A design method of preview controller for discrete-time systems with multiple input delays

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ABSTRACT. This paper presents a design method for preview controller of discrete-time linear systems with multiple input delays based on the discrete lifting technology for single input delay plants. The design approach is featured by the infinite dimensionality of the actuator dynamics and the discrete-time feature of the delay-accumulation feedback operator. Specifically, an augmented error system was constructed to transform the tracking problem into a regulation problem, and the delays were eliminated by a new discrete lifting technique. On this basis, a controller with preview compensation and delay compensation was developed based on the preview control technology. The proposed controller was proved through a case study as suitable for multiple input delays system like robot trajectory control.

RÉSUMÉ. Cet article présente une méthode de conception pour le contrôleur de prévisualisation de systèmes linéaires à temps discret avec des retards d'entrée multiples basés sur la technologie de levage discret pour des installations à retard d'entrée unique. L'approche de conception est caractérisée par la dimension infinie de la dynamique de l'actionneur et la caractéristique de temps discret de l'opérateur de rétroaction d'accumulation de retard. Spécifiquement, un système d'erreur amélioré a été construit pour transformer le problème de suivi en problème de régulation, et les retards ont été éliminés par une nouvelle technique de levage discret. Sur cette base, un contrôleur avec compensation de prévisualisation et compensation de retard a été développé sur la base de la technologie de contrôle de prévisualisation. Une étude de cas a prouvé que le contrôleur proposé convenait à un système à retards d'entrée multiples comme le contrôle de trajectoire de robot.

KEYWORDS: discrete-time system; input delays; preview control; lifting method.

MOTS-CLÉS: système à temps discret, retard d'entrée, contrôle de prévisualisation, méthode de levage.

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1. Introduction

Preview control is an effective approach for systems to tracking reference signal by utilizing the reference signal's future information. For its perfect tracking quality, preview control draws considerable favor in practice such as motorcycle performance (Sharp, 2007; Marzbanrad *et al.*, 2003), robots walking (Kajita *et al.*, 2003; Shimmyo *et al.*, 2013; Czarnetzki 2009), and protection against earthquakes (Marzbanrad *et al.*, 2004). In the last decade, preview control has been extended to stochastic systems (Gershon *et al.*, 2014), time-varying systems (Li *et al.*, 2018), and descriptor systems (Liao *et al.*, 2012). Preview control in delay systems is also a major branch since time delay occurs frequently in engineering (Fridman, 2014). The preview control problems for continuous-time systems with signal input delay (Liao and Liao, 2018) and multiple input delays (Liao and Liao, 2017) are discussed by predictor method and finite spectrum assignment method separately. The preview control problems for discrete-time systems with delay are treated in (Cao and Liao, 2015; Liao and Liao, 2017) where the latter work treats specifically for the input delay systems.

In the present paper, we address the problems of preview control of multiple input delays, discrete-time, linear systems, as formulated in Section 2. Time delay will affect the control result and even destroy the stability of the system. For eliminating the delay, predictor method (Furukawa and Shimemura, 1983; Manitius and Olbrot, 1979; Mondié and Michiels, 2003) is introduced into the discrete-time systems to design a preview controller for discrete-time input delay systems. Discrete lifting technique is also an effective method to transform the delay system into a delay-free one (Cao and Liao, 2015). Furthermore, Liao and Liao (2017) gives a lifting-predictor method to design preview controller for discrete-time linear systems with both state delay and input delay, which reduce the dimension of the augmented system effectively. However, the predictor method is no longer applicable to multiple input delays, meanwhile, discrete lifting technique cannot be extended to the multiple input delays situation directly.

