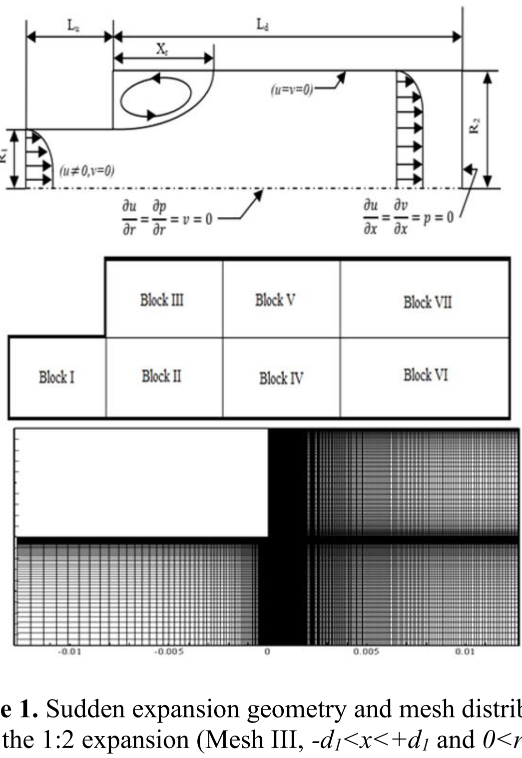

Figure 1. Sudden expansion geometry and mesh distribution near the 1:2 expansion (Mesh III, $-d_1<x<+d_1$ and $0<r<d_2$)

The entrance length of the sudden expansion $L_u= d_1$, where the downstream length $L_d = 120d_1$.

At the entrance, the boundary condition is set to be velocity inlet with a fully developed velocity profile and for the laminar flow of Bingham fluid, this boundary condition is introduced by using a separated geometry of considerable length in order to obtain the fully developed profile. The comparison of the analytical and numerical velocity profile at the inlet of the expansion at $Re = 50$ for different Bingham numbers (0, 0.5, 1 and 2) as shown in Figure 2. The results show an excellent agreement between the analytical and the numerical profiles.

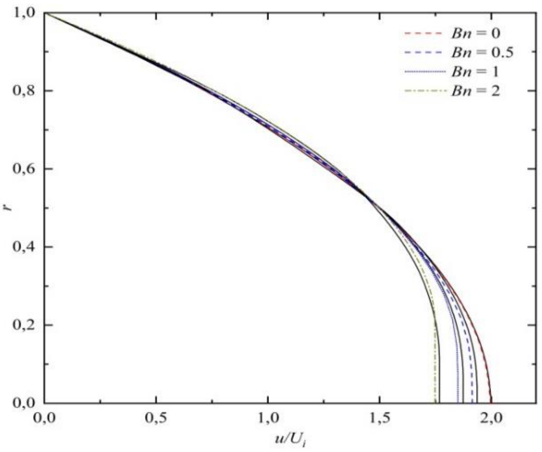

Figure 2. Inlet velocity profiles — Analytical, — Numerical

2.2 Governing equations

The conservation of mass is given by:

$$\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial v}{\partial r}=0 \tag{1}$$

While the conservation of momentum is given by

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial x}+\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rx}\right)\right) \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}\right)=-\frac{\partial p}{\partial r}+\left(\frac{\partial \tau_{rx}}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rr}\right)\right) \tag{3}$$

The extra stress tensor for power-law with yield stress defined by $\tau_{ij} = 2\eta(\dot{\gamma})D_{ij}$.

The plastic materials equation modified by the Bingham model described by the following equation of Papanastasiou [19].

$$\tau=\tau_y\left[1-\exp(-m\dot{\gamma})\right]+\mu\dot{\gamma} \tag{4}$$

For $\tau>\tau_y \rightarrow \dot{\gamma}>0$ and $\tau<\tau_y \rightarrow \dot{\gamma}=0$

The total pressure drop through the expansion written as bellow:

$$\Delta P_{tot}=P_1-P_2=\Delta P_R-\Delta P_I-\Delta P_F \tag{5}$$

The fully developed wall friction terms given by equation bellow for the upstream and downstream of the expansion in which the friction at the wall for Bingham plastic flow given by Swamee and Aggarwal [20].

