Analysis of transmit antenna selection based selective decode forward cooperative communication protocol

Vikash Sachan*, Indrajeet Kumar, Ravi Shankar, Ritesh Kumar Mishra

Department of Electronics and Communication Engineering,
National Institute of Technology Patna, Patna-800005, India

vikash.ece15@nitp.ac.in

ABSTRACT. *In this work, we consider a single relay multiple input multiple output (MIMO) space-time block-code (STBC) based relaying system for two strategies using transmit antenna selection (TAS) technique. We consider the Rayleigh distribution between source to destination (SD), relay to destination (RD) and source to relay (SR) fading channel links. In first selection strategy, we consider selective decode and forward (SDF) protocol between the relay and destination and in second selection strategy, we consider STBC SDF protocol between RD fading channel links. We derive the closed form expressions for SER, SER upper bound and diversity order (DO). The optimal power allocation factors (OPFs) are derived for the both strategies, which minimize the SER of the relaying system. Simulation results show that the second strategy performs better than the first one for the same DO.*

RÉSUMÉ. *Dans ce travail, nous considérons un système de relais à base de code de blocs spatio-temporels (STBC) à entrées multiples et sorties multiples (MIMO) à relais unique pour deux stratégies utilisant la technique de sélection d'antenne d'émission (TAS). Nous considérons les liaisons de canaux à évanouissements de la distribution de Rayleigh entre source vers destination (SD), relais vers destination (RD) et source vers relais (SR). Dans la première stratégie de sélection, nous considérons le protocole de décodage et de transmission sélectifs (SDF) entre le relais et la destination. Et dans la deuxième stratégie de sélection, nous considérons le protocole STBC SDF entre les liaisons de canaux à évanouissements RD. Nous dérivons les expressions de forme fermée pour SER, limite supérieure et ordre de diversité (DO). Les facteurs optimaux d'allocation de puissance (OPFs) sont dérivés pour les deux stratégies, ce qui minimise le SER du système de relais. Les résultats de la simulation montrent que la deuxième stratégie est plus performante que la première pour le même OD.*

KEYWORDS: *multiple input multiple output, space- time-block- code, selective decode and forward, pairwise error probability.*

MOTS-CLÉS: *entrée multiple et sortie multiple, code spatio-temporel, décodage et transmission sélectifs, probabilité d'erreur paire.*

DOI:10.3166/TS.35.47-60 © 2018 Lavoisier

1. Introduction

MIMO is proven, cost-effective technology, high spectral efficiency, provides antenna diversity and reduces channel fading. Cooperative communication attains ominously high data rates in 4G/5G communication systems due to their ability to create a virtual array of antennas (Ibrahim *et al*., 2008). With increasing emphasis on Femto, small and Pico cell networks, cooperative systems are a promising solution for 5G systems. The most famous relaying protocols are amplify-and-forward (AF), decode-and-forward (DF) and SDF protocols (Khattabi and Matalgah, 2015; Ryu *et al*., 2018; Shankar *et al*., 2017; Shankar *et al*., 2017). Also by using MIMO and STBC together, better end-to-end error performance has been achieved and it will enhance the data transmission rate.

In references (Varshney and Puri, 2017; Varshney *et al*., 2015), the author analyzed the pairwise error probability (PEP) of MIMO STBC S-DF cooperative communication protocol. The authors derived the closed-form PEP expressions for dual phase and multiple phase cooperation protocol, derive the DO and OPFs.

In the works (Amarasuriya *et al*., 2011; Yang *et al*., 2014), the authors investigated TAS based cooperation network. In stduy (Amarasuriya *et al*., 2011), AF based relaying is investigated and it is shown that two sub-optimal TAS technique achieves DOs $M_D + M_R\, min(M_S, M_D)$ and $M_R + M_S M_D$. In sdudy (Yang *et al*., 2014), TAS for full duplex AF relaying protocol is extensively investigated.

In study (Krishna and Bhatnagar, 2014), the author investigated the symbol error rate (SER) performance of two sub-optimal TAS strategies having only one relay SDF cooperation network. Closed form and upper bound expressions of SER for SDF systems have been taken for both TAS strategies.

In study (Krishna and Bhatnagar, 2016), the authors investigated the single-relay MIMO DF relaying network with M_S, M_R and M_D number of antennas are employed in source, relay and destination. In sudy (Jin and Shin, 2013), the authors offered the selection of a new source transmit antenna based on the channel state information. It is shown that source transmit antenna selection achieves the full DO of $M_S M_D + MM_R\, min(M_S, M_D)$. In study (Halber and Chakravarty, 2018), the author has investigated the relay for the optimization purpose.

In this paper, investigation of the single relay MIMO STBC based SDF system employing M-ary PSK by deriving the closed form PEP expressions and PEP upper bounds has been done. The closed form SER expression for two sub-optimal selection strategies has been derived. There are consideration two criteria for antenna selection 1) Maximization of SNR of SD and RD fading channel links and 2) Maximization of SNR of RD and SR fading channel links. Also, we investigated the DO and optimal power allocation.