Similar to 15. Liao *et al.* (2016), an augmented error system is constructed firstly, and then the delays should be eliminated in structure which in turn allows one to use the preview control theorem (Katayama *et al.*, 1985; Liao *et al.*, 2003). To eliminate the delays, a new discrete lifting technology is proposed here where we first derive, in Section 3, a delay-dependent matrix is involved. Based on the delay-free augmented error system, a preview controller is derived, and finally the preview controller for the original system is obtained. The problem of design preview controller with compensation for multiple input delays, as far as we know, was studied in (Liao and Liao, 2018) for the first time. The approach in Liao and Liao (2018) is based on integral transformation which results in controllers involving multiple integrals. On the one hand, the integral transformation is no longer applicable to discrete-time systems. On the other hand, this new lifting technology resulting in an explicit and simple augmented system. It is easy to verify that the original lifting technology is a special case of the new lifting technology proposed here.

This paper is organized as follows. In Section 2, a formulation of the problem is described and some basic assumptions are given. In Section 3, an augmented error system is constructed and the delays are eliminated. In Section 4, a preview controller for the original system is derived. In Section 5, a numerical example is provided to verify the effectiveness of the controller. And a brief conclusion is drawn in Section 6.

2. Problem formulation and basic assumptions

Consider a discrete-time system with multiple input delays as follows:

$$\begin{cases} x(k+1) = Ax(k) + \sum_{i=1}^{d} B_i u(k-f_i) \\ y(k) = Cx(k) \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the input vector, $y(k) \in R^p$ is the output vector. The positive integers f_i $(i = 1,2,\cdots,d)$ represent constant input delays of the system satisfying $0 < f_1 < \cdots < f_d$. $A \in R^{n\times n}$, $B \in R^{n\times m}$, and $C \in R^{p\times n}$ are constant matrices. The vectors $u(-f_d)$, $u(-f_d+1)$, $\cdots$, $u(0)$ are initial inputs, and the vector $x(0) = x_0$ is the initial state. Let $r(k) \in R^p$ be the reference signal.

First, we give the following three basic assumptions.

Assumption 1 (A1): The pairs $(A_0, \sum_{i=1}^{d} B_i)$ are stabilizable, namely, for all λ $(|\lambda| \geq 1)$, the matrix $[\lambda I_n - A_0 \quad \sum_{i=1}^{d} B_i]$ is of full row rank.

Assumption 2 (A2): The matrix $\begin{bmatrix} A_0 & \sum_{i=1}^{d} B_i \\ C_0 & 0 \end{bmatrix}$ is of full row rank.

Assumption 3 (A3): The reference signal $r(k)$ is previewable, and its preview length is N_r, namely, at the present time k, the present value $r(k)$ as well as the N_r future values $r(k+1), \cdots, r(k+N_r)$ are available, the future values of the reference signal beyond time $k+N_r$ are assumed as $r(k+N_r)$, namely $r(k+l) = r(k+N_r)$ $(l \geq N_r + 1)$. Moreover, the reference signal $r(k)$ satisfies $\lim_{k\to\infty} r(k) = \bar{r}$.

where $\bar{r}$ is a constant vector. This implies that $r(k)$ reaches a steady state when k growing large enough.

Let $e(k)$ be the error signal defined as the subtraction of $y(k)$ and $r(k)$, i.e.,

$$e(k) = y(k) - r(k) \tag{2}$$

The purpose of this work is to design a controller with preview compensation such that the output $y(k)$ of the closed-loop system tracks the reference signal $r(k)$ without static error, namely $\lim_{k\to\infty} e(k) = 0$.

We apply optimal control method to achieve the goal. The performance index of (1) is defined as

$$J = \sum_{k=1}^{\infty} [e^T(k) Q_e e(k) + \Delta u^T(k) H \Delta u(k)] \tag{3}$$

where $Q_e \in R^{p\times p}$ and $H \in R^{m\times m}$ are positive definite weight matrices, the first-order backward difference operator Δ is defined as follows: $\Delta u(k) = u(k) - u(k-1)$.

The basic method of designing a preview controller for (1) is that an augmented error system is constructed firstly, then the delays are eliminated by a new lifting technology, based on which, a preview controller is derived for the augmented error system by using optimal control theory, and finally the preview controller for the original system is obtained.