$$\Delta P_{f_1} = f_1 L_1 \rho u_1^2 / 2d_1$$
$$\Delta P_{f_2} = f_2 L_2 \rho u_2^2 / 2d_2 \quad (6)$$

The decrease of velocity across the expansion (Bernoulli Effect):

$$\Delta P_R = \frac{1}{2}\rho u_1^2 (\frac{1}{\delta^2} - 1) \quad (7)$$

The irreversible pressure loss coefficient K:

$$K = \frac{\Delta P_I}{\frac{1}{2}\rho U_i^2} \quad (8)$$

3. NUMERICAL PROCEDURE

The numerical solution was obtained using commercial code Ansys-Fluent. The SIMPLE algorithm was used to solve the pressure-velocity coupling. To discretize the convective terms, a Quadratic upwind differencing scheme (QUICK) was used, and three meshes are tested in Table 1, furthermore, the absolute residual values of the continuity, the axial velocity and the radial velocity are set at 10^{-6}. The maximum error between the analytical and numerical calculation is located around the center of upstream sudden expansion, which reaches 1.27% for Bingham fluid, and it fades near the wall where the flow characteristics are of interest.

3.1 Validation

In Figure 3 the comparison of reattachment length L_r of our numerical calculations for a Newtonian fluid Bn = 0 and a 1: 2 sudden expansion with the experimental values obtained in the work of Macagno and Hung [7] and other numerical works are presented. Three mesh configurations are tested. The present calculations show a very good agreement for all the ranges of Reynolds numbers studied.

For the aspect ratio 1:1.5, 1:3 and 1:4 the present calculation was compared with some correlations obtained numerically by Scott et al. [9] and Badekas and Knight [10] the Table 2 shows the length of the backflow region L_r, it appears a very good agreement with the previous studies.

To further establishes the validity of the present results, Figure 4 Representing the dependency of the length of backflow region L_r on the Bingham number for a yield stress fluid and for a variety of Reynolds number, the present result were compared with those of Mitsoulis and Huilgol [18] on 1:2 aspect ratio, a slight different remarkable appears when the fluid goes to high value of the Bingham number, this difference was expected due to the different numerical method using to obtain the solution.

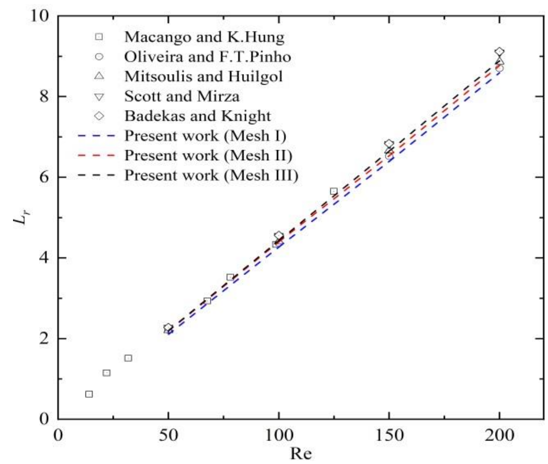

Figure 3. L_r versus Re for Bn = 0, δ = 2

Table 1. Computational domain and mesh characteristics of the sudden expansions

Mesh	Block	δ = 2		δ = 4	
		$N_x \times N_y$	$f_x \times f_y$	$N_x \times N_y$	$f_x \times f_y$
	I	22×13	1.08×1.125	22×13	1.08×1.125
	II	154×13	1.022×1.125	154×13	1.022×1.125
	III	154×13	1.022×1.05	154×39	1.022×1.05
M1	IV	39×13	1×1.125	39×13	1×1.125
	V	39×13	1×1.05	39×39	1×1.05
	VI	92×13	1×1.125	92×13	1×1.125
	VII	92×13	1×1.05	92×39	1×1.05
	I	44×26	1.08×1.125	44×26	1.08×1.125
	II	308×26	1.016×1.125	308×26	1.016×1.125
	III	308×26	1.016×1.125	308×78	1.016×1.125
M2	IV	56×26	1×1.125	56×26	1×1.125
	V	56×26	1×1.125	56×78	1×1.125
	VI	146×26	1×1.125	146×26	1×1.125
	VII	146×26	1×1.125	146×78	1×1.125
	I	64×52	1.057×1.06	64×52	1,057×1.06
	II	464×52	1.05×1.066	464×52	1,011×1.06
	III	464×52	1.05×1.12	464×156	1,011×1.04
M3	IV	99×52	1×1.066	99×52	1×1.06
	V	99×52	1×1.12	99×156	1×1.04
	VI	306×52	1×1.066	306×52	1×1.06
	VII	306×52	1×1.12	306×156	1×1.04

