Study of the mechanical behavior of orthotropic plates with a centered elliptic hole

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ABSTRACT. *The presence of the elliptical holes may lead to the increase in local stress, this phenomenon is known as stress concentration, has dangerous consequences what causes damage to the metal or composite structures. In this analytical and numerical study outlines the effect of the presence of elliptical hole on the mechanical behavior of composite structures. This is to determine the stress distribution around and away from the hole under pure tensile stress. The study focused on the evaluation of the stress concentration factor that is considered as an inducer of the local increase in stress due to the presence of the hole. This factor has been studied as a function of some parameters such as the variation of elliptical ratio (a/b) and the tensile stress of angles to the direction of large axis of the ellipse β (°/), and the variation of the fiber orientation.*

RÉSUMÉ. *La présence des trous elliptiques dans une plaque peut conduire à l'augmentation de la contrainte locale. Ce phénomène, appelé concentration des contraintes, a des conséquences dangereuses et peut provoquer l'endommagement des structures métalliques ou composites. Dans cette étude analytique et numérique on présente l'effet de la présence d'un trou elliptique sur le comportement mécanique des structures composites. Il s'agit de déterminer la distribution des contraintes autour et loin du trou sous sollicitation de traction pure. L'étude est focalisée sur l'évaluation du facteur de concentration des contraintes qui est considérée comme un inducteur de l'augmentation locale de la contrainte due à la présence du trou. Ce facteur a été étudié en fonction de quelques paramètres comme la variation du rapport elliptique (a/b), ainsi que l'angle β de sollicitation de traction par rapport à la direction de grand axe de l'ellipse, et la variation de l'orientation des fibres du matériau composite.*

KEYWORDS: *composite material, plates with a hole, elliptical hole, stress distribution, stress concentration factor.*

MOTS-CLÉS : *matériau composite, plaques trouées, trou elliptique, distribution des contraintes, facteur de concentration des contraintes.*

DOI: 10.3166/rcma.2017.00020

1. Introduction

Composite materials are increasingly used in the manufacturing of parts of structures where lightness and rigidity are required such as aeronautical and aerospace construction, maritime construction, sport…

The presence of elliptical holes can lead to increased local stress. This phenomenon, called stress concentration, is a hazardous phenomenon, which causes damage to metal or composite structures, which negatively affects their rigidity. To study the variation of this stiffness, it is necessary to determine the stress concentration factor due to the presence of the holes. This study deals with the analytical and numerical evaluation by the finite element method of the effect of the presence of an elliptical hole positioned at the center of a plate of composite materials on the stress concentration factor in the Case of pure tensile.

The variation of the stress concentration factor will be studied as a function of some parameters such as the elliptic ratio (a/b), the tensile stress angle with respect to the direction of the major axis of the ellipse, Angle of fiber orientation.

The research of Timoshenko and Goodier (1988) and Forest *et al.* (2009-2010) studied the case of rectangular isotropic plates equipped with an elliptic hole stressed in simple traction. These analytical or numerical studies indicate that the state of stresses is not homogeneous in the vicinity of the hole. Several research studies have studied the case of composite plates equipped with a circular hole and stressed in tension and simple compression. The theories of Lekhnitskii (1968) and Savin (1968) are widely used by researchers to analyze the distribution of stresses in perforated orthotropic plates. The work of Laroze and Barran (1983) and Sharma (2011) is based on solving the equations of the Lekhnitskii complex variable method in order to determine the field of stresses at the edge of Elliptical hole. The two works of Boubeker and Hecini (2015a,b) that the state of stress away from the hole is not affected by the presence of a hole and can therefore be assimilated to the homogeneous state. In this study the analytical determination of the stress distribution around the holes will be made using the approaches proposed by Savin (1968) and Lekhnitskii (1968). The numerical solution will be obtained by the finite element method using the ANSYS software. In this study, we consider the plane state of stress, and that the dimensions of the plate are sufficiently large with respect to the dimensions of the elliptical hole. Volumetric forces are considered negligible.

2. Behavior of isotropic plates with a centered elliptical hole

Let a plate of dimensions supposed to be sufficiently large with respect to the dimensions of the hole, provided with a hole of elliptic cross-section characterized by the parametric dimensions a (semi-major elliptic axis) and b (half elliptic minor axis) $0<b<a$ (figure 1). The plate is subjected to these two ends to a state of pure traction $\sigma\infty$ in a direction making an angle β with the axis 1 coinciding with the half major axis of the ellipse.

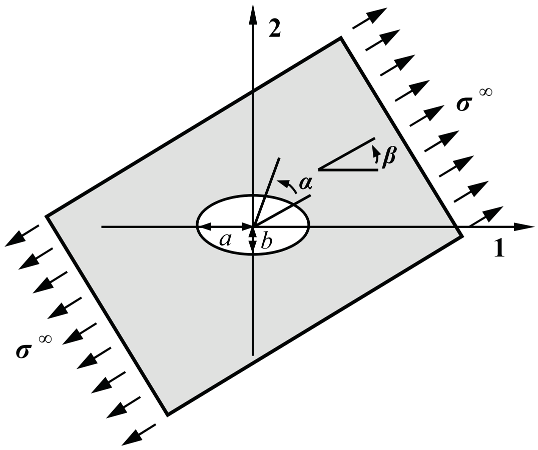

Figure 1. Plate provided with an elliptical hole and subjected to a tensile stress of intensity σ^{∞} inclined by an angle β with respect to the major axis of the ellipse a

2.1. Stress at the edge of the elliptical hole

The Stress $\overline{\sigma}$ at the edge of the ellipse is characterized by the angle α given by the following relation (Forest *et al*., 2009-2010):

$$\overline{\sigma} = \sigma^{\infty} \frac{1 - m_e{}^2 + 2m_e \cos 2\beta - 2\cos 2(\alpha - \beta)}{1 - 2m_e \cos 2\alpha + m_e{}^2} \quad (1)$$

Or $m_e = \frac{a-b}{a+b}$: Parameter of the ellipse with $0 \leq m_e \leq 1$.

