
Physical modeling and deformation simulation of flexible cable under the plane constraint

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ABSTRACT. *The flexible deformation is often neglected in cable modelling on CAD software. To solve the problem, this paper proposes a physical modeling and deformation simulation method for flexible cable under plane constraint based on the nonlinear statics theory of elastic rod. Specifically, a statics equation of the balanced cable was established under plane constraint in the cylindrical coordinate system. Taking the Euler angles as the generalized parameter, the cable space configuration was described through dynamic analogy by the semi-analytical method and the analytical expression in the form of elliptic integral. Then, the simulated cable shapes were compared with the actual shapes under different material parameters and constraint conditions. The results show that the proposed model and numerical solution can fully describe the deformation features of the cable under the plane constraint in an accurate and efficient manner. The research results lay a solid basis for the CAD in cable production.*

RÉSUMÉ. *La déformation flexible est souvent négligée dans la modélisation de câbles par le logiciel de CAD. Afin de résoudre ce problème, cet article propose une méthode de modélisation physique et de simulation de déformation pour câble flexible sous contrainte plane basée sur la théorie statique non linéaire de la tige élastique. Plus précisément, une équation statique du câble équilibré a été établie sous contrainte plane dans le système de coordonnées cylindriques. En prenant les angles d'Euler comme paramètre généralisé, la configuration de l'espace du câble a été décrite par analogie dynamique en appliquant la méthode semi-analytique et l'expression analytique sous forme d'intégrale elliptique. Ensuite, les formes de câbles simulées ont été comparées aux formes réelles sous différents paramètres de matériau et conditions de contrainte. Les résultats montrent que le modèle et la solution numérique proposés peuvent décrire de manière précise et efficace les caractéristiques de déformation du câble sous la contrainte plane. Les résultats de la recherche jettent une base solide pour la CAD dans la production de câbles.*

KEYWORDS: *plane constraint, flexible cable, elastic rod theory, semi-analytical method, deformation simulation.*

MOTS-CLÉS: *contrainte plane, câble flexible, théorie de la tige élastique, méthode semi-analytique, simulation de déformation.*

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1. Introduction

As the complex products such as automobiles and aircraft develop towards mechatronics, the proportion of cables in products is increasing, and the design and assembly of cables become the key factors affecting product quality (Xia *et al.*, 2013). However, traditional cable design and assembly methods are both based on a serial method, which leads to the problem of over-design of cable length and inaccurate routing path.

With the development of computer engineering, graphics and numerical calculation, higher and higher requests have been put forward to computer aided design of electromechanical products. At present, although the design of cable can be modeled through spline curve fitting using CAD software, cables are still considered rigid body. In addition, the actual wiring scheme can only be determined by experience, mode installation and multiple actual wiring, and the optimal scheme cannot be obtained in the design stage. As a result, the reliability of cable in electromechanical products is poor during its usage. Such problems as bending damage, gripping, fatigue, shell abrasion, and entanglement with protruding structural parts frequently occur to cables.

The physics-based modeling can take a full consideration of the physical properties of cable such as materials, quality and stress, and can fully reflect the essential law of real cable movement

combining the computer simulation technology. It has important theoretical significance and practical value for the optimization design of products (Hermansson *et al.*, 2016). Because of the flexible deformation of cable, the modeling based on physical properties is a very challenging problem. The key technologies involved mainly include the mechanics modeling method and efficient and accurate numerical calculation.