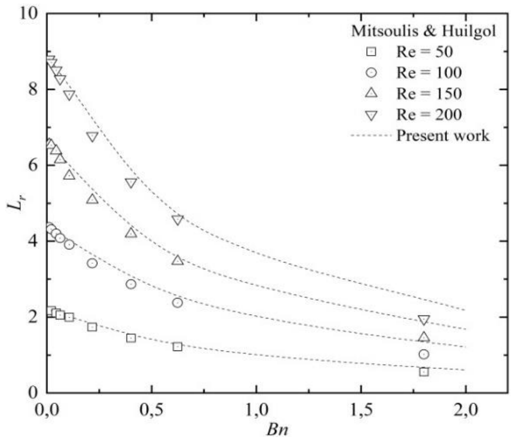

Figure 4. L_r versus *Bn* for δ = 2

Table 2. Comparison of L_r with the previous correlations for Newtonian fluid *Bn* = 0

Re	*δ*	P.S.Scott & F.A.Mirza	Badekas & Knight	Present Work
	1.5	0.625	0.772	0.695
50	3	5.5	5.295	5.285
	4	8.5	8.31	8.130
	1.5	1.25	1.545	1.292
100	3	11.00	10.59	10.773
	4	17.00	16.62	16.53
	1.5	1.875	2.317	1.914
150	3	16.5	15.885	16.341
	4	25.5	24.93	25.12
	1.5	2.5	3.09	2.55
200	3	22	21.18	21.949
	4	34	33.24	33.459

4. RESULTANTS and DISCUSSION

4.1 Effect of Reynolds and Bingham numbers

The solutions of the present analysis of the Newtonian fluid and the Bingham fluid through the axisymmetric sudden expansion are obtained for a number of fluid parameters and geometrical conditions, the results show some flow characteristics at the downstream step of the sudden expansion, in this zone, the Newtonian fluid flow constructs a vortex region their size and intensity dependency on the Reynolds number and the aspect ratio of the geometry, Figure 5 indicates the increase in the reattachment length with increasing Reynolds number and the aspect ratio, this dependence is linear as indicated with Macagno and Hung [7], Scott et al. [9] and Badekas and Knight [10].

In the case of the Bingham fluid flows, Figure 6 present the reattachment length as a function of the Bingham number, the vortex zone is greatly affected by the Bingham number where any increase in this parameter decreases the length and the intensity of the vortex regardless of Reynolds number and aspect ratio values, it's mentioned by Scott et al. [12], Vradis and Otugen [13] and Hammad et al. [14] for 1:2 axisymmetric sudden expansion. The increasing of Bingham number also generates another region at the corner of the sudden expansion known as an un-yielded zone, this region enlarges at the expense of the recirculation, and it changes here size and structure.

The linearity of the reattachment length on the Reynolds number given previously in correlations forms into a Newtonian fluid flow by Scott et al. [9] and Badekas and Knight [10], based on the present calculations and for preserving the linearity as a function of Reynolds number the Eq. (9) and Table 3 be fitted to predict the reattachment length of Newtonian and Bingham fluid flow simultaneously. The dependency on the reattachment length of the Bingham number was also fitted on the exponential form Eq. (10) and Table 4.

$$L_r = (\alpha\delta^2 + \beta\delta + \gamma)Re \tag{9}$$

Table 3. The coefficients of the Eq. (9)

	Bn			
	0	0.5	1	2
α	0	-0.00403	-0.00327	-0.00188
β	0.06037	0.05134	0.0375	0.02032
γ	-0.07576	-0.05971	-0.04292	-0.02185

$$L_r = \lambda + \varepsilon.\exp(\kappa.Bn) \tag{10}$$

with $\lambda = d_1\delta + d_2$ $\varepsilon = e_1\delta + e_2$ $\kappa = k_1\delta + k_2$.