In this paper, section 2 gives the System Model. Section 3, describes SER analysis. Section 4, shows the Simulation results and discussions. Section 5 provides the conclusion for our proposed method.

2. System model

Consider a MIMO SDF cooperative communication system employing single relay, as given in Figure 1. The relay, source, and destination nodes are employed with M_R, M_S and M_D number of antennas, respectively. Only in the case of successful decoding relay node, the signal will be forwarded to the destination node, otherwise it will be inactive state. Let $H_{SR} \in \mathbb{C}^{M_R M_S}, H_{SD} \in \mathbb{C}^{M_D M_S}$ and $H_{RD} \in \mathbb{C}^{M_D M_R}$ denote the channel matrix from SR, SD and RD respectively. Let $h_{d_i s_j} \in H_{SD}, h_{r_i s_j} \in H_{SR}$ and $h_{d_i r_j} \in H_{RD}$ denote the channel coefficients for SD, SR and RD fading links. The channel coefficient is modeled as the zero mean complex Gaussian circular shift (ZMCGCS) random variable (RV) with unit variance.Let $\mathbb{Q}_{d_i r_j} = \left|h_{d_i r_j}\right|^2, \mathbb{Q}_{r_i s_j} = \left|h_{r_i s_j}\right|^2$ and $\mathbb{Q}_{d_i s_j} = \left|h_{d_i s_j}\right|^2$ denote the exponentially distributed instantaneous channel gains from the j^{th} transmitter (Tx) to i^{th}receiver (Rx) antenna in the RD, SR and SD fading channel. $\delta_{SD}^2, \delta_{SR}^2$ and δ_{RD}^2denote the average channel gain for SD, SR and RD fading link respectively. The transmission of signals can be divided into two steps, one transmission phase and one relaying phase. In broadcast phase using Time Division Multiple Access (TDMA), the signal from the source is being transmitted to both destination and relay in T_1 time slots. In relaying phase, the relay node forwards the signal correctly decoding to the destination node using STBC technique.

2.1. The Broadcast phase

Let $X_1 \in \mathbb{C}^{T_1 \times 1}$ denotes the symbol vector, each symbol has unit energy, i.e., $E\left\{{X_1}^H X_1\right\} = 1$. Let $y_{SD} \in \mathbb{C}^{T_1 \times 1}$ and $y_{SR} \in \mathbb{C}^{T_1 \times 1}$ denote received symbol vector at the destination and relay node, modeled as,

$$y_{SD} = \sqrt{P_S} h_{D_i S_j} x + w_{SD} \tag{1}$$

$$y_{SR} = \sqrt{P_S} h_{R_i S_j} x + w_{SR} \tag{2}$$

Where $w_{SD} \in \mathbb{C}^{T_1 \times 1}, w_{SR} \in \mathbb{C}^{T_1 \times 1}$, denote the noise vector, modeled as ZMCGCS RV with noise variance N_0. Let y_{SD}^k denote the received symbol at k^{th} time slot, modeled as,

$$y_{SD}^k = \sqrt{P_S} h_{D_i S_j} x^k + w_{SR}^k \tag{3}$$

Let us define α_{SD}, the weight factor for SD fading link, the SNR is maximized when $\alpha_{SD} = h_{D_i S_j}^H$. Also the maximized SNR is given as, $\lambda_{SD} = \frac{P_S}{N_0}\left|h_{D_i S_j}\right|^2$. Following similar procedure, weight factor and maximized SNR for SR fading link is given as, $\alpha_{SR} = h_{R_i S_j}^H$and $\lambda_{SR} = \frac{P_S}{N_0}\left|h_{R_i S_j}\right|^2$respectively.

2.2. The relaying phase

2.2.1. Strategy I-single Tx and Rx antenna between the relay and destination nodes

In relaying phase, the relay node selects one aerial in a random manner to transmitter and receiver selects one aerial randomly to receive, as given in Figure 2. Let $y_{RD}^{k+T_1}$ denote the received symbol block at the destination at the $k+T_1$ time corresponding to transmission of x^k data, modeled as,

$$y_{RD}^{k+T_1} = \sqrt{P_R} h_{D_i S_j} x^k + w_{RD}^k \tag{4}$$

The weight vector α_{RD} of the RD link and maximum SNR is given as $\alpha_{RD} = h_{D_i R_j}^H$ and $\lambda_{RD} = \frac{P_R}{N_0}\left|h_{D_i R_j}\right|^2$ respectively. Cooperation mode SNR is modeled as,

$$\lambda = \frac{P_S\left|h_{D_i S_j}\right|^2 + P_R\left|h_{D_i R_j}\right|^2}{N_0} \tag{5}$$

