Application of category theory in the generation of meta-ontologies

Maribel Mendonça[1], **Jose Aguilar**[2], **Niriaska Perozo**[1]

*1. Universidad Centroccidental Lisandro Alvarado, Lara, Venezuela
(mmendonca; nperozo) @ucla.edu.ve*

*2. Universidad de Los Andes, Mérida, Venezuela
Premeteo Researcher at the Escuela Politécnica Nacional, Quito
and
Universidad Técnica Particular de Loja, Ecuador
aguilar@ula.ve*

ABSTRACT. Meta-ontologies can be used to define a generic form of meta-concepts, which can be used for the modeling of ontologies and the ontological integration processes also. When there are several ontologies of the same domain, it is possible, from a combination process, to obtain important inputs for the generation of meta-concepts. Moreover, category theory allows defining in a formal way, the structures and the set of data that have common properties. In this article, we apply the category theory, in particular, the definitions of categories and sub-categories, in the process of generating of meta-concepts, as a way for the formalization of the automatic construction of meta-ontologies. The category theory is applied together with a collective intelligence approach based on the Ant Colony Optimization algorithm, during the combination process of multiple ontologies, in order to automate the meta-ontology construction.

RÉSUMÉ. Les méta-ontologies peuvent être utilisées pour définir une forme générique de méta-concepts, qui peut être employée pour la modélisation des ontologies et des processus d'intégration ontologique. Lorsqu'il existe plusieurs ontologies d'un même domaine, il est possible, à partir d'un processus de combinaison, d'obtenir des entrées importantes pour la génération de méta-concepts. De plus, la théorie des catégories permet de définir de manière formelle les structures et l'ensemble des données ayant des propriétés communes. Dans cet article, nous appliquons la théorie des catégories, en particulier les définitions des catégories et des sous-catégories, dans le processus de génération des méta-concepts, comme moyen de formaliser la construction automatique des méta-ontologies. La théorie des catégories est appliquée avec une approche d'intelligence collective basée sur l'algorithme d'optimisation des colonies de fourmis, lors du processus de combinaison de plusieurs ontologies, afin d'automatiser la construction des méta-ontologies.

KEYWORDS: meta-ontologies, meta-concepts, category theory, collective intelligence.

MOTS-CLÉS : méta-ontologies, méta-concepts, théorie des catégories, intelligence collective.

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1. Introduction

To conceptually model a domain, it is rarely the case that a single ontology can satisfy all the conceptual needs of the particular domain. Generally, there are several ontologies in a given domain, so that it is necessary to carry out an integration process of these multiple ontologies. The combination through the alignments among several ontologies, allows obtaining a learning or enrichment from diverse ontologies (Mendonca *et al.*, 2015). During the process of ontological enrichment, using a measure of similarity based on the common properties of the concepts aligned, can be obtained an ontological collective learning (Mendonca *et al*., 2015), which represents an important input in the definition of meta-concepts to define a domain meta-ontology that can be used later in the ontological integration. A meta-ontology is a high level ontology, which provides generic terms in the form of meta-concepts, which can be used to generate ontologies in the same domain, and as an intermediary in the integration processes of multiple ontologies.

On the other hand, category theory allows defining in a formal way, structures and data set that have common properties. The Category theory studies “objects” and “morphisms” among them (Asperti and Longo, 1991). Morphisms in the category theory correspond to the properties of the relationships between the objects. The category theory can describe several concepts in a uniform way. A category models one class and its relationships with others (Aliyu *et al.*, 2015).

There are several works that have studied the category theory and its application in the area of ontologies. In (Aliyu *et al*., 2015) is proposed to categorically model the syntax and semantics of a RDF ontology, as a step towards the formalization of ontological operations using the category theory. In (Zimmermann *et al*., 2006), the category theory is used to represent the alignment of ontologies, regardless of the language used in the ontology and the alignment technique employed, for which they define a categorical structure with objects and morphisms to model the alignment between ontologies.

In the area of meta-ontologies, some studies determine the importance of their use in the generation of ontologies and data integration processes. In (Cho *et al*., 2006), a meta-ontology is used for the integration of data sources with semantic heterogeneity. They propose some meta-concepts that can be used by ontology developers, in order to describe concepts in the domain of electronic parts libraries. In (Yudelson *et al*., 2005), a meta-ontology is proposed for a high level classification in the area of user modeling. To develop this meta-ontology, first, essential concepts of the domain were selected, and then, in a stage of generalization and refinement, similar or synonymous terms are grouped, and the dominant term is selected by the authors.