Table 4. The coefficients of the Eq. (10)

	Re			
	50	100	150	200
d_1	0.2676	0.7633	1.2814	1.7937
d_2	-0.0914	-0.6723	-1.1963	-1.7797
e_1	2.6378	5.2139	7.7346	10.277
e_2	-3.4977	-6.83	-10.107	-13.518
k_1	-0.3187	-0.2113	-0.2092	-0.225
k_2	-0.4864	-0.9087	-0.975	-0.9446

The proposed correlation Eq. (9) give the same approximation with those of Scott et al. [9] and Badekas and Knight [10] for Newtonian flow with a maximum error of 1.35%, the case of Bingham fluid flows Figure 5 do not differ much, however it remains a linear function of Reynolds number, but the plastic force reduces the vortex length regardless of the inertia force or the geometry conditions, Figure 6 show the effect of Bingham number on the reattachment length where there is a contrast to the inertia forces and geometry condition on one side and the plastic force on the other for all aspect ratios studied. The enlargement of the vortex length by increasing Reynolds number and reducing it by increasing the Bingham number was indicated by Vradis and Otugen [13] for 1:2 sudden expansions.

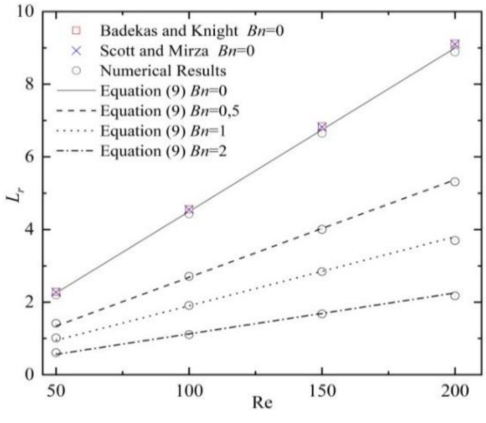

Figure 5. L_r versus Re, $\delta = 2$

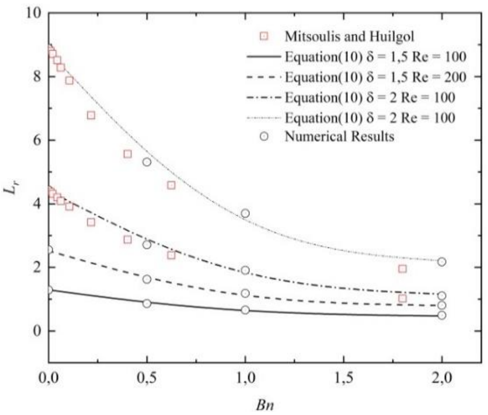

Figure 6. L_r versus *Bn*

The eddy intensity appears as a nonlinear function of the flow parameters, a higher aspect ratio and Reynolds number give higher eddy intensity shown as a function of Reynolds number in Figure 7 and Figure 9. The fitted exponential formulas Eq. (11) appear to a maximum error of 4.3% with the earlier study for Newtonian fluid, this difference is expected with considering the correlation's of Scott et al. [9] are specific and the present are general correlations, and it's also due to the different methods of solution.

For the Bingham fluid flow, the higher Bingham number gives smaller eddy intensity and vice versa as shown in Figure 8, at $Bn = 0$ the eddy intensity values multiply with the increasing the aspect ratio δ but it quickly decreases when the Bingham number increase, the flow go to be creeping flow at the infinite value of Bingham number.

The aspect ratio effect is constant, where it is shown in Figure 10 the enlarges of the un-yielded zones as well as the recirculation zones at the step of the downstream flow.