In recent years, many scholars both at home and abroad have simulated the geometrical configuration of the cable through different mechanical modeling methods and achieved good results. Gilles B conducted cable modeling based on the thought of finite element and used the frame model to simulate the cable shape, which can check the strain of the cable when it is deformed (Gilles *et al.*, 2011). Klimowicz & Mihajlović, (2007). used a spline curve to describe the geometric shape of the centerline and carried out physical modeling of piping combining the thought of energy optimization. Servin & Lacoursière, (2008). analyzed the cable deformation during the cabling process based on the idea of multi-body hybrid dynamics system. Shang W proposed the assembly simulation of cable using the mass-spring method (Wei *et al.*, 2012). The nonlinear mechanical method of elastic rod is considered to be the most effective modeling method for one dimensional flexible parts such as cable. It can meet the requirements of computer aided design and assembly of cable in both precision and efficiency. There are mainly two theories: Kirchhoff theory and Cosserat theory. Mireille Gregoire earlier applied the Cosserat theory to cable laying in a virtual environment and determined the equilibrium position of the cable using the energy optimization principle. This

model can be used in the basic wiring planning and strength checking of cables (Grégoire and Schömer, 2007). Based on Cosserat theory, Hermansson T put forward a simulation model of cable geometry, which considers the position constraint of cable joint. Based on the principle of energy minimization, the position of centerline is calculated and the configuration simulation of the cable in the vehicle door is carried out (Hermansson *et al.*, 2013). According to Kirchhoff's elastic rod theory, J. Linn described the cable configuration of assembly simulation in a virtual environment using energy. The conjugate gradient method is used to numerically solve the equations, with the distance between the two ends and torsional angle as the parameters (Lann *et al.*, 2011).

In the study of the physical modeling of cables, most scholars only consider the situation that the position of both ends of the cable are known, without considering the actual constraints of cables. In fact, cables are often subjected to complex constraints such as plane and surface contact in product. In this paper, the physical modeling and deformation simulation of flexible cable under plane constraint are studied.

Aiming at the particular object of cables under plane constraint, this paper artfully obtained the analytical integral expression of Kirchhoff equation in the cylindrical coordinate system when the constraints at both ends are already known. The Euler angles are taken as the generalized parameter and analytical solution in the form of elementary function is provided. This paper also develops the deformation simulation module of cable under the plane constraint based on existing virtual assembly simulation platform and verifies the validity of the model and algorithm proposed in this paper.

2. Spatial constraint cable modeling

For the curve cable in space, it is assumed that the cross section is a circular section, and the cable material satisfies the linear constitutive relation. The distributed force and contact force imposed to the cable is not considered.

Arc coordinate s of the cable centerline (all variables in this paper are a function of the arc coordinate s) is established, and the spatial configuration of the cable is regarded as consisting of the shape of the centerline and the rotation angle of the cross section relative to the centerline. The centerline is expressed with curvature and torsion. The following three coordinates are established: reference rectangular coordinate system ($O-\xi\eta\zeta$), Frenet coordinate system (P-NBT) and principal axis coordinate system (P-xyz). The geometry configuration of the cable in space can be regarded as the sum of the rotation of the principal axis coordinate system around the Frenet coordinate system and the rotation of Frenet coordinate system based on the fixed reference system. The twisting vector is introduced to describe the curvature of the space cable. The physical significance is the change rate of the infinitesimal angular displacement of the cable cross-section relative to the fixed coordinate system to the arc coordinate s .

The following assumptions are made about the cable:

- 1) The cable material is uniform isotropy and the material coefficient is a constant. The stress and strain satisfy the linear constitutive relation;
- 2) For the time being, the distributed force such as the body force of the cable and the gravitation or contact force between cables are not considered;
- 3) The cable is straight in an unrestrained relaxation state and its original curvature and torsion are not considered;

Based on the basic balance equation of rigid body force and moment, the statics equations of the cable balance described by Euler Angles are obtained, i.e. Kirchhoff equations, which are shown as follows.