With regard to the creation of the concept categories that can form a meta-ontology, there is a work presented in (Kokla and Kavouras, 2001), which generates a set of formal concepts, conceptual classes or categories, in the geographical area, for unifying the different conceptualizations of the geographical space. In this case, they consider the essential properties of the concepts (*i.e.*, the permanent or necessary properties of a concept, since they ensure the semantic definition of the concept). One of the processes that result from the integration of ontologies is the generation of a more general ontology, as a result of merging a set of ontologies (at least 2 ontologies) (Pinto *et al*., 1999). However, this is a process that has not been formalized, and in which there are not concrete proposals. On the other hand, (Rangel *et al.*, 2015) propose a process of integration of ontologies, based on the automatic suggestion of ontology alignments. The automatic approach for the comparison and selection of alignment techniques is based on the ABC algorithm, which uses as criteria the runtime, the number of concepts aligned, and the number of times that the colony chooses each technique.

Among the weaknesses of previous work is the fact that the process of generating of a meta-ontology is not an automatic process, and the elements required and the steps to follow have not been formalized either, to apply to any domain or to allow the ontological emergence. For these reasons, it is considered important to make a proposal in this area.

On the other hand, the algorithms based on collective intelligence inspired by the collective behavior of some living beings, represent a novel paradigm of distributed intelligence, which have been used for the resolution of optimization problems, among other problems. This work proposes the generation of categories of concepts based on the category theory and the application of the collective intelligence algorithm called ACO (Ant Colony Optimization), during the process of combining of multiple ontologies proposed in (Mendonca *et al.*, 2015). The objective is to establish generic concepts that allow forming a meta-ontology of a domain, from the knowledge (traces) generated by the ants during the process of enrichment of an ontology due to the combination with other N ontologies with which already exist an alignment. This process considers a measure of similarity based on the properties of the concepts, in order to select the best alignment among the concepts that offer a greater enrichment for the origin ontology.

Thus, this article aims to apply the category theory, in particular, the definitions of categories and sub-categories, in the process of generating meta-concepts, as a way to formalize the process of automatic generation of meta-ontologies, based on the combination process of multiple ontologies using the ACO algorithm proposed in (Mendonca *et al.*, 2015). This paper is organized as follows: first, the theoretical bases about meta-ontologies and category theory are presented, next, the description of the proposal is made, and finally, a case study and the conclusions are shown.

2. Theoretical Aspects

2.1. Meta-Ontology

A meta-ontology provides generic terms in the form of meta-concepts, which can be reusable to model other ontologies (Guizzardi, 2007; Ramos and Nuñez, 2007). The meta-ontology is composed of a set of meta-concepts, which are defined based on the categories identified (Guizzardi, 2007). The meta-ontologies can help to establish the categories of entities that exist in a specific domain. When a domain is very standard, it is easier the alignment of an ontology with others in the same domain.

A high-level ontology, or meta-ontology, can also be used in the process of integration and alignment of ontologies (Mascardi *et al.*, 2010). On the other hand, in (Milton and Smith, 2004) is stated that a high level ontology can contribute to the integration of data and the interoperability of information systems.

The concepts of a meta-ontology are called meta-concepts. A meta-concept represents a generic concept, which can be used to generate other more concrete concepts during the design of an ontology. The meta-concepts have an explicit ontological semantic, which helps to identify concepts consistently, and to structure them systematically (Cho *et al.*, 2006). The meta-concepts can help to establish categories of entities in a specific domain, which allows grouping similar concepts, and thus, facilitating their manipulation and integration.

The definitions around the meta-ontologies that will be used in this work can be modeled with RDF ("Resource Description Framework"), which is a set of specifications for representing information and resources on the web (Aliyu *et al.*, 2015). The basic structure is a set of triples consisting of a subject, a predicate, and an object.

2.2. Category Theory

According to (Spivak, 2014), a category consists of a set of objects that are related in some way. A category is a collection of data that satisfies a particular property. The category theory is the mathematical theory of structures, of great importance for its ability to express relationships among structures (Aliyu *et al.*, 2015). Based on this theory, the definitions of category and subcategory are presented, both definitions are key to our proposal (Asperti and Longo, 1991; Barr and Wells, 1998; Spivak, 2014).

A **category C** is a structure in which the following elements participate:

– A set of objects **Obj(C),** denoted as A, B, C ...;

– A set of morphisms **Mor(C),** denoted as f, g, h, ...;

– A relation that associates to each morphism to a pair of objects, which is denoted as follows:

$$f: A \rightarrow B$$

A and B represent the domain and co-domain of the morphism f, respectively.