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$$\psi^* = A_1 + B_1.\exp(C_1.\mathrm{Re}) \tag{11}$$

$$\psi^* = A_2 + B_2.\exp(C_2.Bn) \tag{12}$$

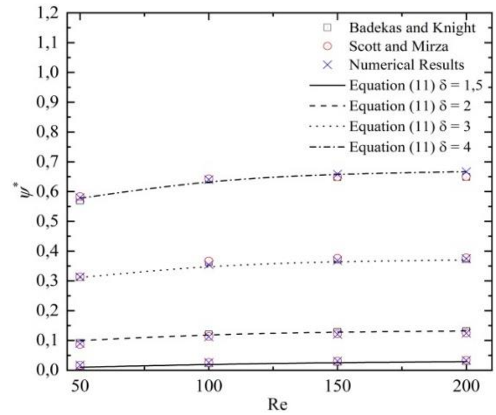


Figure 7. Eddy intensity ψ^* versus *Re* for $Bn = 0$

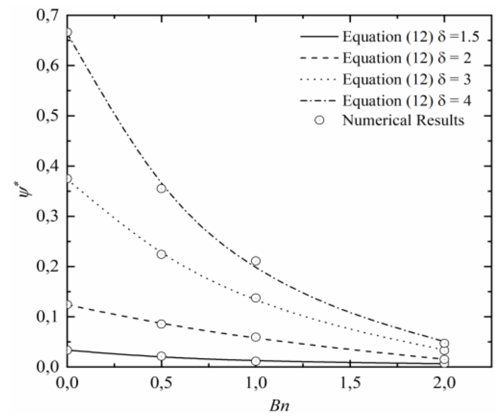
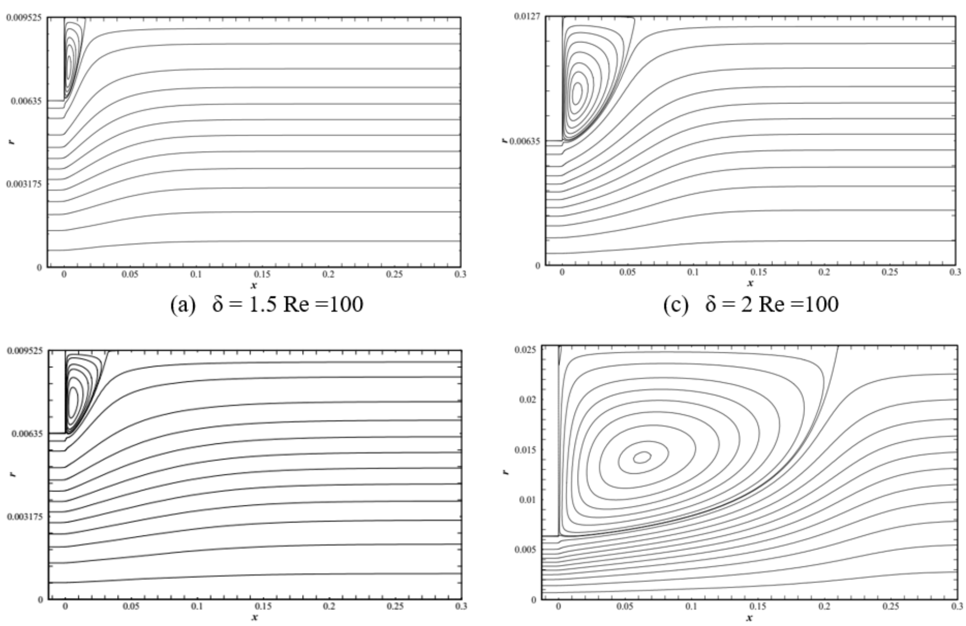


