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# Optimal sizing of permanent magnet synchronous machine for a given profile for hybrid application

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**ABSTRACT.** *The optimal sizing of electrical machines for a given profile is an important issue in automotive industry to reduce volume and so to save scarce material and cost of actuator. Two phenomena limit this volume, i.e. the magnetic saturation and the increase of temperature especially in coils end turns and in permanent magnets. Consequently, an optimal sizing requires the use of multiphysics models, i.e. coupled electromagnetic-thermal. This paper proposes an original method for optimal sizing on a given profile by the use of two models: a fast electromagnetic model where the temperature is indirectly taken into account by the electrical current density in windings. This model is implemented in a SQP optimization method. And a coupled model electromagnetic-thermal model, which provides the time evolution of the temperatures during the given profile inside the machine. This second model, more complex than the previous model requires long time calculation especially for long profile. This article proposes a strategy using the two models in order to reduce the volume of the actuator. The two models will be described and the optimization results will be given and commented.*

**RÉSUMÉ.** *Le dimensionnement optimal de machines électriques sur cycles de fonctionnement variable reste délicat et est un enjeu dans le domaine automobile où il est important de réduire le volume des matériaux utilisés pour des raisons de place limitée sous le capot et de compétitivité. La minimisation de ce volume est limitée par deux phénomènes physiques que sont la saturation magnétique et l'élévation de la température au niveau des enroulements ou des aimants permanents. Ainsi, un dimensionnement optimal en termes de volume doit s'appuyer sur un modèle multi-physique électromagnétique-thermique couplé. L'article propose une démarche de dimensionnement optimal originale s'appuyant sur deux modèles :*

*un modèle électromagnétique rapide dans lequel la température est prise en compte de manière indirecte à travers des densités de courant. Ce modèle est utilisé dans un optimiseur SQP. Un modèle électromagnétique-thermique couplé permettant de déterminer l'évolution thermique en différents points de la machine lors de l'application d'un cycle de fonctionnement connu. Ce modèle plus complexe que le précédent conduit à des temps de calculs très importants, particulièrement lors des cycles de fonctionnement de longue durée. L'article propose de décrire la stratégie d'utilisation conjointe de ces deux modèles dans le cadre d'une réduction du volume extérieur des parties actives d'une machine à aimants permanents. Les deux modèles seront décrits et les résultats d'optimisation seront donnés et commentés.*

*KEYWORDS: optimization, multiphysics simulation, thermal-electromagnetic coupling.*

*MOTS-CLÉS : optimisation, simulation multi-physique, électromagnétique-thermique couplé.*

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## **1. Introduction**

For embedded application as hybrid vehicle, the taking into account of the profile is an important way in order to reduce the volume of the actuator. Indeed, the machine works by intermittence as we can notice on Artemis profile given in Figure 1. The temperatures inside the machine could vary significantly. Its internal temperatures do not reach the steady state during the profile and consequently, we need to calculate it by thermal simulation in order to verify the thermal limits at critical time of the cycle.

The profile of the driver has been taken into account in an optimal sizing of electric machine in Nguyen (2011) for an electric vehicle where the objective was to minimizing losses during the cycle and minimizing the rms current at particular operating point. In this case, the simulation is directly implemented in the optimization process but the consequence is that models are simplified and thermal limits are indirectly taken into account by current density limit in windings. Then, the machine is designed in order to increase the autonomy of the electrical vehicle with high thermal margins and weight. In Dagusé (2013), the profile, originally with high variations, has been simplified by weighting method called partitioning classification. By this way, it has been possible to take into account the simplified profile in optimization process. In the case of this paper, we will do neither profile simplification or model reduction in order to be have a really good fitting with the reality. Consequently, the method requires electromagnetic-thermal model accurate enough with long time computation incompatible with an optimization process. For this reason, we will carry out in a first step an optimal sizing of electrical machine with a fast electromagnetic model minimizing the volume, i.e., iron volume and copper volume by respectively increasing magnetic saturation level and increasing maximal current density in winding. In this step, the thermal behaviour is not checked but indirectly adjusted by the maximal current density in windings. Consequently in the second step, an electromagnetic-thermal simulation is carried

out with the profile and will check the critical temperatures in order to readjust the current density of the first step with optimization process. After several iterations, the whole method will minimize the iron and copper material in the machine until the maximal temperatures during the given profile are close of the given temperature limits. This paper describes the results of the thesis of Küttler (2013).