2.2.2. Strategy II-STBC between relay and destination

In strategy II broadcast phase is similar to strategy I. Relay generated the STBC code-word block $X \in \mathbb{C}^{M_R \times T_2}$ after receiving the transmitted vector $X_1 \in \mathbb{C}^{T_1 \times 1}$ at T_2 time slot. According to the STBC transmission from the relay node, the symbol block received on the destination node has been modeled as,

$$Y_{RD} = \sqrt{P_R / M_R} H_{RD} X + W_{RD} \tag{6}$$

Where $H_{RD} \in \mathbb{C}^{M_D M_R}$ denote channel matrix for RD fading link and W_{RD} denote the noise vector for RD fading link respectively, modeled as ZMCGCS RV with noise varianceN_0. Assuming perfect CSI availability at receiver terminal and uncorrelated noise component, the maximum likelihood (ML) decoding of X is given as [1],

$$\hat{X} = arg \max_{X \in C} \left\| Y_{RD} - \sqrt{P_R/M_R} H_{RD} X \right\|_F^2 \tag{7}$$

Where C denotes the STBC code-word set and $|C|$ denote the cardinality of the code-word set C

3. SER analysis

3.1. SER analysis for strategy I

In Figure 2, it has given the various steps involve in the broadcast phase and in the other phase of selecting the antennas in strategy I. Broadcast phase is comprised of two steps. Broadcast phase involves the selection of i^{th}, j^{th} and k^{th} aerials respectively, shown in Figure 2.

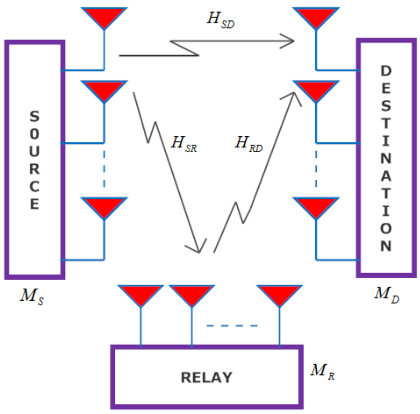

Figure 1. Selection strategy I

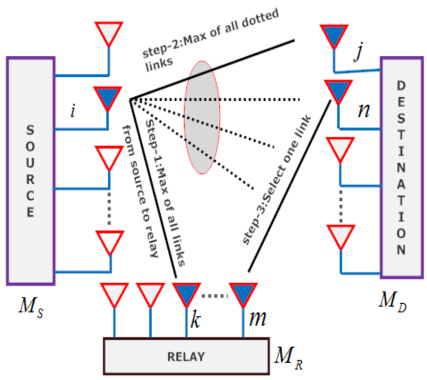

Figure 2. Selection strategy II

In step 1, i^{th} antenna at the source and k^{th} antenna at the relay is selected depending on the maximum instantaneous gain $\mathbb{Q}_{SR}$ of all fading links. In step 2, j^{th}antenna at the destination node has been selected depending on the maximum instantaneous gain $\mathbb{Q}_{SD}$ of the fading channels displayed as dotted lines. Lastly, in step 3, one antenna is selecting between m^{th} antenna at the relay node and n^{th} antenna at the destination node. $\mathbb{Q}_{SR}$, $\mathbb{Q}_{SD}$ and $\mathbb{Q}_{RD}$ are given as,

$$\mathbb{Q}_{SR} = max\left(\mathbb{Q}_{S_1,R_1}, \mathbb{Q}_{S_2,R_2}, \dots \dots \dots, \mathbb{Q}_{S_{M_S},R_{M_r}}\right), 1 \le i \le M_S, 1 \le k \le M_R$$

$$\mathbb{Q}_{SD} = max\left(\mathbb{Q}_{S_iD_1}, \mathbb{Q}_{S_iD_2}, \ldots\ldots\ldots, \mathbb{Q}_{S_iD_{M_D}}\right), 1 \le j \le M_D \mathbb{Q}_{RD} = \mathbb{Q}_{R_i,D_i}, 1 \le m \le M_R, 1 \le n \le M_D \tag{8}$$

The cumulative distribution function (CDF) and probability distribution function (PDF) of the $\mathbb{Q}_{SD}$, $\mathbb{Q}_{SR}$ and $\mathbb{Q}_{RD}$ is modeled as [8],

$$F_{\mathbb{Q}_{SD}}(\mathbb{Q}) = \left(1 - exp\left(\frac{-\mathbb{Q}}{\delta_{SD}^2}\right)\right)^{M_D},$$

$$f_{\mathbb{Q}_{SD}}(\mathbb{Q}) = \frac{M_D}{\delta_{SD}^2} exp\left(\frac{-\mathbb{Q}}{\delta_{SD}^2}\right)\left(1 - exp\left(\frac{-\mathbb{Q}}{\delta_{SD}^2}\right)\right)^{M_D-1},$$

$$F_{\mathbb{Q}_{SR}}(\mathbb{Q}) = \left(1 - exp\left(\frac{-\mathbb{Q}}{\delta_{SR}^2}\right)\right)^{M_S M_R},$$

$$F_{\mathbb{Q}_{RD}}(\mathbb{Q}) = \left(1 - exp\left(\frac{-\mathbb{Q}}{\delta_{RD}^2}\right)\right),$$

$$f_{\mathbb{Q}_{RD}}(\mathbb{Q}) = \frac{1}{\delta_{RD}^2} exp\left(\frac{-\mathbb{Q}}{\delta_{RD}^2}\right), \tag{9}$$