Figure 8. Eddy intensity ψ^* versus *Bn*



(a) δ = 1.5 Re =100

(c) δ = 2 Re =100

(b) δ = 1.5 Re = 200

(d) δ = 4 Re =100

Figure 9. Stream function contours of Newtonian fluid *Bn*=0

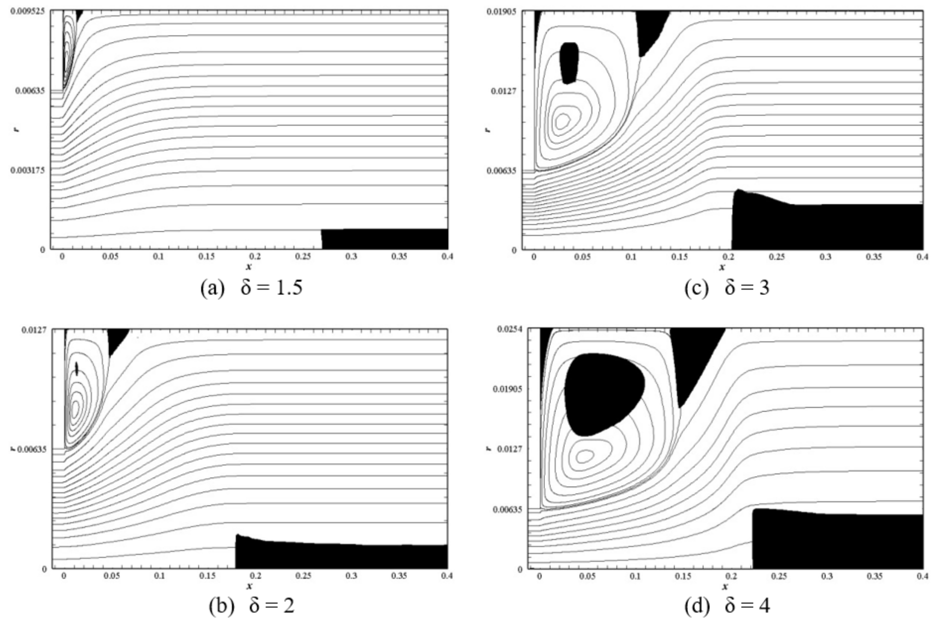


Figure 10. Stream function, yielded and un-yielded contours of non-Newtonian viscoplastic fluid at *Re=150* and *Bn*=0.5

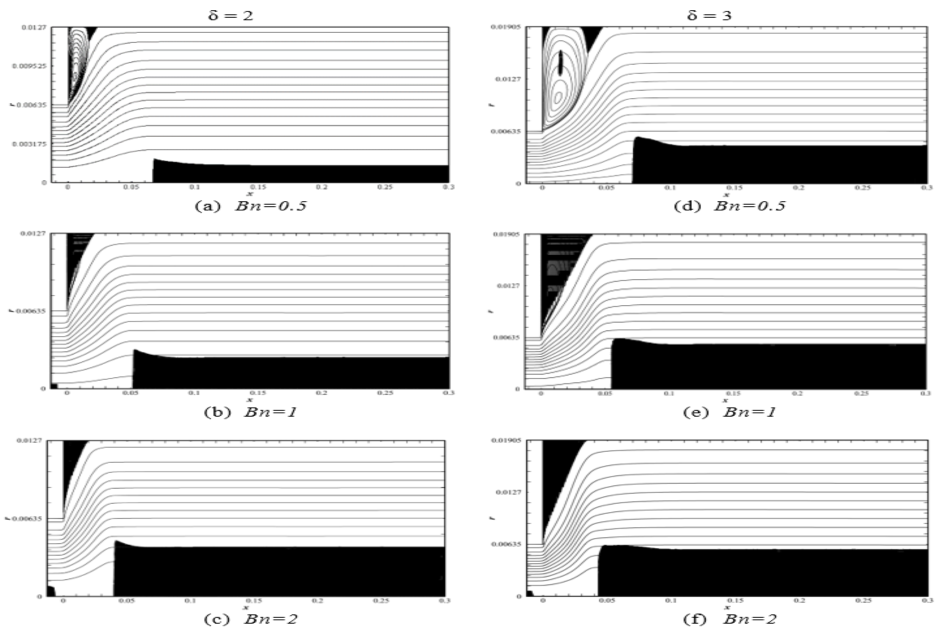


Figure 11. Stream function, yielded and un-yielded contours of non-Newtonian viscoplastic fluid at Re=50

The un-yielded zones start to appear separately at the corner and near the wall at the values of Bn = 0.5, Re = 50 and δ = 2 Figure 11(a) and enlarges to be one solid region with increasing the Bingham number or decreasing the Reynolds number than shown in Figure 11(b)(c) for δ = 3 Figure 11(d)(c)(f) another un-yielded zones appears at the vortex region, this zone depends on a higher aspect ratio or Reynolds number, and lower Bingham number (not 0 value), it's goes to be one region with increasing of the Bingham number. At the centerline of the sudden expansion, the un-yielded zones enlarges also if the Bingham number and the aspect ratio increase, where the Reynolds number affect only the axial direction of this zone.