α (°/): Angle of the stress location in the vicinity of the hole, with respect to the direction of the traction stress.

2.1.1. Cases of a stress parallel to the major axis of the ellipse

In the case of the direction of the tensile stress parallel to the major axis of the ellipse in the direction of axis 1 ($\beta = 0$). By replacing ($\beta = 0$) in equation (1) we obtain:

$$\overline{\sigma} = \sigma^{\infty} \frac{1 - m_e^2 + 2m_e - 2\cos 2\alpha}{1 - 2m_e \cos^e 2\alpha + m_e^2} \quad (2)$$

Given the periodicity of $\overline{\sigma}$, the stress maximum and minimum are:

$$\overline{\sigma}^{min} = \overline{\sigma}(\alpha = 0) = -\sigma^{\infty} \quad (3)$$

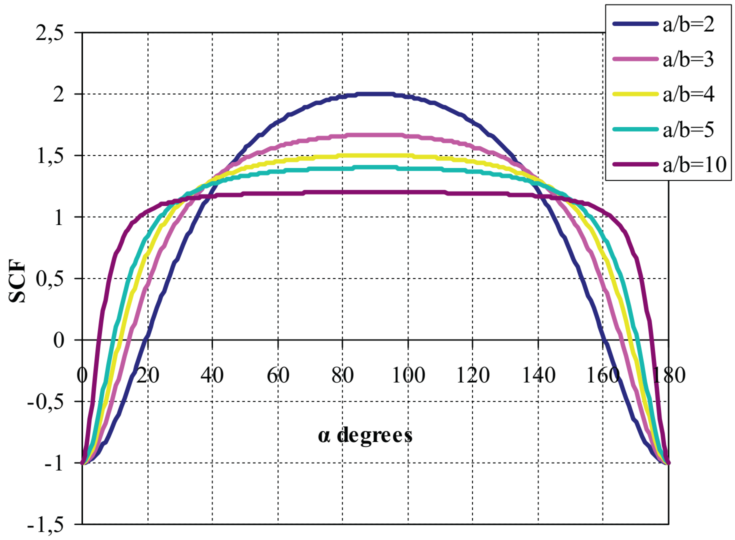

Figure 2. Distribution of the stress concentration factor around the elliptic hole Case of a stress parallel to the major axis of the ellipse (β = 0°)

$$\overline{\sigma}^{\max} = \overline{\sigma}(\alpha = \frac{\pi}{2}) = \frac{3 - m_e}{1 + m_e}\sigma^{\infty} = \left(1 + \frac{2b}{a}\right)\sigma^{\infty} \tag{4}$$

The stress is therefore maximum at the edge of the hole for (α = ± 90°) and minimal at (α = 0, 180°). The stress concentration factor (FCC) is thus:

$$K_t = \frac{3 - m_e}{1 + m_e} = \left(1 + \frac{2b}{a}\right) \tag{5}$$

It remains bounded for $0 \leq \text{me} \leq 1$, contrary to the expression (2) for traction direction the traction along axis 2. The stress concentration at α = 90° is less severe and remains less than 3.

The figure 2 and formula (5) show that for the values of the geometric parameters (a/b) greater than one, the stress concentration factor is always less than the value three which corresponds to the case of a circular hole. Increasing the elliptic ratio (a/b) decreases the maximum value of the stress concentration factor.

2.1.2. Case of a stress perpendicular to the major axis of the ellipse

In this case, it is considered that the traction is in direction 2 (figure 1) perpendicular to the major axis of the ellipse. So for β = 90°, the constraint at the edge of the hole will be given by formula (1) and is:

$$\overline{\sigma} = \sigma^{\infty} \frac{1 - m_e^2 - 2m_e + 2\cos 2\alpha}{1 - 2m_e \cos 2\alpha + m_e^2} \tag{6}$$

Given the periodicity of $\overline{\sigma}$, the stress maximum and minimum are:

$$\overline{\sigma}^{\max} = \overline{\sigma}(\alpha = 0) = \frac{3 + m_e}{1 - m_e}\sigma^{\infty} = \left(1 + 2\frac{a}{b}\right)\sigma^{\infty} \tag{7}$$

$$\overline{\sigma}^{min} = \overline{\sigma}(\alpha = \frac{\pi}{2}) = -\sigma^{\infty} \tag{8}$$

In the case of the elliptical single-tensile hole, the stress concentration factor is therefore:

$$K_t = \frac{\sigma_t^{\max}}{\sigma^{\infty}} = \frac{3 + m_e}{1 - m_e} = 1 + 2\frac{a}{b} \tag{9}$$

The ratio a/b being greater than 1, formula (9) and figure 3 show that this situation is very dangerous because the stress concentration factor proportional to the ratio a/b is always greater than 3.