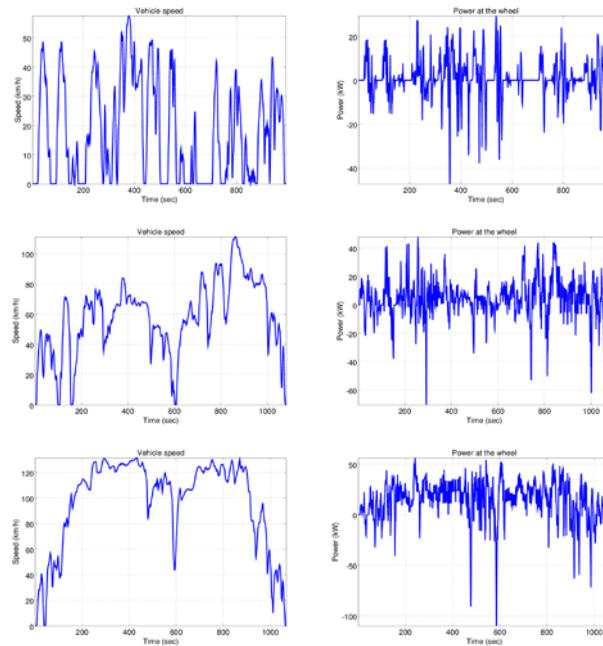


Figure 1. Artemis profiles in urban zone (up), on road (middle) and highway (down)

In the first part, this paper will introduce the state of art about multi-physics modelling coupling electromagnetic and thermal behaviour. In a second part, as the results quality is linked to models quality, a description of fast electromagnetic model and accurate thermal-electromagnetic model will be detailed. Next, a description of the sizing method using together the both models will be detailed. Finally, this method will be applied for an example of hybrid vehicle with profile on a Renault Kangoo for the sizing of the electric machine. The results about volume reduction, mass reduction and efficiency on the profile will be analysed and commented.

## 2. State of art: multi-physics modelling

The machine performances have to be calculated on the whole operating space because of the complex and variable profile including a large speed scale. So, the

electric, magnetic, mechanical and thermal quantities can vary significantly. Consequently, it requires to calculate optimal control laws taking into account nonlinearities as electric, magnetic and thermal phenomena and too, limits imposed by electric devices as inverter and battery to the electrical machine working (voltage and current limits, power limits) as in Chédot (2004). Of course, in order to have a good estimation of electrical machine performances, the developed models require coupling between physics due to influence of temperature on electric resistivity and residual magnetic field of permanent magnet (cf. Jannot (2010)).

$$\begin{aligned}\rho(T) &= \rho_0 \cdot [1 + \alpha_{Co} \cdot (T - T_0)] \\ B_r(T) &= B_{r0} \cdot [1 + \alpha_{Br} \cdot (T - T_0)]\end{aligned}\quad (1)$$

With  $\rho_0$  electric resistivity at 20° C [ $\Omega \cdot m$ ]  
 $\alpha_{Co}$  thermal coefficient of copper [ $10^{-3} \cdot K^{-1}$ ]  
 $B_{r0}$  residual field of permanent magnet at 20°C [T]  
 $\alpha_{Br}$  thermal coefficient of permanent magnet [ $10^{-3} \cdot K^{-1}$ ].

To detail precisely the couplings, there is high coupling between the thermal model and electric model because heat sources from copper losses in windings. These losses are linked to their internal temperatures because of dependency of electric resistance with temperature. The modification of this resistance modify the control laws. The second high coupling, between thermal model and magnetic model, modify the magnetic fluxes  $\Psi_d$  and  $\Psi_q$  with permanent magnet temperatures. As for electric resistance, this evolve modifies the control laws. Futhermore, iron losses, heat sources in thermal model, are really linked to control laws and consequently create another coupling between thermal model and electromagnetic model (cf. Legranger (2009)). It are too slightly dependent to magnet temperatures. Other works about multiphysics modelling as Jannot (2010) with a steady state model and in Lamghari-Jamal (2006), Fasquelle (2007) and Legranger (2009), with a transitory model. In Fasquelle (2007), thermal simulation included the profile.

### 3. Models description

#### 3.1. Fast electromagnetic model

For an electrical machine with radial flux and distributed windings, Park model can be applied the electromagnetic and electromechanical quantities can be simplified by equations given in Table 1.