$$\begin{cases} A \left[\frac{d^2 \beta}{ds^2} - \left(\frac{d\alpha}{ds} \right)^2 \cos \beta \sin \beta \right] + \\ C \frac{d\alpha}{ds} \sin \beta \left(\frac{d\gamma}{ds} + \frac{d\alpha}{ds} \cos \beta \right) - F \sin \beta = 0 \\ A \left[\frac{d^2 \alpha}{ds^2} \sin \beta + 2 \left(\frac{d\alpha}{ds} \right) \left(\frac{d\beta}{ds} \right) \cos \beta \right] - \\ C \frac{d\beta}{ds} \left(\frac{d\gamma}{ds} + \frac{d\alpha}{ds} \cos \beta \right) = 0 \\ C \frac{d}{ds} \left(\frac{d\gamma}{ds} + \frac{d\alpha}{ds} \cos \beta \right) = 0 \end{cases} \quad (1)$$

Where, $\alpha(s)$, $\beta(s)$ and $\gamma(s)$ are the Euler angles that describe the relative rotation of the dynamic and static coordinate system. A, B and C are the bending stiffness and torsional stiffness of the cable section respectively, and F is the force on the cable end.

To sum up, as long as the force F on the cable end, the initial values of Euler angles and the initial value of the derivative of Euler angles are given, the law of change of the three Euler angles $\alpha(s)$, $\beta(s)$ and $\gamma(s)$ with the arc coordinate can be obtained from the integral of formula (1). Furthermore, it is possible to determine the change rule of the posture of an arbitrary cross section of the cable along the arc coordinates. Finally, the spatial configuration of the cable is determined.

However, it is found in the actual analytic solving process that the analytical integral of Euler Angle is easy to obtain, but the transformation from the analytic integration of Euler angles to the integration of the spatial position coordinates of cable is hard to achieve. In view of this problem, the cylindrical coordinate system (ρ, ψ, Z) is introduced to describe the spatial pose of the cable.

According to the Saint-Venant principle, the cylindrical coordinate system is established by boundary force constraints at both ends to replace the original rectangular coordinate system. The cylindrical coordinate system is established according to the following principles. The principles for the foundation of the

coordinate origin O are determined by the end constraints (F_0, M_0) and (F_L, M_L) , which are shown in Figure 1.

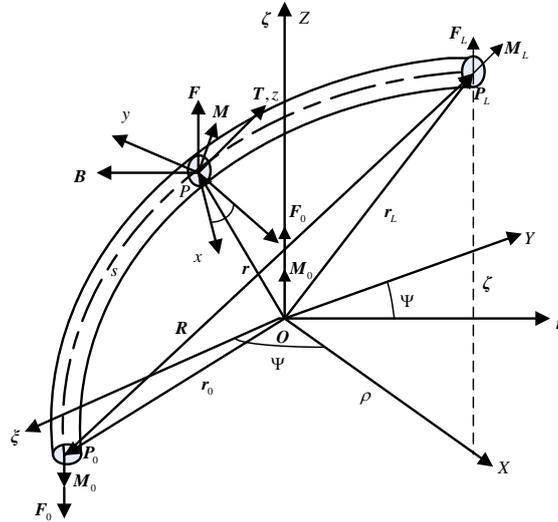


Figure 1. Cable VA system framework

The analytic expressions of the three cylindrical coordinates describing the center line of the curved cable can be obtained based on the basic idea of variable substituting integral and by virtue of the mathematical calculation software, which are shown as follows:

$$\rho(s) = \frac{2}{p} \sqrt{a - p\lambda_1} \left\{ 1 - \frac{p(\lambda_2 - \lambda_1)}{a - p\lambda_1} \times \text{JacobiSN}^2[\Omega(s - s_0), k] \right\}^{1/2} \quad (2)$$

$$\psi(s) = \psi_0 + \frac{l}{2}(s - s_0) - \frac{al - mp}{2\Omega(a - p\lambda_1)} \times \text{EllipticPi} \left[\Omega(s - s_0), \frac{p(\lambda_2 - \lambda_1)}{a - p\lambda_1}, k \right] \quad (3)$$

$$Z(s) = Z_0 + \lambda_3(s - s_0) - 2\sqrt{\frac{\lambda_3 - \lambda_1}{p}} \text{EllipticE} \{ am[\Omega(s - s_0), k] \} \quad (4)$$

Where, a, p, λ_i, m, l and Ω are integral constants which are introduced for the simplicity of the expression. They are related to the material parameters of the cable

and the size of the boundary constraining force. Please refer to literature for details. Although Formula (2-4) present the analytic expressions of the three cylindrical coordinates describing the form of the cable centerline, but the form of the expression is not elementary function. It is necessary to further simplify the expressions according to the constraints to the cable. In literature, the deformation morphological analysis of the symmetric constrained cable is studied. This paper focuses on the cable under plane constraint.