3. Derivation of the augmented error system

Using Δ on both sides of the first equation of (1), we have

$$\Delta x(k+1) = A\Delta x(k) + \sum_{i=1}^{d} B_i \Delta u(k-f_i) \tag{4}$$

According to (2), we have

$$e(k+1) = Cx(k+1) - r(k+1). \tag{5}$$

Using Δ on both sides of (5), we obtain following equation:

$\Delta e(k+1) = C\Delta x(k+1) - \Delta r(k+1)$.

Since $\Delta e(k+1) = e(k+1) - e(k)$, the error signal satisfies

$$e(k+1) = e(k) + CA\Delta x(k) + C\sum_{i=1}^{d} B_i \Delta u(k-f_i) - \Delta r(k+1) \tag{6}$$

Equations (4) and (6) yields

$$\begin{cases} X(k+1) = \bar{A}X(k) + \sum_{i=1}^{d} \bar{B}_i \Delta u(k-f_i) + \bar{D}\Delta r(k+1) \\ e(k) = \bar{C}X(k) \end{cases} \tag{7}$$

where

$$X(k) = \begin{bmatrix} e(k) \\ \Delta x(k) \end{bmatrix}, \bar{A} = \begin{bmatrix} I_p & CA \\ 0 & A \end{bmatrix}, \bar{B}_i = \begin{bmatrix} CB_i \\ B_i \end{bmatrix}, \bar{C} = [I_p \quad 0], \bar{D} = \begin{bmatrix} -I_p \\ 0 \end{bmatrix}.$$

Equation (7) is called the augmented error system of (1). It is realizable to take $e(k) = \bar{C}X(k)$ as the output of (7), because $y(k)$ and $r(k)$ is available.

We note that the reference signal $r(k)$ has been introduced into the dynamic equation (7). However, the input delays still exist. Let

$$X_u(k) = \begin{bmatrix} \Delta u(k-f_d) \\ \Delta u(k-f_d+1) \\ \vdots \\ \Delta u(k-2) \\ \Delta u(k-1) \end{bmatrix}.$$

we have

$$X_u(k+1) = A_u X_u(k) + B_u \Delta u(k) \tag{8}$$

where

$$A_u = \begin{bmatrix} 0 & I_m & 0 & \cdots & 0 \\ 0 & 0 & I_m & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I_m \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, B_u = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ I_m \end{bmatrix}$$

Combining (7) and (8), we obtain the following system without delay

$$\begin{cases} \tilde{X}(k+1) = \tilde{A}\tilde{X}(k) + \tilde{B}\Delta u(k) + \tilde{D}\Delta r(k+1) \\ e(k) = \tilde{C}\tilde{X}(k) \end{cases} \tag{9}$$

where

$$\tilde{X}(k) = \begin{bmatrix} X(k) \\ X_u(k) \end{bmatrix}, \tilde{A} = \begin{bmatrix} \bar{A} & \sum_{i=1}^{d} \Gamma_i \\ 0 & A_u \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0 \\ B_u \end{bmatrix}, \tilde{D} = \begin{bmatrix} \bar{D} \\ 0 \end{bmatrix}, \tilde{C} = \begin{bmatrix} \bar{C} & 0 \end{bmatrix},$$

$$\Gamma_i = \begin{bmatrix} 0 & \cdots & 0 & \underset{\uparrow}{\bar{B}_i} & 0 & \cdots & 0 \end{bmatrix}$$

$the\ (d - f_i)th.$

Now, the performance index (3) can be expressed as

$$J = \sum_{k=1}^{\infty} \left[X^T(k)\tilde{Q}X(k) + \Delta u^T(k) H \Delta u(k) \right] \tag{10}$$

where $\tilde{Q} = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} Q_e & 0 \\ 0 & 0 \end{bmatrix}$ which have corresponding dimensions.

If a controller $\Delta u(k)$ can be derived such that the performance index (10) minimum, then it is easy to get $\lim_{k\to\infty} \tilde{X}(k) = 0$, and immediately the conclusion $\lim_{k\to\infty} e(k) = 0$ holds. Furthermore, we can obtain the input $u(k)$ from $\Delta u(k)$. Thus, the purpose is achieved.

