Free Convective Flow Past a Vertical Cone with Magnetohydrodynamics / Heat Generation / Absorption with Variable Heat Flux

Rayampettai Munisamy Kannan¹, Bapuji Pullepu²*, Mohammad Sajid²

¹ Department of Mathematics, SRMIST, Kattankulathur 603203, Tamil Nadu, India
² Department of Mechanical Engineering, College of Engineering, Qassim University, Buraydah 51452, Al Qassim, Saudi Arabia

Corresponding Author Email: bapujip@yahoo.com

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ABSTRACT

The outcome of heat sink/source on natural convection movement with temperature and Magnetohydrodynamics transport from a vertical cone with variable surface heat flux is discussed. The leading boundary layer equation are obtained to dimensionless equation of the movement are explained by an absolutely stable finite difference system of Crank-Nicholson kind. This scheme transforms the non-dimensional equation into a system of tri-diagonal equation and which are worked out by using well known Thomas algorithm. The numerical solutions are presented in terms of fluid velocity, thermal, local as well as average shear stress and heat transfer rate for values of Prandtl number, magnetic parameter, heat sink/source constraint and the exponent in power law variant in surface thermal h has been analyzed both tabular and graphical forms. The current fallout is comparing with existing outcome in literature as well as be found to be outstanding concurrence.

1. INTRODUCTION


Vajravelu and Nayfesh [8] discussed MHD convection from a cone and a wedge with uneven surface thermal and interior temperature sink/source. Chen et al. [9] attained mathematical solution for mixed convection thermally dispersive movement throughout a fluid-saturated permeable medium to analysis the combination of natural and forced convective system. They presented that non-Darcian and temperature diffusion effect consume a main effect on velocity profiles, thermal and thermal transport rates from the perpendicular surface.

Several authors discussed natural convection precedent a vertical cone, top of cone in porous material. Yih [10, 11] investigated the full and truncated cone in the presence of isothermal/non-isothermal surface heat/mass transfer effects also discussed uniform/variable heat/mass flux effects in the saturated porous medium.

Hossain and Paul [12] discussed the non-similarity solution in favor of the natural convection of a vertical porous cone with inconstant surface heat flux. The laminar steady non-similar free convection movement of gas over a uniform cone has been investigated by Takhar et al. [13]. Affify [14] investigated the cause of emission on natural convective movement and mass transport from a vertical cone surface with element effects in the existence of a transversal magnetic field. Kumari and Nath [15] to discuss the non-Darcy natural convection movement of Newtonian fluid on cone inserted in a porous medium with exponent power difference of the wall temperature/concentration or temperature/concentration flux and injection/suction. El-Kabeir et al. [16] investigated chemical responses; temperature and mass transport on MHD movement of cone over the uniform surface in nano polar fluid with temperature absorption/generation arithmetic solution are obtained by Runga-Kutta scheme of 4th order with shooting technique. Later Pullepu et al. [17] focused on free convection flow from uniform surface of a cone with heat flux conditions. Pullepu and Chamkha [18] drawn-out the earlier exertion under the situation of variable thermal flux by MHD effect.

The outcome of thermal absorption/generation and thermal emission on mixed convection movement over a transient
stretch porous surface considered by Khan et al. [19, 20]. Mohamed et al. [21] examined natural convection flow over vertically hot surface in the presence of heat sink, radiative, MHD and chemical. Theoretical study on laminar natural convection movement over an axi-symmetric bodies have conventional extensive interest particularly in system of similar and non-homogeneous surface thermal flux distribution. Pullapil et al. [22] analyzed a numerical model of chemically reacted and heat source/absorption on transient free mass and thermal convection over incompressible viscous fluid through a vertical cone with non-homogeneous surface concentration and temperature results are drawn.

A little back Elbashbeshy et al. [23] analyzed a outcome of heat source or sink and temperature radiation on natural convective movement and temperature transport over the frustum of cone in the existence of pressure effect is discussed. Sambath et al. [24] explored laminar free convective hydromagnetic flow of radiated fluid from chemically reacted vertical cone under temperature generation/absorption parameter \( \Delta \) with variable temperature flux. When the fluid is incompressible, electrically conducting and combination.

The aim of the current analysis is to learn the combined impacts of MHD and heat generation/absorption parameter \( \Delta \) with variable temperature flux. When the fluid is incompressible, electrically conducting and combination.