4.2 Local loss coefficient

The calculations of the local loss coefficient through the sudden expansion of both Newtonian and Bingham fluid flows are shown in Figure 12, the Newtonian fluid Bn = 0 shows good compatibility with the previous studies of Oliveira et al. [4] and Kfuri et al.[5] for the range of Reynolds number and aspect ratio Re = [50 ~ 200], δ = [1.5 ~ 4] respectively, in which any increase in Reynolds number is equivalent to a decrease in the local loss coefficient, this behavior appears for all values of δ with a remarkable increase in the local loss coefficient.

For the Bingham fluid, the increases in the Bingham number values increase the local loss coefficient, Figure 13 show at higher Bingham number and for the Reynolds number values investigated the fluid become similar to the Newtonian fluid flow at lower Reynolds number that because the flow becomes more plastic, as opposed to the Newtonian fluid Figure 14 and Figure 15 shows when the aspect ratio δ increase the local loss coefficient decrease.

The local loss coefficient K was fitted based on the present numerical solution, the previous model of Oliveira et al. [4]

was modified the logarithmic terms have been saved to be appropriate for the range of Reynolds and Bingham numbers studied.

$$K = m_1 + m_2 \log(\mathrm{Re}) + m_3(\log^2 \mathrm{Re}) \quad (13)$$

The present model shows a good compatibility in the Newtonian fluid flow and predict the local loss coefficient for Bingham fluid flows for the range of [0 ~ 2], Reynolds number [50 ~ 200] and the aspects ratios δ studied. This model can help in calculations of pressure loss through sudden expansion of the oil industry and also help in selecting the appropriate reduction.