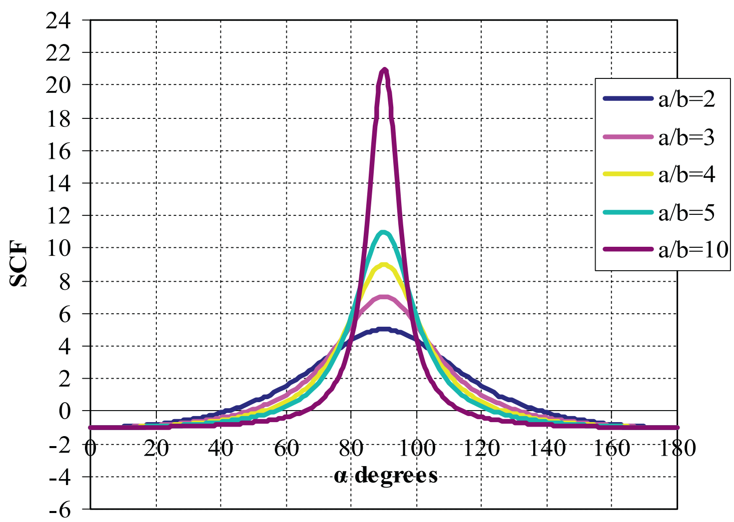

Figure 3. Distribution of the stress concentration factor around the elliptic hole Case of a stress perpendicular to the major axis of the ellipse ($\beta = 90°$)

3. Concentration of the stresses of a composite plate with an elliptical centered hole

The distribution of stresses around an elliptical hole in composite plates has been studied by several researchers (Lekhnitskii, 1968; Savin, 1968; Laroze and Barran, 1983)… They show that there is no exact analytical solution for this case, and that all proposed solutions are approximated. The theoretical study of the stress distribution at the edge and away from the elliptic hole centered in an infinite elastic orthotropic plate is based on the complex function method proposed by Lekhnitskii (1968), Savin (1968) and Laroze and Barran (1983). The purpose of this study is to determine the stress distribution in this plate. Given the limited conditions, it appears that this problem is a problem of plane elasticity and more particularly of plane stress.

3.1. Definition

The present study consists of studying the stress field around and away from the hole in a composite plate. In order to do this, we consider a plate of orthotropic materials, whose stiffness matrix expressed in the reference frame (0, x, y, z) has the component Qij. This plate is pierced at the center of an elliptic hole whose boundary has as equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. It is subjected to a simple tensile stress of intensity $\sigma\infty$ per unit area, in the direction forming an angle β with the axis (figure 4).

The resolution will be performed using the complex variable method. In this case, we consider that the tensile stress is parallel to the major axis of the ellipse ($\beta = 0°$). Thus, the field of constraints is obtained by using the following formulas (Laroze and Barran, 1983):

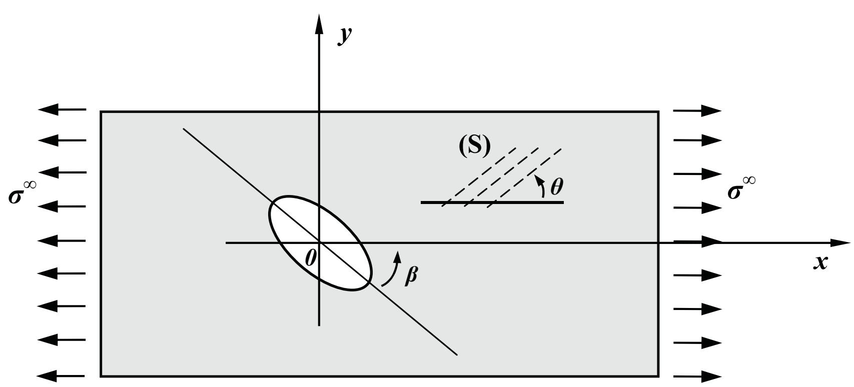

Figure 4. Orthotropic plate provided with an elliptical hole and subjected to tensile stress of intensity σ^∞ inclined at an angle β relative to the major axis of the ellipse

$$\begin{cases} Pour\, x = 0, \sigma_{xx} = \sigma^{\infty}\left[1 + \dfrac{b}{\beta_1 - \beta_2}\left[\dfrac{\beta_1^2}{a - \beta_1 b}\left(1 - \dfrac{\beta_1 y}{\sqrt{a^2 + \beta_1^2(y^2 - b^2)}}\right)\right.\right. \\ \qquad\qquad \left.\left. - \dfrac{\beta_2^2}{a - \beta_2 b}\left(1 - \dfrac{\beta_2 y}{\sqrt{a^2 + \beta_2^2(y^2 - b^2)}}\right)\right]\right] \\ Pour\, y = 0, \sigma_{yy} = -\dfrac{\sigma^{\infty} b}{\beta_1 - \beta_2}\left[\dfrac{1}{a - \beta_1 b}\cdot\left(1 - \dfrac{x}{\sqrt{x^2 - a^2 + \beta_1^2 b^2}}\right)\right. \\ \qquad\qquad \left. - \dfrac{1}{a - \beta_2 b}\left(1 - \dfrac{x}{\sqrt{x^2 - a^2 + \beta_2^2 b^2}}\right)\right] \end{cases} \tag{10}$$