2. MATHEMATICAL MODELING

The basic equations used to interpret and analyze natural convection are the PDE of the motion of the boundary layer which are due to conservation of momentum, conservation of energy and molecular species. To obtain these equations on the physical grounds, we adopt the following premise: Initially, it is presumed surface of the cone and the surrounding fluid which are at rest possess the unchanged thermal\( T''_c \) when time increases, the thermal of the cone surface is rapidly increased to \( q_a(x)=\alpha x^n \).

The geometric assumptions are made for the magnetic field.

➢ The effect of pressure ramp is assumed small.
➢ The Joule heating of the fluid is ignored.
➢ The stable electrical conductivity co-efficient is maintained all over the fluid.
➢ The magnetic field is stable and is apply in a direction vertical to the cone surface.
➢ The magnetic Reynolds numeral is little so as to the stimulated magnetic field is neglected and consequently does not alter the magnetic field.
➢ The magnetic field equations are the electromagnetic and hydro magnetic equations, also the dealings involving the flow and the magnetic field is taken.
➢ Maxwell’s dislocation currents are disregarded, so that electric currents are regard as flow in sealed circuits.

A system that uses coordinates (as depicted in Figure 1) has been considered as \( x \)-axis is taken towards the cone surface from the vertex \( (x=0) \) and \( y \) to the distance directed normally outward. The fluid behaviours are unchanged except density difference. We can study the impacts of the equations which governs continuity, thermodynamic quantity and energy equivalent to the capacity of a physical system by an approximation given by Boussinesq are stated below.

\[
\frac{\partial}{\partial x} (ur) + \frac{\partial}{\partial y} (vr) = 0 \quad (1)
\]

\[
\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta (T' - T''_c) \cos \phi + v \frac{\partial^3 u}{\partial y^3} - \frac{\sigma B^2 u}{\rho} \quad (2)
\]

\[
\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \alpha \frac{\partial^2 T'}{\partial y^2} + \frac{Q}{\rho C_p} (T' - T''_c) \quad (3)
\]

The boundary, initial conditions are prescribed as:

\[
\begin{align*}
t' & \leq 0: u = v = 0, T' = T''_c, \text{for all} \quad x \text{ and } y \\
t' & > 0: u = v = 0, \frac{\partial T'}{\partial y} = -\frac{q_a(x)}{k} \text{ at } y = 0 \\
u & = 0, T' = T''_c \text{ at } x = 0 \\
u & \to 0, T' \to T''_c \text{ as } y \to \infty
\end{align*}
\]

Nusselt number, local skin friction are derived by:

\[
\tau_s = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} \quad (5)
\]

\[
Nu_x = -x \left( \frac{\partial T'}{\partial y} \right)_{y=0} \quad (6)
\]

Scheduling with discussion, we introduce the subsequent dimensionless quantity to make a possible numerical solution for boundary value problem established by Eqns. (1) to (3) under condition (4):

\[
X = x/L, \quad Y = \left( y/L \right) Gr_L^{1/5}, \quad 1 = \left( \frac{v T'}{L^2} \right) Gr_L^{-1/5}, \quad R = \frac{r}{L}
\]

where \( r = x \sin \theta \)

\[
T = \frac{(T' - T''_c)}{q_a(L)/k} U = \left( \frac{uL}{v} \right) Gr_L^{-2/5}, \quad V = \left( \frac{vL}{v} \right) Gr_L^{-1/5}
\]

\[
Gr_L = g \frac{q_a(L)/k}{\beta L^2 \cos \phi}, \quad Pr = \frac{v}{L}
\]

\[
\Delta = \frac{Q L^2}{C_p L^2} (Gr_L)^{1/5}, \quad M = \frac{\sigma B^2 L^2}{\mu} (Gr_L)^{-2/5}
\]

Non-dimension expression of Eqns. (1)-(4) are as follows:

\[
\frac{\partial}{\partial X} (UR) + \frac{\partial}{\partial Y} (VR) = 0
\]
\[
\frac{\partial U}{\partial T} + \frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} V = (T-MU) + \frac{\partial^2 U}{\partial Y^2}
\]  
(9)

\[
\frac{\partial T}{\partial t} + \frac{\partial T}{\partial X} U + \frac{\partial T}{\partial Y} V = 1 \frac{\partial^2 T}{\partial Y^2} + \Delta T
\]  
(10)