Table 1. Electrical machine modeling

<b>Equations in sinusoidal working</b>	
<b>Electromagnetic quantities</b>	$\begin{cases} v_d = R_S \cdot i_d - \omega_e \cdot \Psi_q \\ v_q = R_S \cdot i_q + \omega_e \cdot \Psi_d \\ \Gamma_{em} = p \cdot (\Psi_d \cdot i_q - \Psi_q \cdot i_d) \\ P_e = v_d \cdot i_d + v_q \cdot i_q \end{cases}$
<b>Electromechanical quantities</b>	$\begin{cases} P_{méca} = \Gamma_{em} \cdot \omega_m - P_{fer} - P_{roul} \\ \Gamma_{fer} = \frac{P_{fer}}{\omega_m} \\ \Gamma_{roul} = \frac{P_{roul}}{\omega_m} \\ \Gamma_{utile} = \Gamma_{em} - \Gamma_{fer} - \Gamma_{roul} \end{cases}$

Although the analytical equations are simple, its require to calculate nonlinear magnetic fluxes  $\Psi_d$  et  $\Psi_q$  because of magnetic saturation and iron losses depending of flux density waveforms in material, which are particularly sensitive to control laws  $i_d$ - $i_q$ . For the optimal sizing of electric machine, a magnet temperature is fixed for magnetic fluxes calculation and a mean winding temperature is fixed for the calculation of winding resistance. So, we have the following relations:

- $\Psi_d = f(i_d, i_q)$ , direct flux magnetic [Wb],
- $\Psi_q = f(i_d, i_q)$ , quadratic flux magnetic [Wb],
- $P_{fer} = f(i_d, i_q, \omega_e)$ , iron losses in stator [W].

For the fluxes calculation, a nodal network of nonlinear reluctances with 70 elements and 48 nodes has been developed in order to calculate rapidly the flux tables with  $i_d$ - $i_q$  relationship for a given magnet temperature. This model provides the flux density waveforms for the calculation of iron losses and then takes into account the impact of control machine in field weakening operation. The iron losses in rotor, mainly due to slot harmonics and inverter modulations are low for this machine topology and are neglected. In table 1, in the model of machine, iron losses are used in the mechanical equations considering the high relationship with the speed rotor although it are too dependent of control laws  $i_d$ - $i_q$ . These models are detailed in Küttler (2013) and has been validated by finite elements approach. A control laws optimization implements the autopilot of electric machine with the limitations of electric devices (rms current in machine, rms current in inverter, battery voltage modelled by E-R network, power limit) and model nonlinearities (magnetic fluxes and iron losses). For the optimal sizing, these models with control

laws optimization provide the maximal torque at a given speed rotor. The calculation of control laws is formalized by the following Table 2.

Table 2. Autopilot electrical machine modeling

Control laws optimization ( $i_d^{opt}, i_q^{opt}$ )	
<b>Objective function</b>	$f = \min(-\Gamma_{utile}^*)$
<b>Constraints</b>	$\begin{cases} I_{rms}^* < \min(I_{max}, I_{inverter}) \\ V_{rms}^* < V_{available}(P_e^*) \\ P_e^* < P_{bat} \end{cases}$

Thus this control is used to check the limit of operating space by checking two operating points, *i.e.* the maximal torque at speed  $N_b$  and the maximal torque in field weakening operation at maximal speed  $N_c$ . So we can check the feasibility of operating space, which includes all the operating points of profile in motor mode (generator mode is not constrained because the fact that energy reverse can be limited by mechanical brakes).

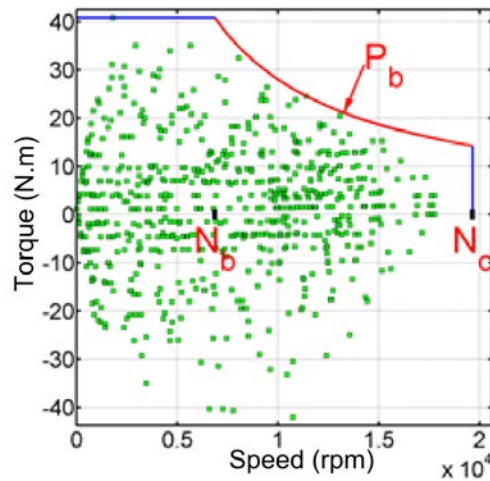


Figure 2. Working space definition for optimal sizing of electrical machine

The fast electromagnetic model is described in Figure 3:

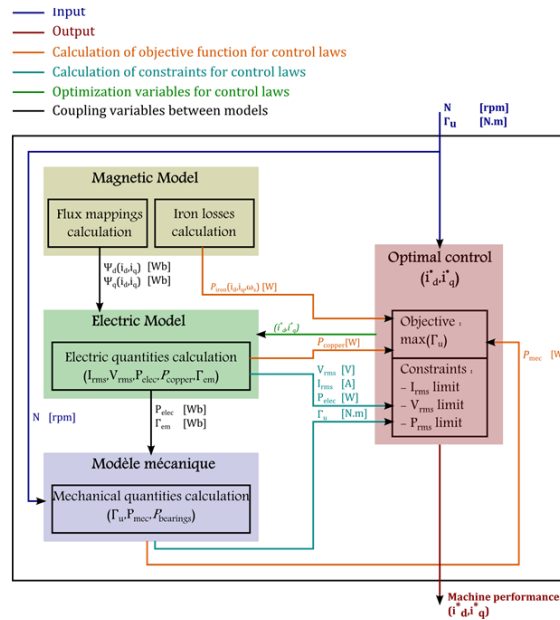


Figure 3. Fast electromagnetic model

### 3.2. Slow coupled Thermal-electromagnetic model

An accurate thermal model by nodal network with nonlinear resistances, capacities and heat sources including 258 elements and 145 nodes describes the machine geometry with rotor, stator, frame and cooling system. This model has been validated by experimental tests in Küttler (2013). The electric machine has been disassembled and rewound in order to put thermal sensor inside. Consequently, we had a well know of thermal material like dielectric insulation (with size and thermal properties), the resin and other materials decreasing uncertainties. The copper losses are coupled with copper temperatures by the change of local copper resistivity, the calculation of these losses is directly coupled with the thermal nodal network by 12 nodes of temperature distributed in slots and coils end turns and solved with the thermal model for each time step. Then the mean value of winding resistance is evaluated for time step and send to Park model. Also, with the magnet temperature, the coupling thermal-electromagnetic is carried out on flux tables. Previously in order to realize couplings during simulation, the fluxes tables and iron losses tables are calculated for several pair of currents  $i_d$ - $i_q$  and magnet temperature  $T_{mag}$ . The iron losses are calculated locally and adapted to thermal model discretization. Consequently, we have the following relationships:

- $\Psi_d = f(i_d, i_q, T_{mag})$
- $\Psi_q = f(i_d, i_q, T_{mag})$
- $P_{iron} = f(i_d, i_q, \omega_e, T_{mag})$  locally in teeth and yoke.

An optimization of control laws minimizes the total losses in machine and warrants, for each time steps, the respect of operating torque-speed points while taking account of changes of characteristics fluxes, iron losses, winding resistivity with the different temperatures and other model nonlinearities due to the control.

Table 3. Electrical machine modeling

Control laws optimization ( $i_d^{opt}, i_q^{opt}$ )	
Objective function	$f = \min(Losses^*)$
Constraints	$\begin{cases} I_{rms}^* < \min(I_{max}, I_{inverter}) \\ V_{rms}^* < V_{available}(P_e^*) \\ P_e^* < P_{bat} \\ I_u^* = \Gamma_{consign} \end{cases}$

The coupled thermal-electromagnetic model is detailed in Figure 4:

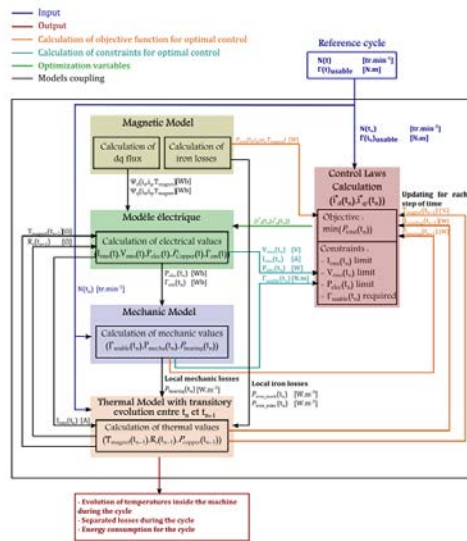


Figure 4. Coupled thermal-electromechanical model



#### 4. Sizing strategy

Finally, the idea is to solve the global sizing problem by divide it in two sub-processes. The first is a reverse problem obtaining a minimized machine with direct thermal considerations. In this optimization process, we will check that the machine could be satisfying magnetically every operating point of the profile. Although the thermal limits will not check in this step, complex phenomena as magnetic saturation and iron losses will take into account with accuracy. The second sub-process is a direct problem and gives by simulation, the evolving of temperatures inside the machine for a given profile. The purpose of the global strategy is to readjust the thermal parameter not optimized in the first step, *i.e.*, especially the current density. We can compare this global strategy with space-mapping method where the accurate model adjust the current density, which is one of sizing parameters of optimizer using the fast model.