3. Solution of configuration of cable under plane constraint

For the cable under completely plane constraint, it is assumed that the center line of the cable is the plane curve passing the Z-axis of the fixed coordinate system, and then the rate of change of the cylindrical coordinate Ψ to the arc coordinate is zero. Formula (5) can be inferred from formula (3).

$$\begin{cases} \frac{d\Psi}{ds} = \frac{p}{2} \left(\frac{m - l\lambda(s)}{a - p\lambda(s)} \right) = 0 \\ m = l = 0 \end{cases} \quad (5)$$

In the physical sense, the component of the applied moment of the cross-section on Z-axis in the fixed coordinate system and in the principal-axis coordinate system Z_1 is both zero, i.e. only when the two ends of the cable are subjected to the independent effect of the axial force F_0 , will the cable be plane bending in two dimensions. Formula (6) can be obtained through inference.

$$\frac{d^2\beta}{ds^2} - \frac{p}{2} \sin \beta = 0 \quad (6)$$

According to the above analysis, the connection line between the two endpoints of the cable can be taken as the Z-axis of the fixed coordinate system, and the midpoint of the connection line is viewed as the origin of the fixed coordinate system. Because of the symmetry, the center line of the cable is symmetrical to the middle point C, and the tangent line axis at the C point is parallel to the Z-axis. The Euler angle β corresponding to C is equal to 0.

The coordinate system is established as shown in Figure 2. Let $\Psi = \pi/2$, then the X_3 axis of the cylindrical coordinate system coincides with the Y-axis of the Cartesian coordinate system. According to equation (5), $a=h$. It is substituted into formula (2) and (4) and formula (7) can be inferred.

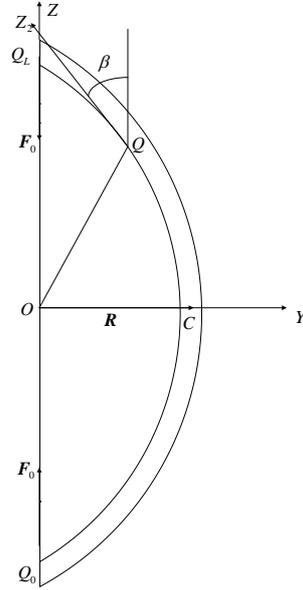


Figure 2. Establishment of coordinate system of cable under plane constraint

$$\frac{d\alpha}{ds} = \frac{d\gamma}{ds} = 0 \tag{7}$$

The above formula shows that the first and third Euler angles remain constant along the arc coordinates, that is, there is no torsion occurring to the cable under plane constraint in the plane (Y, Z), i.e. $\alpha=\gamma=0$. When the two ends of the cable are subjected to pressure, i.e. the acting force of the cross section is negative along the Z-axis, the integral constant p is negative, which is shown as follows:

$$p = -\frac{2F_0}{A} \tag{8}$$

The above equation is substituted to (4) and formula (8) is obtained. Let $\lambda=\cos\beta>0$.