4. Main results and their derivation

Before designing preview controller, we discuss the stabilizability of $(\tilde{A}, \tilde{B})$ and detectability of $(\tilde{Q}^{\frac{1}{2}}, \tilde{A})$.

Theorem 1. The necessary and sufficient condition for $(\tilde{A}, \tilde{B})$ to be positive-definite is to assume that **A1** and **A2** are simultaneously valid.

Proof: The PBH rank criterion shows that $(\tilde{A}, \tilde{B})$ is positive-definite if and only if matrix $[\lambda I - \tilde{A} \quad \tilde{B}]$ is row full rank for any complex number λ ($|\lambda| \geq 1$). According to the structures of $\tilde{A}$ and $\tilde{B}$, we have:

$$[\lambda I - \tilde{A} \quad \tilde{B}] = \begin{bmatrix} \lambda I_{p+n} - \bar{A} & -\sum_{i=1}^{d} \Gamma_i & 0 \\ 0 & \lambda I_{f_d m} - A_u & B_u \end{bmatrix}$$

$$= \begin{bmatrix} (\lambda-1)I_p & -CA_1 & 0 & 0 & \cdots & 0 & 0 & -C_0A_0 & C_0B_1 \\ 0 & \lambda I_n & -I_n & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda I_n & -I_n & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda I_n & \ddots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ddots & \lambda I_n & -I_n & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & \lambda I_n & -I_n & 0 \\ 0 & -A_1 & 0 & 0 & \cdots & 0 & 0 & \lambda I_n - A_0 & B_1 \end{bmatrix}$$

In the last matrix above, each row from the second row was multiplied by λ before adding to the next row. Then, the second to last row of the resulting matrix was pre-multiplied by $-C_0A_0$ before adding to the first row and pre-multiplied by $-A_0$ before adding to the last row. In this way, we have:

$$[\lambda I - \bar{A} \quad \bar{B}]$$

$$\rightarrow \begin{bmatrix} (\lambda-1)I_p & -\lambda^d C_0A_0 - C_0A_1 & 0 & 0 & \cdots & 0 & 0 & 0 & C_0B_1 \\ 0 & \lambda I_n & -I_n & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & \lambda^2 I_n & 0 & -I_n & \ddots & 0 & 0 & 0 & 0 \\ 0 & \lambda^3 I_n & 0 & 0 & \ddots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & \lambda^{d-1} I_n & 0 & 0 & \ddots & 0 & -I_n & 0 & 0 \\ 0 & \lambda^d I_n & 0 & 0 & \cdots & 0 & 0 & -I_n & 0 \\ 0 & \lambda^{d+1}I_n - \lambda^d A_0 - A_1 & 0 & 0 & \cdots & 0 & 0 & 0 & B_1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} (\lambda-1)I_p & -\lambda^d C_0A_0 - C_0A_1 & 0 & 0 & \cdots & 0 & 0 & 0 & C_0B_1 \\ 0 & 0 & -I_n & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_n & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ddots & 0 & -I_n & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -I_n & 0 \\ 0 & \lambda^{d+1}I_n - \lambda^d A_0 - A_1 & 0 & 0 & \cdots & 0 & 0 & 0 & B_1 \end{bmatrix}$$

Since the matrix rank does not change in the elementary transformation, the row full rank of matrix $[\lambda I - \bar{A} \quad \bar{B}]$ is equivalent to that of matrix $\Phi = \begin{bmatrix} (\lambda-1)I_p & -\lambda^d C_0A_0 - C_0A_1 & C_0B_1 \\ 0 & \lambda^{d+1}I_n - \lambda^d A_0 - A_1 & B_1 \end{bmatrix}$.

Then, the following two cases were discussed:

(1) If $\lambda \neq 1$, then $(\lambda-1)I_p$ is reversible. In this case, matrix $\Phi$is row full rank if and only if matrix $[\lambda^{d+1}I_n - \lambda^d A_0 - A_1 \quad B_1]$ is row full rank. Thus, A1 is valid.