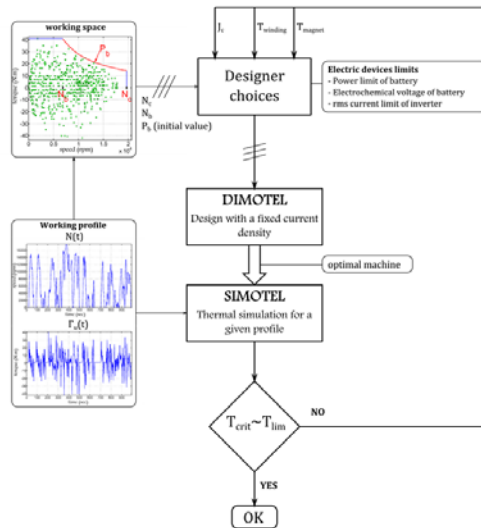


Figure 5. Design strategy

In order to save time and use complex models, the optimization algorithm is deterministic, *i.e.*, SQP method. The sizing process has to be initialized by the calculation of an initial machine. This first machine is calculated from state of art in Pyrhönen *et al.* (2008).

## 5. Results

Requirements come from demonstrator of hybrid vehicle Kangoo. The electrical machine have a cooling system with water around the yoke. The final machine, designed on its thermal limits for a given profile can work with a current density in conductors which can reach  $65 \text{ A/mm}^2$ . The fixed limit of temperature is  $100^\circ\text{C}$  on coils end turns. The initial machine has been designed in order to include all the operating points for a thermal steady state limits, so for an established temperature in coils end turns at  $100^\circ\text{C}$  at most critical operating point. The initial current density is to  $14 \text{ A/mm}^2$ .

### 5.1. Requirements

The vehicle characteristic are given below:

- Weight: 1572 kg.
- Bridge ratio: 0,2203.
- Gearbox ratio: 0,2927 – 0,5526 – 0,7805 – 1,0256 – 1,3030.
- Electrical machine bridge ratio: 4,5.

In order to increase the power density of the electrical machine, we have to increase it rotor speed and so the electrical machine bridge ratio. However, the ratio is limited for higher speed by technological limit and cost.

The torque to the wheel has been calculated for urban profile and the vehicle have to work in ZEV (Zero Emission Vehicle) with second ratio speed of gearbox. This mode is the most constrained mode for the electrical machine design.

Requirements for the electrical machine is:

- Mechanical power at point  $P_b$ : 29,328 kW.
- Maximal speed at maximal torque  $N_b$ : 6 865 rpm.
- Maximal speed  $N_c$ : 19 661 rpm.

The maximal speed is fixed by the maximal rotor speed when the vehicle works at 150 km/h at fifth speed ratio of gearbox.

The other parameters for the electrical machine sizing are given below:

- Pole pairs: 3.
- Slots number per pole and per phase  $q_s$ : 2.
- Mean temperature in copper:  $130^\circ\text{C}$ .
- Magnet temperature:  $60^\circ\text{C}$ .
- Maximal RMS current in inverter: no limit.
- Electrochemical battery voltage: 300 V.

- Power limit of battery: 70 kW.
- Maximal flux density in yoke: no limit.
- Power factor in  $P_b$  operating point: 0,9.
- Water temperature of cooling system: 22,7°C.
- External temperature: 20 °C.

No bound is imposed about the length machine or external diameter.

## 5.2. Results

The table 4 shows a comparison of volume, weight, efficiency, and maximal temperature in coil end turns of urban profile between the initial machine and the final machine. The steady-state is reached after 30 to 50 minutes and the urban Artemis profile lasts 15 minutes. So the simulation includes 12 urban Artemis profile, which represents 191 minutes for all the profiles, in order to estimate the real maximal temperatures when the thermal evolution is established periodically. The time computation by time step is around 0,25 second and the time simulation step is 1 second. The time simulation of the 12 profiles is between 15 to 20 minutes and justifies the sizing strategy in two sub-processes.