$$\rho(\beta) = \frac{2}{|p|} \sqrt{h + |p| \cos \beta} \tag{9}$$

It can be seen that the maximum curvature radius R of the center line of the cable under plane constraint is at the end point C . The upper equation is simplified using the half Angle formula and the following equation is obtained:

$$\rho(\beta) = \sqrt{R^2 - \frac{8}{|p|} \sin^2 \frac{\beta}{2}} \quad (10)$$

According to equation (9), it can be learned that h is greater than or equal to $-p$, and the parameter k is introduced, which is defined as follows:

$$k^2 = \frac{1}{8}|p|R^2 = \frac{1}{2} \left(1 + \frac{h}{|p|} \right) \quad (11)$$

In order to solve the cylindrical coordinate ρ conveniently, the following variable substitution is carried out.

$$\sin \frac{\beta}{2} = k \sin \phi \quad (12)$$

Formula (10) can be changed to

$$\rho(\phi) = R \cos \phi \quad (13)$$

At the starting end Q_0 of the cable, because $\rho=0$, the corresponding variable $\Phi=\pi/2$ and the corresponding second Euler angle is recorded as β_0 , the following equation is obtained:

$$k = \sin \frac{\beta_0}{2} \quad (14)$$

Substitute formula (13) into (3) and (4) and the formula (15) is obtained:

$$ds = -\frac{d\rho}{\sin \beta} = \sqrt{\frac{2}{|p|}} \frac{d\phi}{\sqrt{1-k^2 \sin^2 \phi}} \quad (15)$$

The analytic solution of the first kind of elliptic integrals is found in the above formula. In addition, the change rule of the variable Φ with the arc coordinate can be confirmed, which is shown in formula (16) and (17). EllipticF is the first class of incomplete elliptic integral and JacobiAm is the amplitude of the elliptic function.

$$\Omega(s-s_0) = \text{EllipticF}(\phi, k) \quad (16)$$

$$\phi(s) = \text{JacobiAm}[\Omega(s-s_0), k] \quad (17)$$

In order to get the Z coordinate of an arbitrary point Q on the plane cable's center line, the formula (12) and (17) can be substituted to formula (5) and equation (18) can be obtained.

$$dZ = \frac{1 - 2k^2 \sin^2 \phi}{\Omega \sqrt{1 - k^2 \sin^2 \phi}} d\phi \quad (18)$$

When $\Phi = 0$, $Z = 0$. The above formula can be integrated and the following formula can be obtained:

$$Z(\phi) = \frac{1}{\Omega} \left[\int_0^\phi \frac{1}{\sqrt{1 - k^2 \sin^2 \phi}} d\phi - 2k^2 \int_0^\phi \frac{\sin^2 \phi}{\sqrt{1 - k^2 \sin^2 \phi}} d\phi \right] \quad (19)$$

i.e.

$$Z(\phi) = \frac{1}{\Omega} \left[2 \times \text{EllipticE}(\phi, k) - \text{EllipticF}(\phi, k) \right] \quad (20)$$

To sum up, because of the symmetry of the centerline of the cable under plane constraint, the endpoint C is selected as the starting point of the arc coordinate, and then $s_0=0$. In summary, for the cable under plane constraint, as long as the cable constraining force F_0 and the initial value of the Euler angle β_0 is provided, the integral constants k , p and R can be calculated under the condition that the material constant is already known. When these constants are substituted into formula (13), (17) and (20), the three cylindrical coordinates describing the spatial form of the cable under plane constraint are determined eventually.

4. Simulation and verification of the cable form under plane constraint

Based on the model and algorithm proposed in this paper, the wiring module of the cable under plane constraint is developed based on the strength of existing virtual assembly system. In addition, this method is very suitable for the interactive cable wiring application program. The geometric expression of the cable is realized based on Open Cascade, an open source geometric kernel system. The OpenGL engine is used for contour rendering. When the cable is drawn, the cable is skinned with b-spline interpolation surface mesh. Rigid parts are modeled in Pro/E and are introduced into this system in the format of *.step. Based on the above algorithm, the geometric shape of the cable is calculated. On the account of the Visual Studio development environment, the hybrid programming idea is adopted to achieve the purpose. Mouse and keyboard are the input devices to operate the cable.