In study (Shankar *et al*., 2017), the end-to-end error Probability is given as,

$$P_E^I = P_{S\to D,R\to D} \times (1 - P_{S\to R}) + P_{S\to D} \times P_{S\to R}. \tag{10}$$

Let $\psi(\lambda_{SD})$ represents the instantaneous symbol error rate of M-PSK modulation, given as (Varshney *et al*., 2015),

$$\psi(\lambda_{SD}) = \frac{1}{\pi}\int_0^{\left(\frac{M-1}{M}\right)\pi} exp\left(\frac{-b}{sin^2\theta}\lambda_{SD}\right) d\theta, \tag{11}$$

Where $b = sin^2(\pi/M)$, $\theta(\alpha)$ denotes the Gaussian Q function, defined as (Varshney *et al*., 2008), $\theta(\alpha) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} exp\left(\frac{x^2}{sin^2\theta}\right) d\theta$ and $b = sin^2\left(\frac{\pi}{M}\right)$.

The SER for SD link can be derived as (Varshney et al., 2017),

$$P_{S\to D} = E_{\mathbb{Q}_{SD}}\{\psi(\lambda_{SD})\} \quad = \int_0^\infty \psi(\mathbb{Q}_{SD}) f_{\mathbb{Q}_{SD}}(\mathbb{Q}) d\mathbb{Q}$$

$$= \frac{M_D}{\Pi\delta_{SD}^2}\int_0^\infty \int_0^{\frac{(M-1)\Pi}{M}} exp(-(\frac{bP_S}{N_0\, sin^2\theta} + \frac{1}{\delta_{SD}^2})\mathbb{Q})(1 - exp(\frac{\mathbb{Q}}{\delta_{SD}^2}))^{M_D-1} d\theta d\mathbb{Q} \tag{12}$$

By using the expression, $(1-x)^M = \sum_{m=0}^{M_R-1}\binom{M}{m}(-1)^m x^m$, we further simplify $P_{S\to D}$ as,

$$P_{S\to D} = M_D \sum_{j=0}^{M_D-1}\binom{M_D - 1}{j}(-1)^j F\left(\frac{bP_S\delta_{SD}^2}{N_0\, sin^2\theta} + j + 1\right), \tag{13}$$

Following the similar procedure, SER for the SER link can be derived as,

$$P_{S\to R} = E_{\beta_{SR}}\{\psi(\lambda_{SR})\}$$
$$= M_S M_R \sum_{i=0}^{M_S M_R-1} \binom{M_S M_R - 1}{i} (-1)^i F\left(\frac{bP_S\delta_{SR}^2}{N_0 \sin^2\theta} + i + 1\right), \quad (14)$$

The SER for the cooperation mode can be written as,

$$P_{S\to D,R\to D} = E_{f_{\mathbb{Q}_{SD}} f_{\mathbb{Q}_{RD}}}\{\psi(\lambda)\}$$
$$= \frac{1}{\prod}\int_0^{\frac{(M-1)\prod}{M}} \int_0^{\infty} \psi(\mathbb{Q}_{SD}) f_{\mathbb{Q}_{SD}}(\mathbb{Q}_{SD}) d\mathbb{Q}_{SD} \int_0^{\infty} \psi(\lambda_{RD}) f_{\beta_{RD}}(\mathbb{Q}_{RD}) d\mathbb{Q}_{RD}$$
$$= M_D \sum_{j=0}^{M_D-1} \binom{M_D - 1}{j} (-1)^j F\left(\left(\frac{bP_S\delta_{SD}^2}{N_0 \sin^2\theta} + j + 1\right)\left(\frac{bP_R\delta_{RD}^2}{N_0 \sin^2\theta} + 1\right)\right), \quad (15)$$

Substituting (13), (14) and (15) into (10), end to end SER for selection strategy I is expressed in (16).

$$P_E^I = M_D \sum_{j=0}^{M_D-1} \binom{M_D - 1}{j} (-1)^j F\left(\frac{bP_S\delta_{SD}^2}{N_0 \sin^2\theta} + j + 1\right)$$
$$\times M_S M_R \sum_{i=0}^{M_S M_R-1} \binom{M_S M_R - 1}{i} (-1)^i F\left(\frac{bP_S\delta_{SR}^2}{N_0 \sin^2\theta} + i + 1\right)$$
$$+ M_D \sum_{j=0}^{M_D-1} \binom{M_D - 1}{j} (-1)^j F\left(\left(\frac{bP_S\delta_{SD}^2}{N_0 \sin^2\theta} + j + 1\right)\left(\frac{bP_R\delta_{RD}^2}{N_0 \sin^2\theta} + 1\right)\right)$$
$$\times\left(1 - M_S M_R \sum_{n=0}^{M_S M_R-1} \binom{M_S M_R - 1}{n} (-1)^n F\left(\frac{bP_S\delta_{SR}^2}{N_0 \sin^2\theta} + n + 1\right)\right), \quad (16)$$