Hence, on the edge of the hole:

$$\begin{cases} x = 0, y = b, \sigma_{xx} = \sigma^{\infty}\left[1 + (\beta_1 + \beta_2)\dfrac{b}{a}\right] \\ x = a, y = 0, \sigma_{yy} = -\sigma^{\infty}\dfrac{1}{\beta_1\beta_2} \end{cases} \tag{11}$$

with

$$\begin{cases} \beta_1 + \beta_2 = \sqrt{\dfrac{2Q_{12} + Q_{66}}{Q_{11}} + 2\sqrt{\dfrac{Q_{22}}{Q_{11}}}} = \sqrt{2\left(\sqrt{\dfrac{E_1}{E_2}} - \nu_{12}\right) + \dfrac{E_1}{G_{12}}} \\ \beta_1 - \beta_2 = \sqrt{\dfrac{Q_{22}}{Q_{11}}} = \sqrt{\dfrac{E_1}{E_2}} \end{cases} \tag{12}$$

and

$$\begin{cases} Q_{11} = \dfrac{E_1}{1 - \nu_{21}\nu_{12}} \\ Q_{12} = Q_{21} = \dfrac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} = \dfrac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{22} = \dfrac{E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{66} = G_{12} \end{cases}$$

The constraint at the edge of the hole is therefore worth:

$$\begin{cases} x = 0, y = b, (_{\sigma_{xx}})_{\max} = \sigma^{\infty}\left[1 + \left(\sqrt{2\left(\sqrt{\frac{E_1}{E_2}} - \nu_{12}\right) + \frac{E_1}{G_{12}}}\right)\frac{b}{a}\right] \\ x = a, y = 0, \sigma_{yy} = -\sigma^{\infty}\sqrt{\frac{E_1}{E_2}} \end{cases} \quad (13)$$

The stress concentration factor then takes the following form:

$$K_t^{\infty} = \left[1 + \left(\sqrt{2\left(\sqrt{\frac{E_1}{E_2}} - \nu_{12}\right) + \frac{E_1}{G_{12}}}\right)\frac{b}{a}\right] \quad (14)$$

3.2. Finite element modeling

The composite material plate of the study is rectangular in shape with an elliptical hole in the center and subjected to pure traction. The mechanical characteristics of the material used are given in table 1.

Table 1. Mechanical characteristics of the unidirectional composite material (Berthelot, 2010)

Material	E_1 (Gpa)	E_2 (Gpa)	G_{12} (Gpa)	v_{12}	v_{21}
Glass/epoxy	46	10	4.7	0.31	0.067

We used the ANSYS finite element method for the analysis of the stress field around and away from the elliptic hole.

The element used for the mesh is PLANE 82 two-dimensional, rectangular with eight nodes or triangular with six knots with four degrees of freedom per node (two translations and two rotations). According to the study of the convergence of the mesh, the size of the adapted element is 0,25 mm (figures 5 and 6). The boundary conditions and loads applied are those of a pure tensile test (ANSYS, 2014).

3.3. Effect of the orientation of the fibers on the distribution of the stress concentration factor

The aim of this study is to determine the value and location of the maximum and minimum stress concentration factor at the edge of the elliptic hole as a function of the

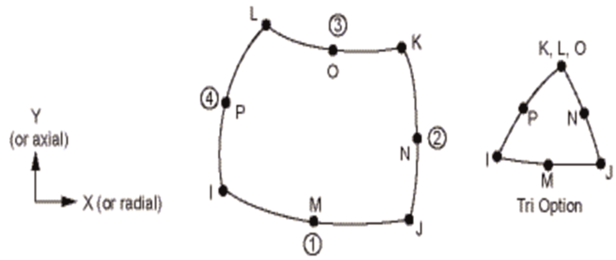

Figure 5. Planar element geometry 82

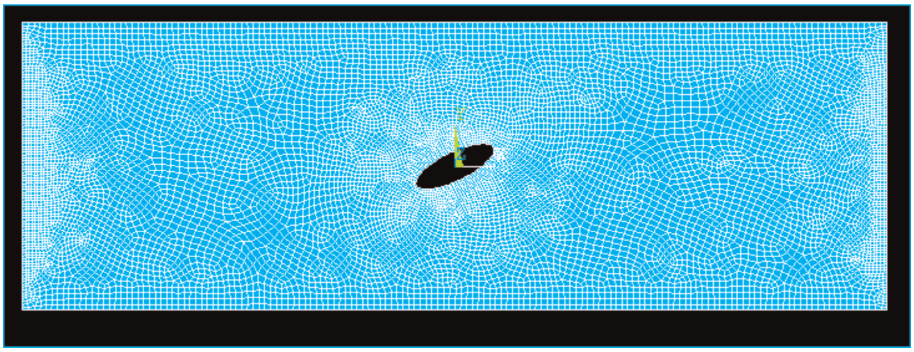

Figure 6. Finite element model of the perforated plate

elliptic ratio (*a/b*) and the different orientations of the fibers. The results are obtained by the formula (14) and the finite element method by ANSYS.

The figures 7 and 8 show respectively the cases of a stress parallel and perpendicular to the major axis of the ellipse ($\beta = 0°$).