The corresponding dimensionless quantities are:

\[
t \leq 0: U = V = 0, T = 0, \text{for all } X \text{ and } Y
\]

\[
t > 0: U = V = 0, \frac{\partial T}{\partial Y} = -X'' \text{ at } Y = 0
\]

\[
U = T = 0 \text{ at } X = 0
\]

\[
U \to 0, T \to 0 \text{ as } Y \to \infty
\]  
(11)

\[\tau_X \text{ (shear stress) and } Nu_X \text{ (heat transfer rate) in the dimensionless form are given by:}\]

\[
\tau_X = (Gr_L)^{1/5} \left( \frac{\partial U}{\partial Y} \right)_{T=0}
\]  
(12)

\[
Nu_X = \frac{X}{T_{Y=0}} \left( \frac{-\partial T}{\partial Y} \right)_{T=0} (Gr_L)^{1/5}
\]  
(13)

Dimensionless average quantities \( \bar{\tau} \) (shear stress) and \( \bar{Nu} \) (heat transport rate) are:

\[\bar{\tau} = 2(Gr_L)^{3/5} \int_0^1 X \left( \frac{\partial U}{\partial Y} \right)_{T=0} dX \]  
(14)

\[\bar{Nu} = 2(Gr_L)^{3/5} \int_0^1 X \left( \frac{-\partial T}{\partial Y} \right)_{T=0} dX \]  
(15)

3. NUMERICAL APPROACH

The transitory non-linear coupled PDE’s (8)-(10) with (11) are resolved by the finite difference system of Crank-Nicolson method. In this system, the non-dimensional ruling equation are obtained to a tri-diagonal structure of equation that is resolved by the Thomas algorithm, which is rapid convergent and absolutely stable [17, 18].

The area of integration is regarded as a rectangle with \( X_{max}=1 \) and \( Y_{max}=26 \) (where \( Y_{max} \) corresponds to \( Y=\infty \)) it is located exterior of together the fluid rate and thermal boundary layer. The quantities of \( Y_{max}=15 \) is used after some preliminary examination, so that the final two penultimate condition (11) where fulfilled within an acceptance limit of \( 10^{-4} \). The network size was permanent as \( \Delta X=0.05, \Delta Y=0.05 \) with the moment step \( \Delta t=0.01 \). The shortness ignorance is \( O(\Delta X+\Delta t+\Delta Y^2) \) approaches to null value as \( AX, \Delta Y \) and \( \Delta t \) approaches to null. Thus, the system is well-matched, as explained in Refs. [17, 18] and its discussions the quality of being enduring and free from change or variation we can establish the convergence.

4. DISCUSSIONS OF RESULTS

The present work establishes the behaviour of velocity and temperature parameter ranges at stable condition \( X=1.0 \) with approachable similar solutions in literature. Boundary layer, the momentum of cone with uniform surface heat flux when \( Pr=0.72 \) the numerical quantities of \( \tau_X \) and \( Nu_X \), for different values of \( Pr \) presented in Table 1 are investigated with resemblance solutions of Lin [2] in steady-state using the following transformation:

\[
Y = \left( \frac{20^3}{9} \right)^{1/5}, T = \left( \frac{20^3}{9} \right)^{1/5} \left[ -\theta(0) \right], U = \left( \frac{20^3}{9} \right)^{1/5} f' (\eta), \\
\tau_X = \left( \frac{20^3}{9} \right)^{1/5} f'' (0).
\]

In addition, with, \( \tau_X \) (shear stress), \( Nu_X \) (heat transfer rate) for distinguishable numerical aggregates of \( Pr \) when heat flux gradient \( n=0.50 \) and \( M=0 \) at \( X=1.0 \) unstable situation are related with the non-similarity result of Hossain and Paul [12] in Table 2. It is remarked that the outcomes are in excellent conformity with both other. It is also viewed that the current outcomes accordance with Pop and Watanabe [7] as pointed out in Table 2.

Table 1. Correlation of stable – state \( \tau_X \) (shear stress), thermal value at \( X=1.0 \) with of Lin [2] for isothermal surface thermal flux

<table>
<thead>
<tr>
<th>Pr</th>
<th>Temperature Lin [2] results</th>
<th>Existing Results</th>
<th>Shear stress Lin [2] results</th>
<th>Existing Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( -\theta(0) ) ( \left( \frac{20^3}{9} \right)^{3/5} )</td>
<td>( T ) ( f' (0) ) ( \left( \frac{20^3}{9} \right)^{2/5} )</td>
<td>( \tau_X )</td>
<td></td>
</tr>
<tr>
<td>0.72</td>
<td>1.522878 1.7864 1.7714 0.88930 0.88930*</td>
<td>1.224 1.2105</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.39174 1.6327 1.6182 0.78446 1.0797 1.0669</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.16209 1.3633 1.3499 0.60252 0.8293 0.8182</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.98095 1.1508 1.1385 0.46307 0.6373 0.6275</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.89195 1.0464 1.0344 0.39688 0.5462 0.5371</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.83497 0.9796 0.9677 0.35563 0.4895 0.4808</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.79388 0.9314 0.9196 0.32655 0.4494 0.4411</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.48372 0.5675 0.5531 0.13371 0.184 0.1778</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Values observed from Na and Chiou [3] with solution for the full cone.
Table 2. Relationship of stable-state $\tau_X$ and $Nu_X$ at $X=1.0$ with those of Hossain and Paul [12] for various values of $Pr$ when $n=0.5$, $M=0$, and Suction is zero