3.1.1. SER upper bound

For deriving the SER upper bound, we assume $1 - P_{S\to R} \approx 1$ at high SNR regimes. Tight SER upper bound of P_E^I can be given in (17),

$$P_E^I = \frac{M_S M_R M_D}{\pi^2} \int_0^{\left(\frac{M-1}{M}\right)\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \left(\sum_{n=0}^{M_S M_R-1} \binom{M_S M_R - 1}{n} \times \frac{(-1)^n}{\left(\frac{bP_S\delta_{SR}^2}{N_0 \, Sin^2\theta} + n + 1\right)}\right) \times$$
$$\left(\sum_{m=0}^{M_D-1} \binom{M_D - 1}{m} \times \frac{(-1)^m}{\left(\frac{bP_S\delta_{SD}^2}{N_0 \, Sin^2\theta} + m + 1\right)}\right)$$
$$\times d\theta_1 d\theta_2 +$$
$$M_D \sum_{j=0}^{M_D-1} \binom{M_d - 1}{j} \times (-1)^j \times \left[\frac{1}{\pi}\int_0^{\left(\frac{M-1}{M}\right)\pi} \frac{1}{\left(\frac{bP_R\delta_{RD}^2}{N_0 \, Sin^2\theta} + j + 1\right)} \times \frac{1}{\left(\frac{bP_R\delta_{RD}^2}{N_0 \, Sin^2\theta} + 1\right)} d\theta\right] \quad (17)$$

Applying the approximation,

$$\sum_{n=0}^{N} \binom{N}{n} (-1)^n \frac{1}{x+ny} = \frac{N! y^N}{\prod_{n=0}^{N}(x+ny)} \tag{18}$$

We can write tight SER upper bound for the selection strategy I as,

$$P_E^I \leq \frac{M_S M_R M_D (M_S M_R - 1)!(M_D - 1)! B_0 B_1}{\pi^2 \left(\frac{bP_S \delta_{SR}^2}{N_0}\right)^{N_S N_R} \left(\frac{bP_S \delta_{SD}^2}{N_0}\right)^{N_D}} + \frac{M_D (M_D - 1)! B_1}{\pi \left(\frac{bP_S \delta_{SD}^2}{N_0}\right)^{N_D} \left(\frac{bP_R \delta_{RD}^2}{N_0}\right)} \tag{19}$$

Where $B_0 = \int_0^{\left(\frac{M-1}{M}\right)\pi} (\sin \theta_1)^{2M_S M_R} d\theta_1$, $B_1 = \int_0^{\left(\frac{M-1}{M}\right)\pi} (\sin \theta_2)^{2M_D} d\theta_2$.

3.1.2. Optimal power allocation

Substituting $P_R = P_0 - P_S$ in (19) and differentiating the resultant expression w.r.t. P_S and after equating it to zero, we can get,

$$K_1 (M_S M_R + M_D) P_S^{-(M_S M_R + M_D + 1)}$$

$$+ K_2 (P - P_S)^{-1} M_D P_0^{-(M_D + 1)} - K_2 P_S^{-(M_D)} (P - P_S)^{-2} = 0, \tag{20}$$

Where K_1 and K_2 are appropriately defined constant terms, given below,

$K_1 = \frac{M_S M_R M_D (M_S M_R - 1)!(M_D - 1)! B_0 B_1}{\pi^2 \left(\frac{b\delta_{SR}^2}{N_0}\right)^{M_S M_R} \left(\frac{b\delta_{SD}^2}{N_0}\right)^{N_D}}, K_2 = \frac{M_D (M_D - 1)! B_1}{\pi \left(\frac{b\delta_{SD}^2}{N_0}\right)^{N_D} \left(\frac{b\delta_{RD}^2}{N_0}\right)}.$

Solution of (20) will give optimal powers for source and relay nodes.

3.1.3. DO calculation

Substituting $\mathbb{Q}_0 = \frac{P_S}{P}, \mathbb{Q}_1 = \frac{P_R}{P}$, (19) can be written as, (21),

$$P_e \leq \frac{M_S M_R M_D (M_S M_R - 1)!\,(M_D - 1)!\,B_0 B_1}{\pi^2 (b\delta_{SR}^2)^{M_S M_R} (b\delta_{SD}^2)^{M_D} \mathbb{Q}_0^{-(M_S M_R + M_D)} (P/N_0)^{-(M_S M_R + M_D)}}$$

$$+ \frac{M_D (M_D - 1)! B_2}{\pi \mathbb{Q}_0^{--M_D} (b\delta_{SD}^2)^{M_D} (b\beta_1 \delta_{RD}^2)(P/N_0)^{-(M_D + 1)}}, \tag{21}$$

DO expression is given as,

$$DO = - \underbrace{lim}_{SNR \to \infty - \frac{log(P_E^I)}{log(SNR)_D} min(_{S_R}} .$$