(2) If $\lambda = 1$, then $rank\Phi = rank\begin{bmatrix} -C_0A_0 - C_0A_1 & C_0B_1 \\ I_n - A_0 - A_1 & B_1 \end{bmatrix} = rank\begin{bmatrix} C_0 & 0 \\ I_n - A_0 - A_1 & B_1 \end{bmatrix}$.

The above derivation shows that, for any complex λ ($|\lambda| \geq 1$), matrix $[\lambda I - \bar{A} \quad \bar{B}]$ is row full rank if and only if matrix $[\lambda^{d+1}I_n - \lambda^d A_0 - A_1 \quad B_1]$ and matrix $\begin{bmatrix} C_0 & 0 \\ I_n - A_0 - A_1 & B_1 \end{bmatrix}$ are row full rank. Q.E.D.

Theorem 2. The $(Q^{\frac{1}{2}}, \bar{A})$ is detectable if **A3** is valid.

Proof: The PBH rank criterion shows that $(Q^{\frac{1}{2}}, \bar{A})$ is detectable if and only if matrix $\begin{bmatrix} \lambda I - \bar{A} \\ Q^{\frac{1}{2}} \end{bmatrix}$ is column full rank for any complex number λ ($|\lambda| \geq 1$). According to the structures of $\tilde{A}$ and Q, the matrix $\begin{bmatrix} \lambda I - \bar{A} \\ Q^{\frac{1}{2}} \end{bmatrix}$ can be written as:

$$\begin{bmatrix} (\lambda - 1)I_p & -CA \\ 0 & \lambda I_{(d+1)n} - A \\ Q_e^{\frac{1}{2}} & 0 \\ 0 & 0 \end{bmatrix}.$$

Since Q_e is positive-definite, the column full rank of matrix $\begin{bmatrix} \lambda I - \bar{A} \\ Q^{\frac{1}{2}} \end{bmatrix}$ is equivalent to the column full rank of matrix $\Psi = \begin{bmatrix} -CA \\ \lambda I_{(d+1)n} - A \end{bmatrix}$. According to the structures of matrices A and C, matrix Ψ can be expanded as:

$$\Psi = \begin{bmatrix} -C_0A_1 & 0 & 0 & \cdots & 0 & 0 & -C_0A_0 \\ \lambda I_n & -I_n & 0 & \cdots & 0 & 0 & 0 \\ 0 & \lambda I_n & -I_n & \ddots & 0 & 0 & 0 \\ 0 & 0 & \lambda I_n & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & \lambda I_n & -I_n & 0 \\ 0 & 0 & 0 & \cdots & 0 & \lambda I_n & -I_n \\ -A_1 & 0 & 0 & \cdots & 0 & 0 & \lambda I_n - A_0 \end{bmatrix}$$

In matrix Ψ, each column from the last column was multiplied by λ before adding to the previous column. Then, the last row was pre-multiplied by $-C_0$ before adding to the first row. In this way, we have:

$$\Psi \rightarrow \begin{bmatrix} -\lambda^{d+1}C_0 & -\lambda^{d}C_0 & -\lambda^{d-1}C_0 & \cdots & -\lambda^{3}C_0 & -\lambda^{2}C_0 & -\lambda C_0 \\ 0 & -I_n & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -I_n & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -I_n & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & -I_n \\ \lambda^{d+1}I_n-\lambda^{d}A_0-A_1 & \lambda^{d}I_n-\lambda^{d-1}A_0 & \lambda^{d-1}I_n-\lambda^{d-2}A_0 & \cdots & \lambda^{3}I_n-\lambda^{2}A_0 & \lambda^{2}I_n-\lambda A_0 & \lambda I_n-A_0 \end{bmatrix}$$

Obviously, the column full rank of matrix Ψ is equivalent to the column full rank of matrix $\begin{bmatrix} -\lambda^{d+1}C_0 \\ \lambda^{d+1}I_n-\lambda^{d}A_0-A_1 \end{bmatrix}$, and to the column full rank of matrix $\begin{bmatrix} C_0 \\ \lambda^{d+1}I_n-\lambda^{d}A_0-A_1 \end{bmatrix}$. Q.E.D.