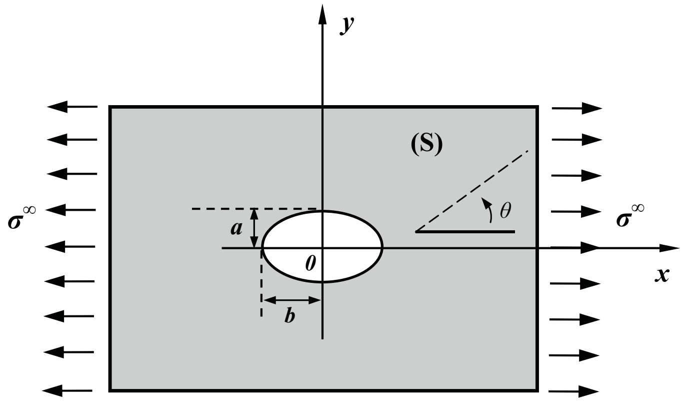

Figure 7. Case of a stress parallel to the major axis of the ellipse ($\beta = 0°$)

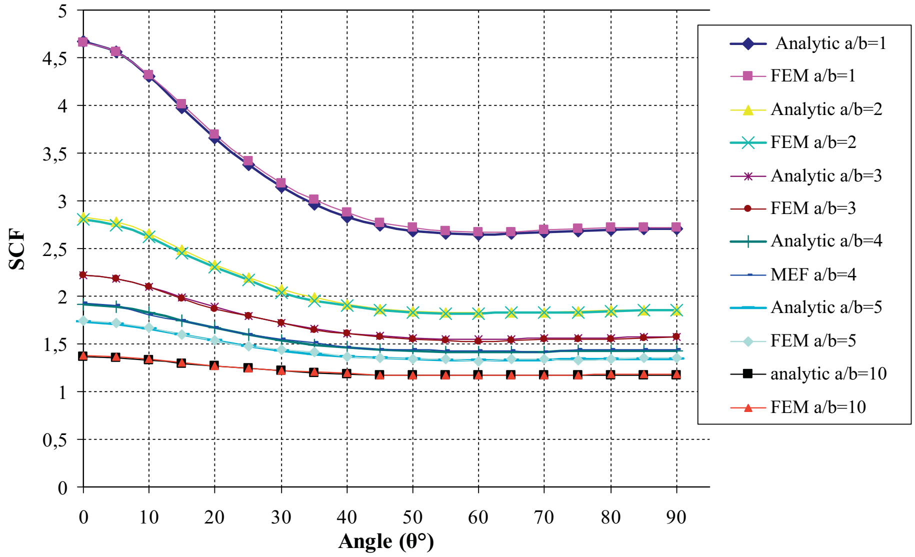

Figure 8. Variation of the maximum SCF as a function of the orientation of the fibers for the tensile stress parallel to the major axis of the elliptical hole ($\beta = 90°$)

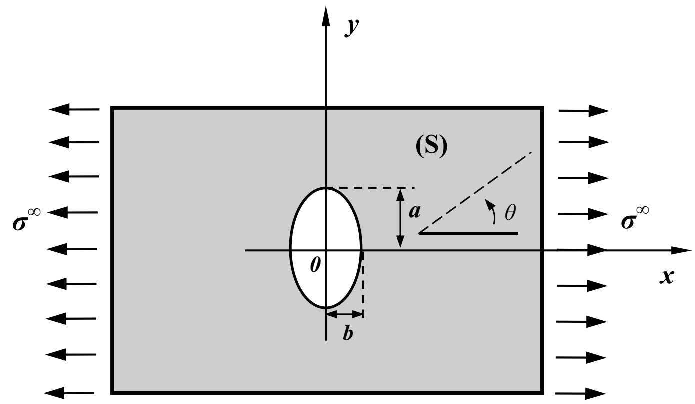

Figure 9. Case of a stress perpendicular to the major axis of the ellipse $\beta = 90°$.

The results of the curves of figures 8 and 10 respectively show the variation of the maximum stress concentration factor as a function of the orientation angle of the fibers for the two tensile stresses parallel and perpendicular to the major axis of the ellipse. These two cases show that the maximum value of the stress concentration factor is

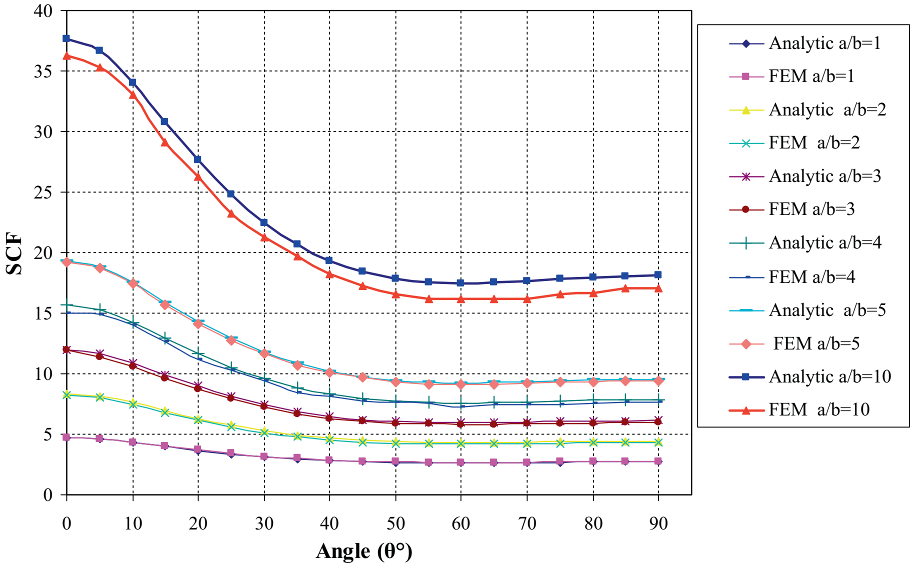

Figure 10. Variation of the maximum SCF as a function of the orientation of the fibers for the tensile stress perpendicular to the major axis of the elliptic hole ($\beta = 90°$)

located in the plate whose fiber orientation is at 0° and the minimum value is in the orientation Plate 60°. For the case of traction parallel to the major axis of the ellipse ($\beta = 0°$), we note that in the different cases of orientation of the fibers when the elliptic ratio (a/b) is increased the concentration factor of ($\beta = 90°$) when the elliptic ratio (a/b) is increased, the stress concentration factor increases. In the case of the tensile force perpendicular to the major axis of the ellipse, it should also be noted that the increase in the elliptic ratio (a/b) decreases the influence of the orientation angle on the maximum value of stress concentration of α in the case of ($\beta = 0°$) and that this influence increases in the case of ($\beta = 90°$). The results obtained by the semi-empirical formulas and the results obtained by finite elements are very close.