3.2. SER strategy II

In this selection strategy, the SD and SR link is having a similar error probability to the previous strategy. In this strategy between the relay and destination fading link we apply STBC. Let us define the PEP as the error probability when STBC codeword X_n is of confused with STBC codeword X_l. The PEP can be modeled as,

$$P(X_n \to X_l|H_{RD}) = Q\left(\sqrt{\frac{P_R}{2M_R N_0} H_{RD}\|X_n - X_l\|_F^2}\right).$$

Averaging the PEP over the probability distribution of the fading channel, the average pairwise error probability can be derived as,

$$P(X_n \to X_l) = G\left(\prod_{k=1}^{M_R}\left(1 + \frac{P_R \lambda_{k,l}\sigma_{RD}^2}{4N_0 M_R \sin^2\theta}\right)^{M_D}\right) = E_{H_{RD}}\{P(X_n \to X_l|H_{RD})\},$$

Where $\lambda_{1,l}, \lambda_{1,l}, \ldots\ldots\ldots\ldots \lambda_{M_R,l}$, are the non-zero singular values of $(X_n - X_l)(X_n - X_l)^H$ and $G(x(\theta)) = \int_0^{\frac{\pi}{2}} \frac{1}{x(\theta)} d\theta$. For RD link the PEP can be upper bounded using union bound, which is very tight on high SNR,

$$P_{R\to D} \le \sum_{X_l\in C, l\neq n}^{|C|} P_{R\to D}(X_n \to X_l) =$$

$$\sum_{X_l\in C, l\neq n}^{|C|} G\left(\prod_{k=1}^{M_R}\left(1 + \frac{P_R \lambda_{k,l}\delta_{RD}^2}{4N_0 M_R \sin^2\theta}\right)^{M_D}\right) \quad (22)$$

$$P_{S\to D, R\to D} = E_{f_{\mathbb{Q}sd} f_{\mathbb{Q}rd}}\{\psi(\lambda)\}$$

$$= \frac{1}{\Pi}\int_0^{\frac{(M-1)\Pi}{M}} \int_0^\infty \psi(\lambda_{SD}) f_{\mathbb{Q}_{SD}}(\mathbb{Q}_{SD}) d\mathbb{Q}_{SD} \times \int_0^\infty \psi(\lambda_{RD}) f_{\mathbb{Q}_{RD}}(\mathbb{Q}_{RD}) d\mathbb{Q}_{RD} d\theta$$

$$\simeq \frac{1}{\Pi}\int_0^{\frac{(M-1)\Pi}{M}} \int_0^{\frac{(M-1)\Pi}{M}} \int_0^\infty \psi(\lambda_{SD}) f_{\mathbb{Q}_{SD}}(\mathbb{Q}_{SD}) d\mathbb{Q}_{SD} \times$$
$$\int_0^\infty \psi(\lambda_{RD}) f_{\mathbb{Q}_{RD}}(\mathbb{Q}_{RD}) d\mathbb{Q}_{RD} d\theta_1 d\theta_2$$

$$= M_D \sum_{m=0}^{M_D-1} \binom{M_D - 1}{m} (-1)^m F\left(\frac{bP_S\delta_{SD}^2}{N_0 \sin^2\theta} + m + 1\right) \times$$

$$\sum_{X_l\in C, l\neq n}^{|C|} G\left(\prod_{k=1}^{M_R}\left(1 + \frac{P_R \lambda_{k,l}\sigma_{RD}^2}{4N_0 M_R \sin^2\theta}\right)^{M_D}\right), \quad (23)$$

The end-to-end error probability for selection strategy II is given as,

$$P_E^{II} = P_{S\to D} \times P_{S\to R} + P_{S\to D, R\to D} \times (1 - P_{S\to R}) \quad (24)$$

Substituting (13), (14) and (23) into (24), we are getting (25),

$$P_E^{II} = M_D \sum_{j=0}^{M_D-1} \binom{M_D - 1}{j} (-1)^j F\left(\frac{bP_S\delta_{SD}^2}{N_0 \sin^2\theta} + j + 1\right)$$

$$\times M_S M_R \sum_{i=0}^{M_S M_R - 1} \binom{M_S M_R - 1}{i} (-1)^i F\left(\frac{bP_S\delta_{SR}^2}{N_0 \sin^2\theta} + i + 1\right)$$

$$+M_D \sum_{m=0}^{M_D-1} \binom{M_D-1}{m} (-1)^m F\left(\frac{bP_S\delta_{SD}^2}{N_0 \sin^2\theta} + m + 1\right)$$

$$\times \sum_{X_l \in C, l \neq n}^{|C|} G\left(\prod_{k=1}^{M_R} \left(1 + \frac{P_R \lambda_{k,l} \sigma_{RD}^2}{4N_0 M_R \sin^2\theta}\right)^{M_D}\right) \tag{25}$$

Table 1. Optimal power allocation for SR link for selection strategy I

Number of Antennas	$\mathbb{Q}_0$	$\mathbb{Q}_1$
$M_S = 1, M_R = 1, M_D = 1,$	0.60	0.40
$M_S = 2, M_R = 2, M_D = 2$	0.70	0.30
$M_S = 2, M_R = 3, M_D = 3$	0.73	0.27