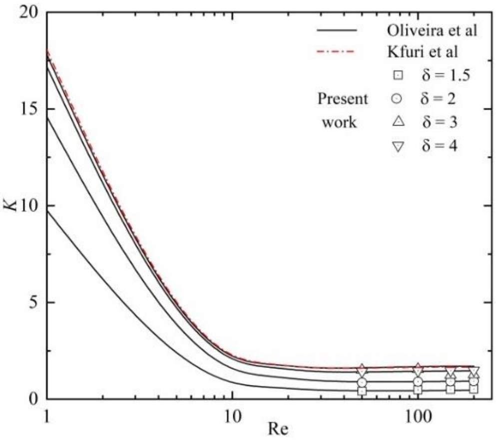


Figure 12. K versus Re for Bn = 0

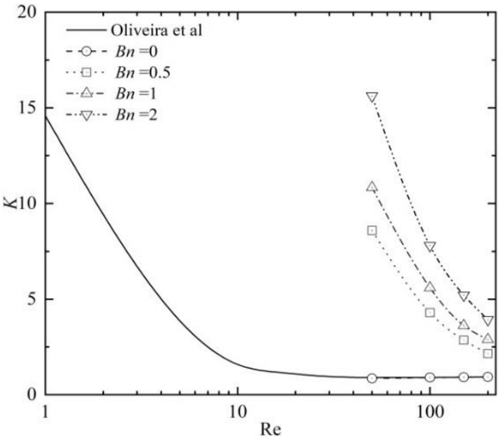


Figure 13. K versus Re for δ = 2

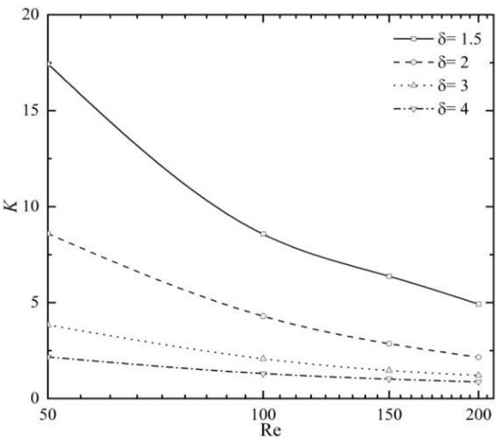


Figure 14. K versus Re for Bn = 0.5

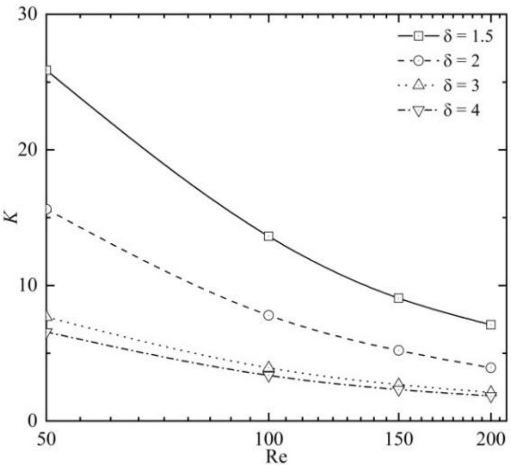


Figure 15. K versus Re for Bn = 2

The m_i values at the proposed model were fitted polynomial as function of Bingham number for all aspects ratios as appearing in Table 5.

$$m_i = a.Bn^3 + b.Bn^2 + c.Bn + d \quad (14)$$

Table 5. Fitted coefficients m_i polynomial 3rd order equation

δ	m_i	a	b	c	d
	m_1	138.2	-488.4	525,7	-0.423
1.5	m_2	-119.3	413.4	-429	0.724
	m_3	29.77	-99.94	96.74	-0.137
	m_1	58.033	-197.15	227.89	0.359
2	m_2	-47.414	160.48	-183.8	0.413
	m_3	9.748	-32.88	37.46	-0.071
	m_1	25,97	-86.29	102	-2.267
3	m_2	-23.05	76.78	-89.13	4.256
	m_3	5.061	-16.827	19.36	-1,191
	m_1	10.98	-27.78	38.86	1.149
4	m_2	-9.246	23.91	-33.16	0.214
	m_3	1.808	-4.627	6.617	-0.028

5. CONCLUSION

A numerical simulation was carried out for the Bingham fluid flow through a sudden expansion of different aspects ratios, the present results confirm the earlier studies regarding the reduction of the length and the eddy intensity of the recirculation zones as a function of Bingham number for all aspects ratios investigated, it has also been clarified the difference between the presence of the entrance length or not. Based on the numerical results dimensionless equations were built for the reattachment length and the eddy intensity for Bingham fluid flows at a range of Reynolds number [50 ~ 200] and Bingham number [0 ~ 2], these equations are effective with a maximum error of 1.35% for a Newtonian fluid. It turns out also that the local loss coefficient for Bingham fluid flow is higher at high Reynolds number than the Newtonian fluid flows; the equation of Oliveira et al. [4] has been modified according to the range of Reynolds number studied. The aspect ratio affects adversely on the local loss coefficient for Bingham fluid flow, in which a higher aspect ratio reduces the K values, for that it is clear to us the effect of the geometry conditions in the transport of this type of fluid, it is recommended to using a sudden expansion (concentric reducer) of higher aspect ratios to minimize the pressure drop.

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NOMENCLATURE

Bn	Bingham Number, $\tau_y. d_1/ \mu. U_i$
K	Friction Coefficient
L_d	Length of expansion section, m
L_r	Dimensionless reattachment length, m
L_u	Entrance length, m
d_1	Expansion upstream diameter, m
d_2	Expansion downstream diameter, m
Re	Reynolds number, $\rho d_1 U_i/ \mu$
U_i	Inlet velocity, m.s^{-1}
u,v	Cylindrical coordinates

Greek symbols

δ	Expansion ratio, d_2/d_1
ρ	Density (kg/m^3)
τ_y	Yield Stress, Pa
μ	plastic viscosity, m^2.s^{-1}
ψ^*	Eddy intensity $(\psi_{max} - \psi_{wall}) / (\psi_{cl} - \psi_{wall})$