*Table 4. Design review for an urban profile*

	<b>Initial machine</b>	<b>Final machine</b>
Current density [A.mm <sup>2</sup> ]	14	65
External volume [dm <sup>3</sup> ]	1,57	0,82
Weight [kg]	13,29	5,46
Global efficiency [%]	90,03	89,30
Maximal temperature in end-winding during the profile [°C]	50,6	105,7

Between the initial to final machine, the volume reduction is 48 % as described in Figure 6. This reduction is mainly due to surface slot reduction and teeth reduction and so reducing the external stator diameter.

The Figure 7 shows the weight of active parts between the machines. We can notice a weight reduction of 10 kg. This is due to weight copper reduction from 6,36kg to 1,16kg. The slot height is largely shorter and consequently the iron volume of teeth are also reduced.

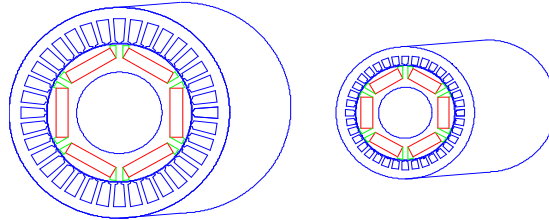


Figure 6. Dimensions of initial and final machine

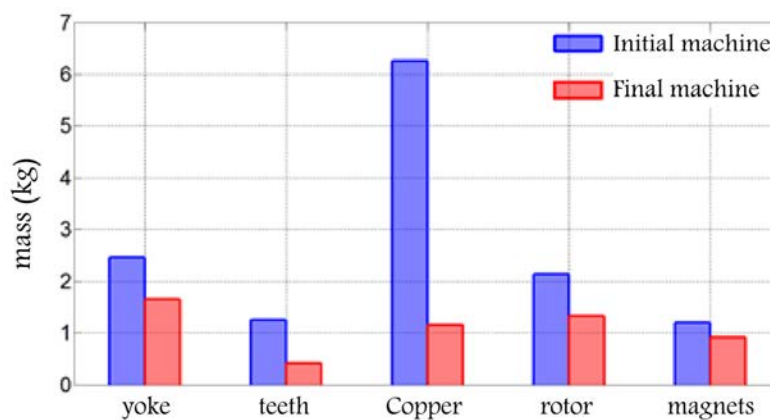


Figure 7. Mass comparison of active parts between the both machines

### 5.3. Analyse of results

The Figure 8 shows temperatures in coil end turns during the 12<sup>th</sup> urban Artémis profile.

The volume minimization of electrical machine by copper minimization volume has the effect to increase copper losses density especially during high torque demand during the profile, i.e., during accelerations. Consequently, the final machine is more sensitive to temperature increases and a difference of temperatures of 50 °C is observed in coil end turns during the 12<sup>th</sup> profile. The losses and energetic review for a profile is given in Figure 9.

The Figure 9 shows a high level of copper losses for the final machine in comparison with the initial machine, almost 5 times more than the initial machine because the fact the increase of winding resistance. Contrariwise, the iron losses have been divided by almost two by reducing the teeth volume. However, for energetic point of view on one profile, consumed energy associated to copper losses is not so high because of the fact that its occurrences are punctual during the profile

whereas iron losses are permanent as soon as the vehicle is in movement. Consequently, though iron losses have increased, the iron losses reduction, even slighter, the integral of iron losses has been largely reduced and have the effect to preserve a good efficiency on the profile.