3.2.1. SER upper bound

Replacing $1 - P_{S\to R} \approx 1$ at high SNR, we can write $P_{S\to D,R\to D}$ as (26),

$$P_{S\to D,R\to D} \leq \frac{M_D}{\pi^2} \int_0^{\frac{(M-1)\pi}{M}} \sum_{m=0}^{M_D-1} \binom{M_D-1}{m} (-1)^m \frac{1}{\left(\frac{bP_S\delta_{SD}^2}{N_0 \sin^2\theta} + m + 1\right)} d\theta$$

$$\times \sum_{X_l \in C, l \neq n}^{|C|} \int_0^{\frac{(M-1)\pi}{M}} \frac{1}{\prod_{k=1}^{M_R} \left(1 + \frac{P_R \lambda_{k,l} \sigma_{RD}^2}{4N_0 M_R \sin^2\theta}\right)^{M_D}} d\theta, \tag{26}$$

Using an expression given in (18), $P_{S\to D,R\to D}$ is approximated as,

$$P_{S\to D,R\to D} \leq \frac{M_D (M_D-1)! I_2^2}{\pi^2 \left(\frac{bP_S\sigma_{SD}^2}{N_0}\right)^{M_D} \left(\frac{P_R \sigma_{RD}^2 z}{4N_0 M_R}\right)^{M_D}} \tag{27}$$

Table 2. Optimal power allocation for SR link for selection strategy II

Number of Antennas	$\mathbb{Q}_0$	$\mathbb{Q}_1$
$M_S = 1, M_R = 1, M_D = 1$	0.60	0.40
$M_S = 2, M_R = 2, M_D = 2$	0.53	0.47
$M_S = 3, M_R = 3, M_D = 3$	0.48	0.52

Finally, the SER upper bound for strategy II can be written as,

$$P_E^{II} \leq \frac{M_S M_D M_R (M_S M_R - 1)! (M_D - 1)! B_0 B_1}{\pi^2 \left(\frac{bP_S\delta_{SR}^2}{N_0}\right)^{M_S M_R} \left(\frac{bP_S\delta_{SD}^2}{N_0}\right)^{M_D}} + \frac{M_D (M_D-1)! I_2^2}{\pi^2 \left(\frac{bP_S\sigma_{SD}^2}{N_0}\right)^{M_D} \left(\frac{P_R\sigma_{RD}^2 z}{4N_0 M_R}\right)^{M_D}}, \tag{28}$$

Where, $z = \sum_{X_l \in C, l \neq n}^{|C|} \lambda_l$, $\lambda_l = \prod_{k=1}^{M_R} \lambda_{k,l}$, $I_2 = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{2}} (\sin\theta)^{2M_D} d\theta$.

3.2.2. DO analysis

Substituting $P_S = \mathbb{Q}_0 P, P_R = \mathbb{Q}_1 P$, we can express SER upper bound as,

$$P_E^{II} \leq \frac{M_S M_D M_R (M_S M_R - 1)!(M_D - 1)! B_0 B_1}{\pi^2 \left(b\mathbb{Q}_0 \delta_{SR}^2\right)^{M_S M_R} \left(b\mathbb{Q}_0 \delta_{SD}^2\right)^{M_D} \left(\frac{P}{N_0}\right)^{M_S M_R + M_D}} + \frac{M_D (M_D - 1)! I_2 I_4}{\pi^2 \left(b\mathbb{Q}_0 \sigma_{SD}^2\right)^{M_D} \left(\frac{\mathbb{Q}_1 \sigma_{RD}^2 z}{4M_R}\right)^{M_D} \left(\frac{P}{N_0}\right)^{2M_D}}, \tag{29}$$

DO expression is given as,

$$DO = -\lim_{SNR \to \infty} \frac{\log(P_E^I)}{\log(SNR)} = M_D\, min(M_S M_R, M_D). \tag{30}$$

3.2.3. Optimal power allocation

Substituting $P_R = P_S - P$ in (28) and differentiating it with respect to P_S, after equating it to zero, the resultant expression we get,

$$K_1 \times (M_S M_R + M_D)(P_S)^{-M_S M_R - M_D - 1} +$$

$$K_2 \begin{bmatrix} M_D (P_S)^{M_D} \times (P - P_S)^{-(M_D+1)} \\ -M_D (P_S)^{-(M_D+1)} (P - P_S)^{M_D} \end{bmatrix} = 0 \tag{31}$$

Where K_1 and K_2 are appropriately defined constant terms. Solution of equation (31) will provide the optimal power for source and relay nodes.

4. Simulation results and discussions

We analyzed the SER performance of the single relay relaying network for the identical and best possible power factors $\mathbb{Q}_0$ and $\mathbb{Q}_1$. In Figure 3, the plots show that analytical results are in close agreement with simulated results. Also, SER simulated matches the SER upper bounds at high SNR regimes. It can be seen from the given graph that S2 performance is better than that of S1 and SER for optimal power outperforms SER for equal power allocation.