Tables 2–4 show the tilting pull direction of angles β (°/) of 30°, 45° and 60° with respect to the major axis of the elliptical hole. The analysis of these tables confirms the influence of the angle of inclination β on the value and the location of the maximum stress concentration factor at the edge of the elliptic hole. Figures 11–13 show the influence of the elliptic ratio (a/b), the angle of orientation θ and the angle of inclination θ on the distribution of the stress concentration factor.

Table 2. Variation of the maximum stress concentration factor under tensile stress in a direction making an angle $\beta = 30°$ with respect to the major axis of the ellipse

Angle (θ°)	a/b = 1 (circular hole)	a/b = 2	a/b = 3	a/b = 4	a/b = 5	a/b = 10
0	4.659	4.045	4.410	5.035	5.323	9.425
5	4.564	3.979	4.341	4.973	5.284	9.296
10	4.318	3.871	4.214	4.708	5.429	8.951
15	4.011	4.543	4.988	5.744	5.985	10.72
20	3.696	3.367	3.741	4.331	4.873	7.982
25	3.421	3.167	3.545	4.087	4.712	7.498
30	3.186	2.989	3.364	3.864	4.561	7.064
35	3.012	2.841	3.208	3.671	4.626	6.695
40	2.877	2.737	3.114	3.619	4.239	6.396
45	2.773	2.653	3.025	3.513	4.109	6.196
50	2.721	2.592	2.957	3.392	4.141	6.003
55	2.689	2.555	2.912	3.359	4.087	5.895
60	2.667	2.542	2.874	3.326	4.047	5.834
65	2.672	2.522	2.841	3.291	4.021	5.808
70	2.691	2.538	2.863	3.309	3.939	5.804
75	2.703	2.531	2.839	3.226	3.985	5.811
80	2.714	2.547	2.842	3.208	3.973	5.821
85	2.721	2.540	2.844	3.211	3.965	5.828
90	2.714	5.541	2.844	3.211	3.962	5.831

Table 3. Variation of the maximum stress concentration factor under tensile stress in a direction making an angle $\beta = 45^\circ$ with respect to the major axis of the ellipse

Angle (θ°)	a/b = 1 (circular hole)	a/b = 2	a/b = 3	a/b = 4	a/b = 5	a/b = 10
0	4.659	5.209	6.450	7.709	8.738	10.801
5	4.564	5.129	6.361	7.616	8.631	10.709
10	4.318	4.942	6.073	7.271	8.967	10.455
15	4.011	5.941	7.438	8.628	9.941	12.325
20	3.696	4.322	5.425	6.681	7.505	9.668
25	3.421	4.079	5.085	6.343	7.352	9.231
30	3.186	3.769	4.746	6.018	6.667	8.811
35	3.012	3.573	4.475	5.744	6.314	8.427
40	2.877	3.433	4.258	5.517	6.022	8.094
45	2.773	3.325	4.108	5.338	5.795	7.820
50	2.721	3.246	4.039	5.204	5.629	7.611
55	2.689	3.195	3.976	5.122	5.521	7.458
60	2.667	3.169	3.991	4.855	6.066	7.356
65	2.672	3.151	3.873	5.017	5.422	7.293
70	2.691	3.146	3.875	4.997	5.409	7.255
75	2.703	3.146	3.885	4.986	5.406	7.232
80	2.714	3.145	3.895	4.979	5.405	7.216
85	2.721	3.192	3.969	4.864	5.883	7.207
90	2.714	3.147	3.904	4.973	5.404	7.203

Table 4. Variation of the maximum stress concentration factor under tensile stress in a direction making an angle $\beta = 60°$ with respect to the major axis of the ellipse

Angle (θ°)	a/b = 1 (circular hole)	a/b = 2	a/b = 3	a/b = 4	a/b = 5	a/b = 10
0	4.659	6.102	8.275	09.404	7.921	13.607
5	4.564	6.228	7.941	10.172	9.297	13.502
10	4.318	5.994	7.666	10.240	10.273	13.231
15	4.011	7.023	9.554	10.812	10.631	14.772
20	3.696	5.188	7.082	8.113	7.233	12.535
25	3.420	4.832	6.528	8.386	9.877	12.228
30	3.186	4.568	6.243	7.191	6.675	11.971
35	3.012	4.315	5.899	6.792	6.417	11.753
40	2.877	4.111	5.618	6.456	6.199	11.568
45	2.773	3.958	5.401	6.194	6.120	11.404
50	2.721	3.853	5.247	6.004	6.048	11.254
55	2.689	3.790	5.148	5.881	5.984	11.115
60	2.667	3.761	4.991	6.425	7.453	10.989
65	2.672	3.750	5.069	5.776	5.881	10.875
70	2.690	3.753	5.065	5.766	5.841	10.778
75	2.703	3.761	5.068	5.767	5.809	10.697
80	2.714	3.792	5.074	5.771	5.786	10.635
85	2.721	3.799	5.077	5.772	7.488	10.596
90	2.714	3.774	5.078	5.772	5.767	10.583