The simulation module of the cable configuration under the plane constraint is shown in Figure 3.

The basic thought of cable configuration simulation is to obtain the specific expression of formula (4) based on the provided boundary condition and initial condition firstly, and then the numerical integration of three cylindrical coordinates of the cable space position is solved using the Gauss-Legendre quadrature formula. The obtained series of point coordinates are scanned by the one-dimensional pipelines provided by OCC to generate function, and then the actual spatial configuration of the cable is obtained.

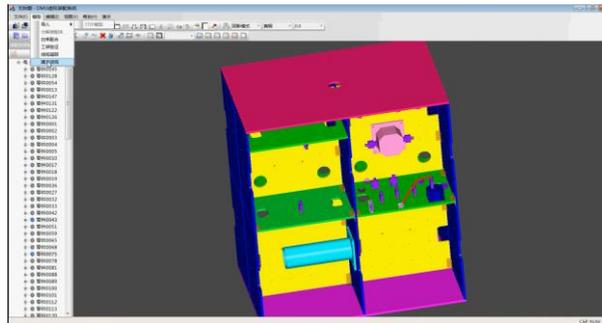


Figure 3. Simulation interface of cables under plane constraint

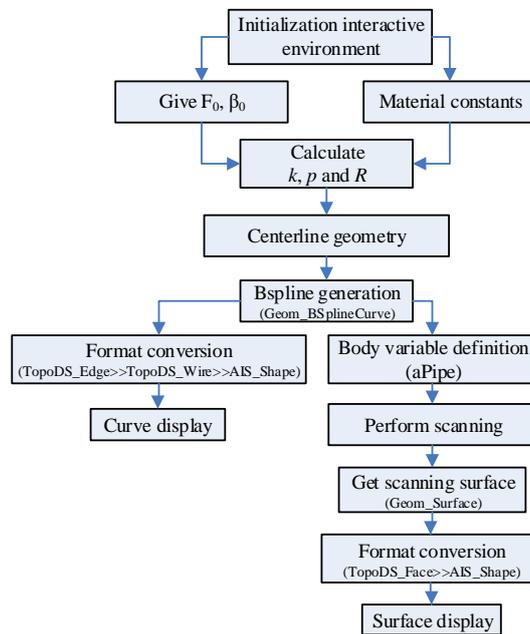


Figure 4. Development flow chart of cable module under plane constraint

This function first reads the spatial coordinates of the discrete points of a given cable series. The center line shape of the cable is obtained through using the B-spline curve to interpolate the adjacent nodes. Then, scanning and enveloping of the circle is carried out along the center line and the circle is of the actual size of the cross section. Finally, the three-dimensional model of the cable is obtained. The actual design process is shown in Figure 4.

In order to verify the model establishment proposed in the first and second part of the paper, the correctness of the algorithm is solved and simulated with the given cable parameters and initial boundary conditions as shown in Table 1.

Table 1. Material parameters for cable

Parameters	Value
Starting external Force, N	$(0,0,4.75 \times 10^{-3})$
End external Force, N	$(0,9.5 \times 10^{-3},0)$
Young's module, N/m ²	3.89×10^9
Passion ratio	0.25
Diameter, mm	4
Length, mm	350
Euler angle β_0 , degree	20

According to the given boundary conditions, the integration constant can be calculated, and the concrete expression of the centerline of the cable under plane constraint can be obtained. These three coordinates are the Cartesian coordinates under the reference coordinate system, which are shown as follows:

$$\begin{cases} x = 0 \\ y = 0.945 \text{Cos}[\text{JacobiAmplitude}[0.724 x, 0.342]] \\ z = 2.76 \text{EllipticE}[\text{JacobiAmplitude}[0.724 x, 0.342], 0.342] - \\ \quad 1.38 \text{EllipticF}[\text{JacobiAmplitude}[0.724 x, 0.342], 0.342] \end{cases} \quad (21)$$

The shape of the center line is shown in Figure 5.