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>$F_\gamma(0)$</th>
<th>$\tau_X/G_r^{3/5}$</th>
<th>$1/\Phi_0(0)$</th>
<th>$Nu/XGr^{1/5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>5.1345</td>
<td>5.1155</td>
<td>0.14633</td>
<td>0.1458</td>
</tr>
<tr>
<td>0.05</td>
<td>2.93993</td>
<td>2.9297</td>
<td>0.26212</td>
<td>0.2630</td>
</tr>
<tr>
<td>0.1</td>
<td>2.29051</td>
<td>2.2838</td>
<td>0.33174</td>
<td>0.3324</td>
</tr>
</tbody>
</table>

Figure 2. Velocity profiles for Various Values of $Pr$, $M\Delta$ and $n$

Figures 2(a), (b), (c) concerned the effects of velocity for various quantities of $Pr$, $M$, $\Delta$ and $n$ correspondingly at a stable point $X=1.0$. It is viewed that the velocity profiles be drops by cumulative quantities of $Pr$, $M$, $n$ and decreasing values of $\Delta$.

Figure 3. Temperature profiles for Various Values of $Pr$, $M\Delta$ and $n$

Figures 3(a), (b), (c) illustrate the effects of heat for various quantities of $Pr$, $\Delta$, $M$ and $n$. As increasing quantities of $Pr$, $n$ and decreasing quantities of $M$, $\Delta$ the temperature decreases. In the electrically conducting fluid due to Lorentz force, gives rise to resistive force that acts in the opposite direction.
Figure 4. Effects of Local Skin friction for Various Values of Pr, $M \Delta$ and $n$

Figures 4(a), (b), (c) demonstrates the effects of shear stress coefficient for different parameters $Pr$, $M \Delta$, $M$ and $n$. It is noticed that, the decreasing values of $Pr$, $M \Delta$ and $M$ and increasing values of $n$ decreases the coefficient of local skin friction.

Figure 5. Effects of Local Nusselt number for Various Values of Pr, $M \Delta$ and $n$

Effects of $Nu_{\mu}$ (Nusselt number) variations are sketched in Figures 5(a), (b), (c). It shows that the growth of local Nusselt number for lesser quantities of $M \Delta$, $M$ and bigger quantities of $Pr$. $M \Delta$.
Figure 6. Average Skin friction Effects for Various Values of $Pr, M \Delta$ and $n$

Inspection of Figures 6(a), (b), (c), that the average skin-friction rises for bigger quantities of $\Delta$, $Pr$ and lesser quantities of $n$, $M$.

Figure 7. Average Nusselt number Effects for Various Values of $Pr, M \Delta$ and $n$

Figure 7(a), (b), (c) plotted to show the effect of heat source/sink parameter $\Delta$, flux parameter $M$, $Pr$ and $n$. The average Nusselt number rises for greater quantities of $Pr$, $n$ and smaller quantities of $\Delta$, $M$. 

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5. CONCLUDING REMARKS

A numerical study has been obtained for the natural convection flow from a vertical cone with temperature source/sink and MHD. The dimensionless governing equation for momentum and energy are converted to non-similar equation by using a dimensionless transformation. These equations are solved mathematically using the well-known Crank-Nicolson method.

(1) It is clear that the velocity and temperature distribution falls with raising the values of Pr, n and lower values of Δ.

(2) In the existence of magnetic field, rises of the strength of the magnetic field, slows the fluid motion along the wall of the cone inside and boundary layer but the velocity decreases and thermal increases.

(3) Shear stress τx and heat transfer rate Nux quantities decreases as M, n and Pr increases.

(4) Increasing quantities of thermal source/sink constraint Δ leads to increases in the value of the shear stress coefficient and heat transfer rate declines.

(5) The average shear stress increases for bigger quantities of Pr, Δ and smaller quantities of n and M.

(6) The average Nusselt numeral rises for larger quantities of Pr and lesser quantities of Δ, n and M.

REFERENCES