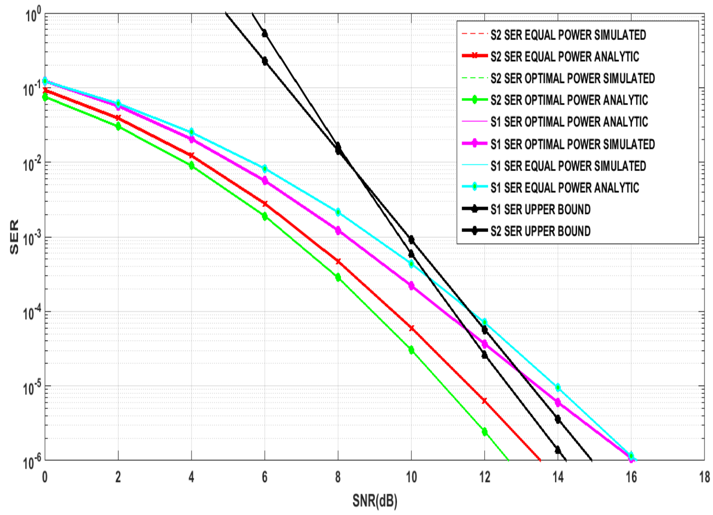

Figure 3. SER Simulated, SER Analytic, SER Upper bound for single relay relaying network with $M = 4, N_0 = 1, M_S = M_R = M_D = 2, \delta_{SD}^2 = \delta_{SR}^2 = \delta_{RD}^2 = 2, \mathbb{Q}_0 = 0.70, \mathbb{Q}_1 = 0.30$ *for selection strategy S1 and* $\mathbb{Q}_0 = 0.53, \mathbb{Q}_1 = 0.47$ *for selection strategy S2*

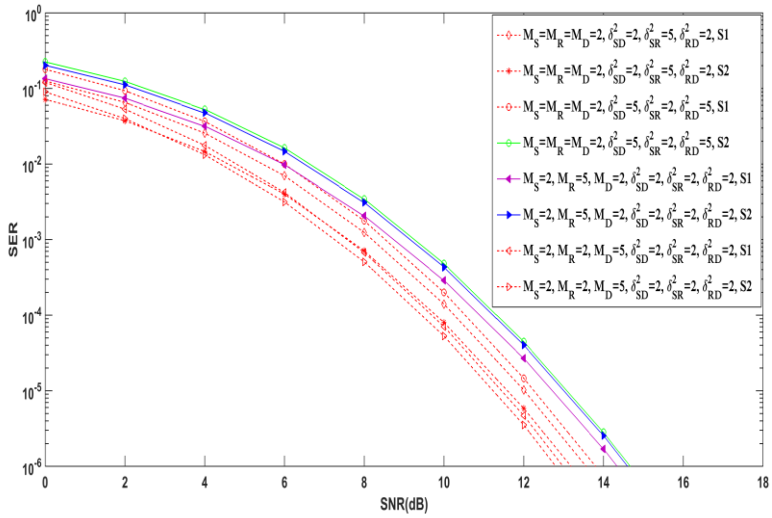

Figure 4. Comparison between selection strategy S1 and selection strategy S2 for various antenna configurations and various values of channel variables

In Figure 4, we show a comparison between selection strategy S1 and S2 in different channel conditions and different antenna configurations with $M = 4, N_0 = 1$. Figure 4, shows that when$M_S = 2, M_R = 5, M_D = 2, \delta_{SD}^2 = \delta_{SR}^2 and \delta_{RD}^2 = 2$, SER performance of strategy S2(DO=10) is better than strategy S1 (DO=8); and when $M_S = 2, M_R = 2, M_D = 5, \delta_{SD}^2 = \delta_{SR}^2 and \delta_{RD}^2 = 2$, SER performance of strategy S1(DO=10) are better than strategy S2 (DO=8). Lastly, when $M_S = 2, M_R = 2, M_D = 2, \delta_{SD}^2 = 2, \delta_{SR}^2 = 5, \delta_{RD}^2 = 2$ i.e., strategy S1 and S2 have same DO, SER performance of S1 is better than S2 by 3dB because it is based on the maximization of the SNR of source-to-destination fading link which is strong in this scenario; when $M_S = 2, M_R = 2, M_D = 2, \delta_{SD}^2 = 5, \delta_{SR}^2 = 2, \delta_{RD}^2 = 5$, the strategy S2 performs better (1dB gain) because it is based on maximization of SNR of the relay to the destination and source-to-relay fading link which are strong in this scenario.

5. Conclusion

We investigated the SER performance of two antenna selection strategy S1 and S2 for S-DF relaying network over time invariant Rayleigh fading links. We presented the closed form expression for SER analytic and SER upper bound for both antenna selection strategies. Analytical outcomes have been validated with simulated outcomes. We have conducted simulations for various antenna configurations and various channel gains. We can select the antenna selection strategy which has maximum DO for that particular antenna configuration. In case both the strategies give the same DO then one can choose strategy I, when source-to-destination link is strong and strategy II, when source-to-destination link is weak as compared to source-to-relay and relay-to-destination fading links.

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