In order to further illustrate the effectiveness of the proposed model and algorithm, aiming at the cable under plane constraint, the constraints to both ends are changed, and the deformation shape of the cable is calculated in real time to simulate the change rule of cable under plane constraint in the actual assembling process. The constraining force F_0 at both ends of the cable and the initial value of the Euler angle β_0 are changed, making the pressure on the plane cable increase constantly. The deformation process of the cable from straightness to the gradual rise of curvature is studied, which is shown in Figure 6.

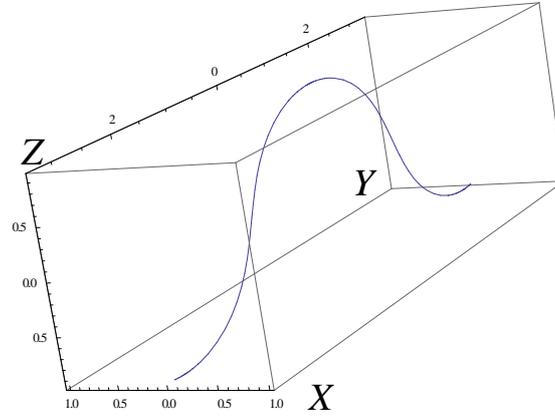


Figure 5. Shape of the center line of cable under plane constraint

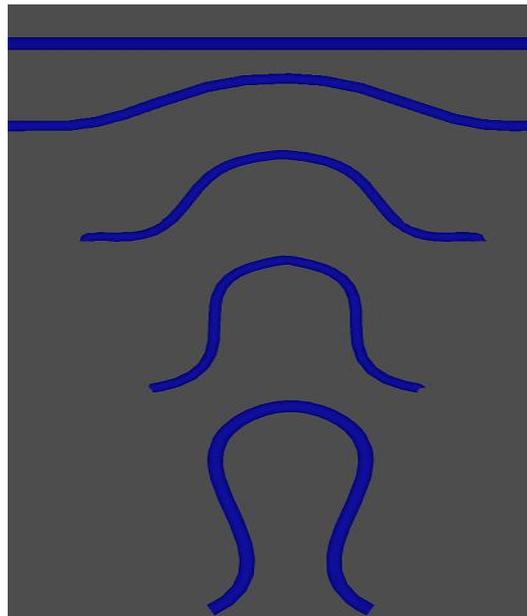


Figure 6. Dynamic change process of the cable under plane constraint

5. Concluding remarks

Considering the nonlinear mechanical properties of plane constrained cables in mechanical and electrical products, this paper proposes a physical modeling and

geometric shape simulation method of the cable under plane constraint based on the theory of nonlinear statics theory of elastic rod. On the basis of arc coordinate, the statics equations of the double-end constrained cables are described in the form of Euler angle. Skillfully following the Saint-Venant principle of elastic mechanics, this paper establishes the cylindrical coordinate system and obtains the analytical solution describing the geometric shape of the cables under plane constraint. The expression is in the form of elliptic integral, and the elliptic integral is calculated using the Gauss-Legendre quadrature formula. This method is efficient and simple and solves the difficulty of the transformation from arc coordinate to Cartesian coordinate for the cable under plane constraint and can ensure the accuracy and efficiency of the simulation of geometric shape. Finally, in view of the cables under plane constraints and based on the existing virtual assembly system, the cable geometry simulation module has been developed and the dynamic deformation pattern of the cable when the boundary conditions are changing is studied. The results show that the model and algorithm are effective enough.

The proposed method in this paper can be used as the physical modeling method of the cable geometry under different constraint conditions. The key is to solve the relation between the constraint condition and the integral constant. In the future, we will focus on more complex constraints, such as surfaces, overpasses, and clamps that appear during cable assembly. The results of this paper have strong guiding significance and reference value for the computer aided design and assembly verification of flexible cable.

Acknowledgements

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