The control system (9) is a delay-free system. Furthermore, it is known from A3 that the reference signal $\Delta r(k)$ is previewable in the sense that the future value $\Delta r(l)$ $(k \le l \le k+N_r)$ is available at each instant of time k. We can get the following preview control theorem according to the results of (Katayama *et al.*, 1985).

Theorem 1. If A1-A3 hold, and Q_e is positive definite, then the preview controller of (9) that minimizes criterion (10) is given by

$$\Delta u(k) = -G_X\tilde{X}(k) - \sum_{l=1}^{N_r} G_d(l)\Delta r(k+l) \tag{11}$$

where

$$G_X = [H + \tilde{B}^T P \tilde{B}]^{-1}\tilde{B}^T P \tilde{A}$$

$$G_d(1) = -[H + \tilde{B}^T P \tilde{B}]^{-1}\tilde{B}^T P \begin{bmatrix} I_p \\ 0 \end{bmatrix}$$

$$G_d(l) = [H + \tilde{B}^T P \tilde{B}]^{-1}\tilde{B}^T \Upsilon(l-1),\ l = 2,\cdots,N_r$$

where $P \in R^{(p+n+mf_d)\times(p+n+mf_d)}$ is the positive semi-definite solution of the algebraic Riccati equation

$$P = \tilde{A}^T P \tilde{A} - \tilde{A}^T P \tilde{B}[H + \tilde{B}^T P \tilde{B}]^{-1}\tilde{B}^T P \tilde{A} + \tilde{Q}.$$

Furthermore, the matrices $\Upsilon(l) \in R^{(n+p+md)\times p}$ are given by

$$\Upsilon(1) = -\tilde{A}_c{}^T P \begin{bmatrix} I_p \\ 0 \end{bmatrix};\ \Upsilon(l) = \tilde{A}_c{}^T \Upsilon(l-1),\ l = 2,\cdots,N_r$$

where $\tilde{A}_c$ is the closed-loop matrix defined by

$$\tilde{A}_c = \tilde{A} - \tilde{B}[H + \tilde{B}^T P \tilde{B}]^{-1}\tilde{B}^T P \tilde{A}$$

Remark 1: the future reference signal value $\Delta r(k+l)$ $(l = 1,\cdots,N_r)$ appearing in $\sum_{l=1}^{N_r} G_d(l)\Delta r(k+l)$ acts as a preview compensation in the controller.

The preview controller $u(k)$ can be derived from Theorem 1 by the following method.

First, we denote G_X as

$$G_X = [G_e \quad G_x \quad G_1 \quad \cdots \quad G_{f_d}] \tag{12}$$

where $G_e \in R^{r\times p}$, $G_x \in R^{r\times n}$, and $G_i \in R^{r\times m}$ $(i = 1,2,\cdots,f_d)$, then (14) can be rewritten as

$$\Delta u(k) = -G_e e(k) - G_x \Delta x(k) - \sum_{i=1}^{f_d} G_i \Delta u(k - f_d - 1 + i) - \sum_{l=1}^{N_r} G_d(l)\Delta r(k + l) \tag{13}$$

It is assumed that the initial values of system (1) and the reference signal are zeros, namely, for $k \le 0$, the vectors $x(k) = 0$, $y(k) = r(k) = 0$ and $u(k) = 0$. Then, the following result from (13) will be obtained:

$$u(k) = -G_e \sum_{j=1}^{k} e(j) - G_x x(k) - \sum_{i=1}^{f_d} G_i u(k - f_d - 1 + i) - \sum_{l=1}^{N_r} G_d(l) r(k + l) \tag{14}$$

Thus, a preview control theory for system (1) will be obtained as follows:

Theorem 2. Let A1-A3 hold, Q_e be positive definite, the performance index be defined as (3). Assume that $x(k) = 0$, $y(k) = r(k) = 0$, $u(k) = 0$ for $k \le 0$. Then the preview controller of (1) is given by (14), where G_e, G_x and $G_i \in R^{r\times m}$ $(i = 1,2,\cdots,f_d)$ are determined by (12), G_X and G_d are given by Theorem 1.