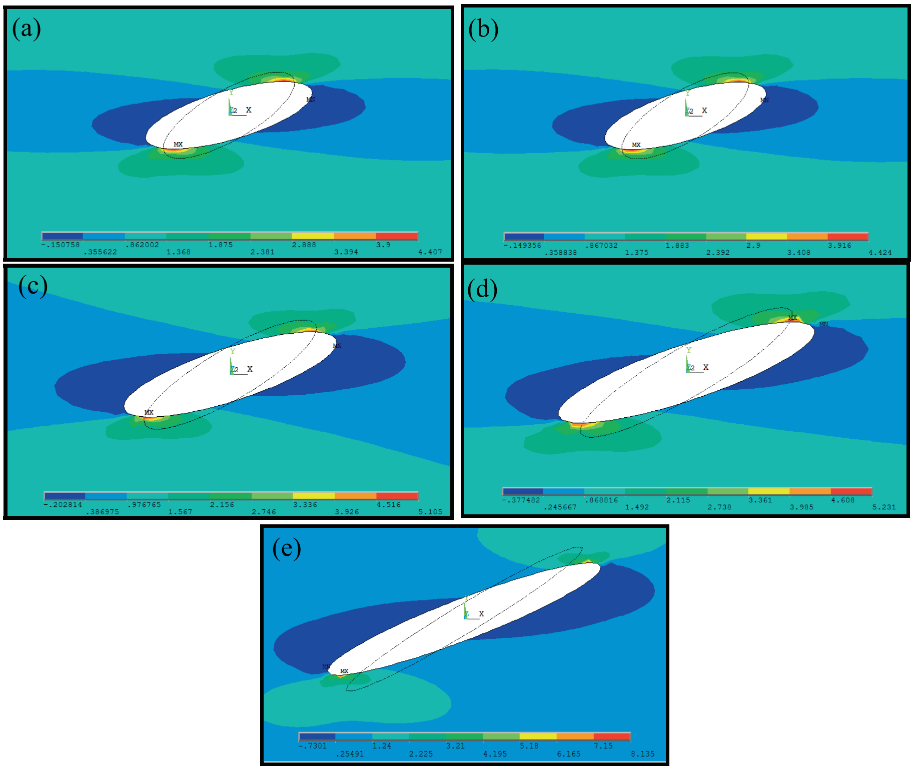

Figure 11. Distribution of the SCF for the angle of tensile stress $\beta° = 30°$ and the angle of orientation of the fibers $\theta = 0°$, and for different values of the elliptic ratio a/b. (a) a/b = 2, (b) a/b = 3, (c) a/b = 4, (d) a/b = 5, (e) a/b = 10

4. Conclusion

This study revealed the disturbance of the stress field in a sheet of composite material subjected to pure traction caused by the presence of an elliptical hole in its center. This perturbation is expressed by the stress concentration factor.

This work was introduced by the case of the isotropic plate. In this case, the dangerous situation corresponds to the direction of the tensile stress perpendicular to the major axis of the ellipse.

In the case of composite plates, this study showed the influence of the elliptic ratio (a/b), the angle of orientation θ and the angle of inclination θ on the distribution of the stress concentration factor and the location of its maximum value.

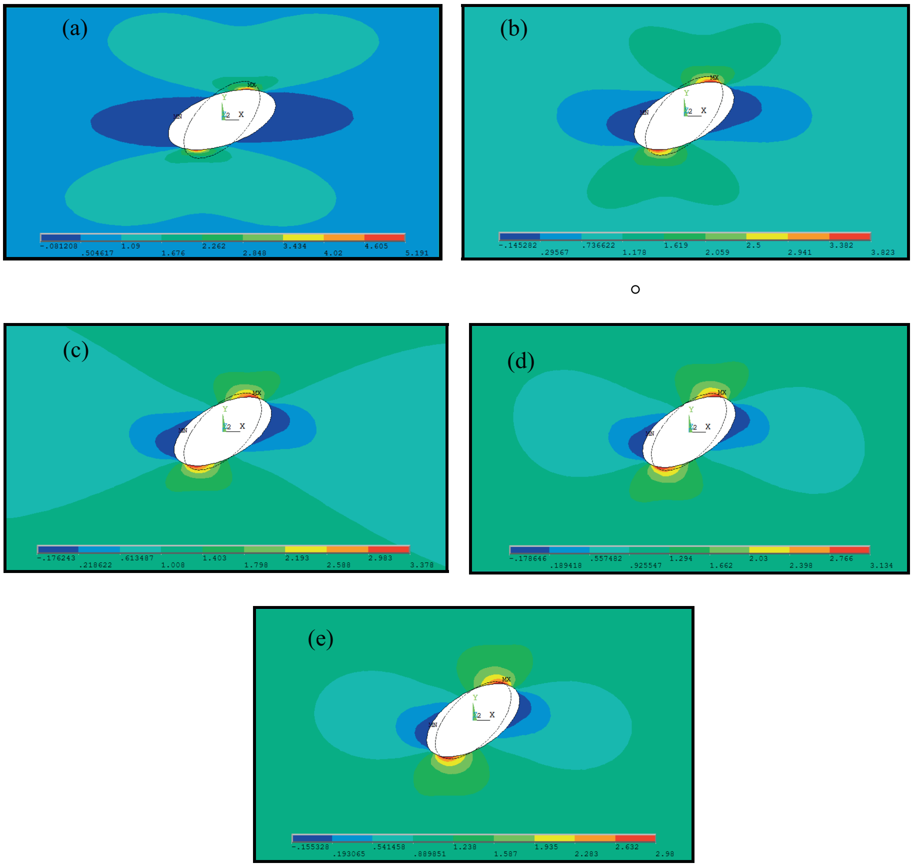

Figure 12. Distribution of the SCF for the tensile stress angle $\beta° = 45°$ and for the elliptic ratio $a/b = 2$ and for different values of the angle of orientation of the fibers θ. (A) $\theta = 0°$, (b) $\theta = 30°$, (c) $\theta = 45°$, (d) $\theta = 60°$, (e) $\theta = 90°$