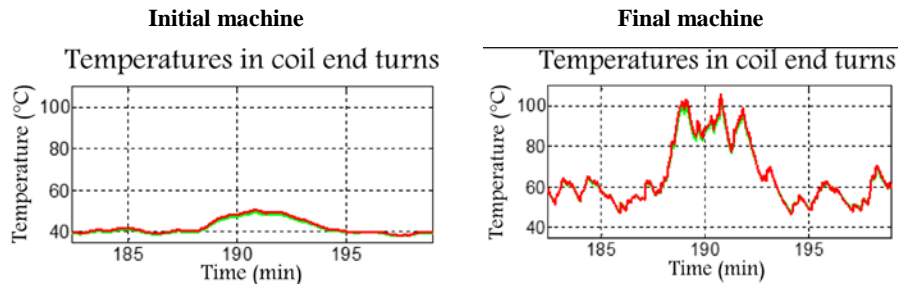


Figure 8. Time evolution of temperatures in coil end turns for the both machines

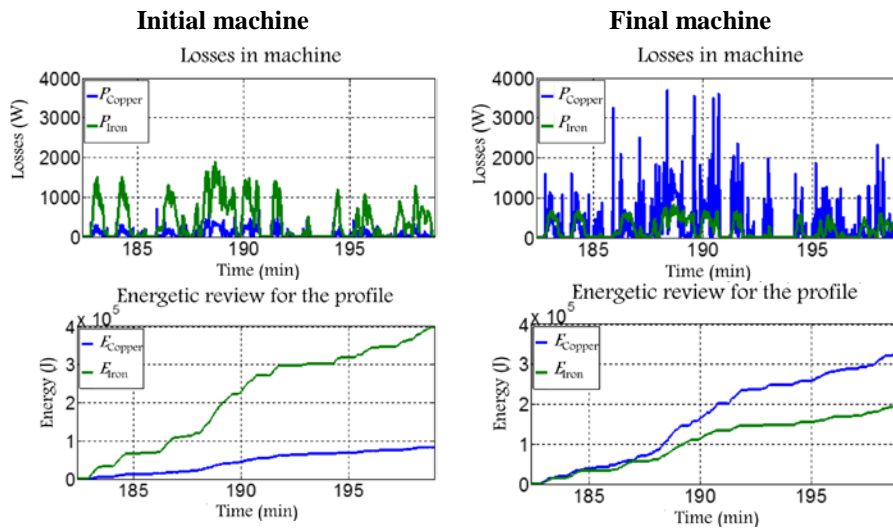


Figure 9. Energetic review for urban profile for the both machines

In other words, the iron losses are mainly in teeth and their integral in function of time during profile is large. So by reducing slot, we reduce teeth and so, we reduce iron losses. Contrariwise, the copper losses increase because of wire diameter but the integral in function of time is short almost punctual during profile.

## 6. Conclusion

The sizing strategy shows that the final machine sized with an accurate thermal model coupled with an electromagnetic model, has most of operating points out of thermal steady-state working as described in Figure 10 (represented by the green line). The ratio between rated current, defined by an established temperature of 100 °C inside end-winding and maximal current for urban profile, is 2,4.

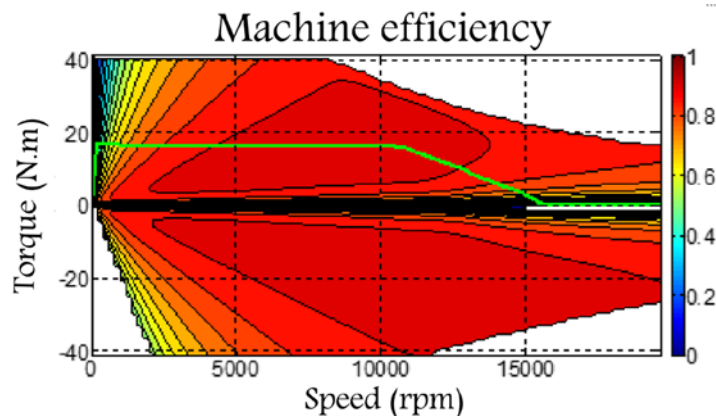


Figure 10. Efficiency mapping on torque-speed space of the final machine

By this strategy, machine volume has been largely reduced by reaching high level of magnetic saturation in iron and high current density in copper winding. This current density can reach 65 A/mm<sup>2</sup> while remaining below the limits of temperature during the profile.

Furthermore, we can notice performances about global efficiency are maintained. Indeed, although the copper losses level has been largely increased with reduction of conductor surface, these losses have short occurrences only during acceleration phases of the profile. Contrariwise, the iron losses level has been reduced, more slightly, with the iron volume reduction of teeth. However, during the profile, these losses occur during the large time where the vehicle is in movement. Consequently, with an energetic point of view, the increase of copper losses is balanced by the reduction of iron losses.

Finally, the results show that for requirements of electrical machine on a transitory profile as Artemis profile, the design taking into account the thermal limits of the actuator provides a significant gain in terms of volume and cost reduction and gives an interesting prospect for competitiveness in automotive industry.

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