Remark 2: Theorem 2 shows that the preview controller is composed of four terms. The first one$-G_e \sum_{j=1}^{k} e(j)$ is the accumulation of the tracking error, which ensures that the output of the closed-loop system tracks the reference signal without static error. The second one $-G_x x(k)$ is a state feedback. The third one $-\sum_{i=1}^{f_d} G_i u(k - f_d - 1 + i)$ is the compensation of the input delay. The last one $-\sum_{l=1}^{N_r} G_d(l) r(k + l)$ is the preview compensation of the reference signal which used to improve the performance of tracking.

It should be noted that, the preview control method is also applicable to some irregular reference signals which cannot be modeled by the dynamic system’s outputs.

5. A simulation example

Now, let us apply present theory to following linear discrete-time model.

Example 1 Consider the system described in (1), where

$$A_0 = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, B_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, C_0 = [3 \quad 1 \quad 1].$$

The delays are $f_1 = 3$, $f_2 = 5$ and $f_3 = 6$ respectively. The reference signal $r(k)$ is given as following:

1) Step signal

$$r(k) = \begin{cases} 0, & k \leq 20 \\ 1, & k > 20 \end{cases}$$

2) Slope signal

$$r(k) = \begin{cases} 0, & k \leq 20 \\ \frac{t-2}{2}, & 20 < k \leq 40 \\ 1, & k > 40 \end{cases}$$

Through verification, assumptions A1-A2 are satisfied. Taking the preview lengths be $N_r = 0$, $N_r = 10$ and $N_r = 20$ respectively. Taking the weight matrices be $Q_e = 1.0$ and $H = 0.05$. By Theorem 2, there exists a preview controller described as (14). The responses of the closed-loop system are showed as follows.

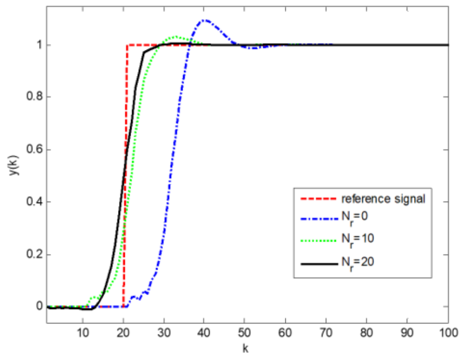

Figure 1. The output responses to the step signal with different preview lengths

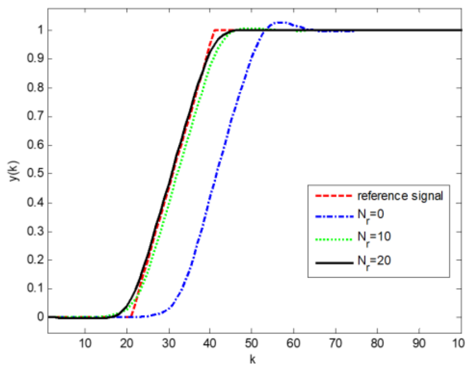

Figure 2. The output responses to the slope signal with different preview lengths

The output responses of the closed-loop system tracking to the step signal are shown in Figure 1 and to the slope signal are shown in Figure 2. From figures 1-2 it can be seen that the closed-loop's output can track the reference signal asymptotically.

We take the preview length $N_r = 0$, $N_r = 10$, and $N_r = 20$ respectively. The output responses show that a longer preview length can take the tracking performance be better.

6. Conclusion

In this paper, a class of preview control problems of discrete-time linear systems with multiple input delays is discussed. A preview controller with delay compensation and preview compensation is derived. The difficulties of input delay are successfully overcome by a new lifting technique. Numerical results show the effectiveness of the present method.

Further studies are needed on selecting a proper preview length according to the delays of the system, which is a complex problem but a significant one.

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