In all cases, increasing the angle of inclination increases the stress concentration factor. Its maximum and minimum values are located in the orientation Plates 0° and 60° respectively. When the elliptic ratio increases, the stress concentration factor decreases in the case of the stress parallel to the major axis of the ellipse and decreases in the case of the stress perpendicular to the major axis.

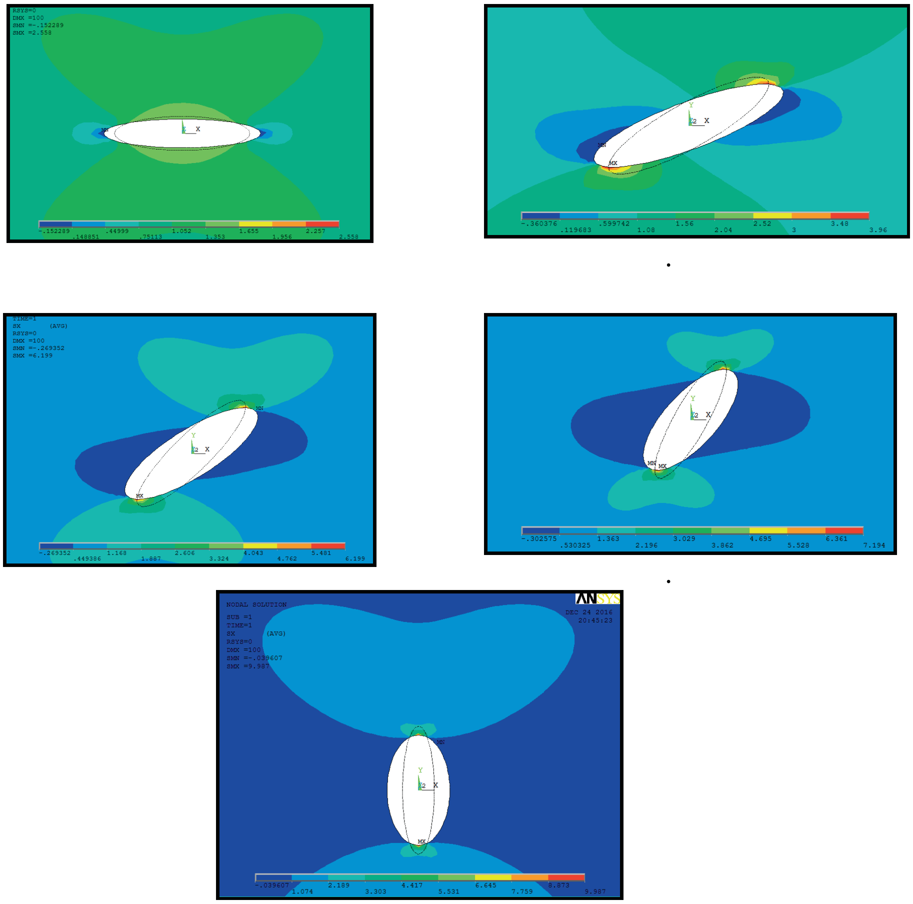

Figure 13. Distribution of the SCF for fiber orientation θ° = 30°, and for the elliptic ratio a/b = 4. And for different tensile stress angle values β. (A) β = 0°, (b) β = 30°, (c) β = 45°, (d) β = 60°, (e) β = 90°

Bibliographie

ANSYS Help System. (2014). *Analysis Guide and Theory*. Reference, version 12.1.

Berthelot J.-M. (2010à. *Matériaux composites*. Éditions Eyrolles.

Boubeker R., Hecini M. (2015). Analyse de la concentration des contraintes dans les plaques orthotropes munies d'un trou circulaire. *Revue des composites et des matériaux avancés*, vol. 5, n° 1, p. 47-68.

Boubeker R., Hecini M. (2015). Étude analytique et numérique de la concentration des contraintes dans les plaques composites avec trou circulaire. *Revue des composites et des matériaux avancés*, vol. 25, n° 3-4, p. 1-20.

Laroze S., Barran J.J. (1983). *Mécanique des structures, Tome 4 : Calcul des structures en matériaux composites*. Eyroues, Masson.

Lekhnitskii S.G. (1968). *Anisotropic plates*, translated from the 2nd Russian Edition Bay. Tsai S. W., Cheron T. Gordon, Breach Science Publishers Inc. N4.

Forest S., Amestoy M., *et al*. (2009-2010). *Mécanique des milieux continus*. École des mines, Paris.

Savin G.N. (1968). *Stress distribution around Holes, NASA Technical translation*. “Naukova, Dumka” Press, Kiev.

Sharma D.S. (2011). Stress Concentration around circular/elliptical/triangular cutouts in infinite composite plat. *Proceedings of the world Congress on Engineering*, London

Timoshenko S., Goodier J.N. (1988). *Theory of Elasticity*. Mc Graw Hill Book Co, New York.