One category **D** is a **sub-category** of another category **C** if:

– $Obj(D) \subseteq Obj(C)$

– $Mor(D) \subseteq Mor(C)$

In particular, a morphism is a function, property or characteristic that associates an object with another. For example, 2 objects can be Vehicle(A) and Person(P), and the morphism Move(T) associates these 2 objects:

$$T: A \rightarrow P$$

2.3. Ant Colony Optimization

The algorithms of artificial ant colonies are algorithms of collective intelligence that are directly inspired by the behavior of the real colonies of ants to solve different problems, such as the combinatorial optimization problems (Aguilar, 2001, 2014 ; Bonabeau *et al.*, 1999). They are based on a colony of artificial ants, that is, simple computational agents, which cooperatively work and communicate through artificial pheromone traces. The ACO algorithm is one of the more known collective intelligence algorithm, with a vast literature. The environment where the ants walk can be modeled by a graph. Each arc of the graph represents the possible steps that the ant can give, and the selection of the next step (movement of an ant) is guided by a heuristic information that measures the heuristic preference of moving from one node to another, and the quantity of artificial pheromone in the traces that measures the “learned desirability” of the movement. The trace information is modified during the execution of the algorithm, depending on the quality of the found solutions by the ants (see (Aguilar, 2001; 2014; Bonabeau *et al*., 1999) for more details about the ACO algorithm).

3. Our Proposal

In this article, we apply the concepts of the category theory, in the process of generating a meta-ontology for a specific domain, based on the combination process of multiple ontologies using the ACO algorithm proposed in a previous work (Mendonca *et al.*, 2015). Besides, we also use a measure of similarity based on common properties of the concepts, to establish the categories and subcategories, as the input for the generation of meta-concepts, which will form a meta-ontology of the domain.

Thus, to define our proposal, first of all a summary of the application of the ACO algorithm in the combination process of multiple ontologies is presented (see section 3.1), then we define the measure of property-based similarity (see section 3.2). Next, the macro-algorithm for the generation of meta-ontology is described (see section 3.3). The definition of the concepts of the category theory, on which the previous macro-algorithm is based is presented (see section 3.4). Next, the macro-algorithm for the generation of meta-concepts is described, which is the central element for the generation of meta-ontologies (see section 3.5), to culminate with the definition of metrics to evaluate the quality of the meta-ontologies generated (see section 3.6).

3.1. Application of the ACO Algorithm in the Multiple Combination Process

The problem of enrichment of an objective ontology from multiple ontologies can be defined as a combinatorial optimization problem, where the ACO algorithms can be used to find its solution. The work presented in (Mendonca *et al.*, 2015) defines the alignments between an O ontology and a set of N ontologies belonging to the same domain, based on the ACO algorithm. The objective is the generation of a meta-ontology of domain, using the trace information generated by the ants as input, seeking the highest enrichment of the O ontology. The next paragraph describes the solution process based on the ACO algorithm, proposed in (Mendonca *et al.*, 2015)

The process of comparison of the O ontology with the O'_1, O'_2, ... O'_N ontologies, with which already exists a series of alignments (A_1, A_2, ... A_N), consists in selecting for each concept C belonging to O, the best possible alignment of the N alignments, based on the greater similarity and the greater enrichment that can obtain the O ontology. The ants walk through a graph that is defined based on all possible combinations of alignments for each concept in the O Ontology. During the search process, the ants select for each concept in the O Ontology one of the alignments as part of the solution, from an information that comes given by a measure of similarity between the concepts, and the traces of the pheromones that allow exploring all possible solutions. The enrichment degree is used to update the pheromone throughout the ant path. An evaporation process is evenly performed in all arcs present in the solution space. Thereby, this algorithm allows the enrichment of an ontology with ontologies in the same domain (see (Mendonca *et al.*, 2015) for more detail).

In this paper, we propose the utilization of the information generated by the ACO algorithm during the search process, for the generation of a meta-ontology for that domain. Particularly, each ant in a cycle is guided by the previous experiences of the ants available through the pheromones dispersed in the graph, and by the similarity measure. After several iterations, the search of the ant colony converges due to that the combination of alignments selected by the ants does not change during the last iterations, or after a maximum number of iterations.

Initially, the similarity measure plays a very important role in the search process, since the pheromone values are practically zero. It allows selecting the alignment that offers a greater measure of similarity between two concepts. The ACO algorithm allows exploring all the alignments of the concepts, in order to detect one that offers a greater similarity, as also a greater enrichment degree. This enrichment degree is defined by the number of new concepts that the origin ontology obtains by selecting an alignment for a given concept. Besides, the enrichment degree determines the quality of the solution, which is used to guide the search process. Once the solution is constructed by an ant, then its quality is determined in order to update the pheromone in the search space. Higher the quality of the solution, then greater the amount of pheromone that is concentrated in the alignments among selected concepts (arcs of the search space). The alignments with greater amount of pheromone will be more attractive for the ants in the following iterations. At the end, the ACO algorithm converges with the best alignment for each concept of the O ontology.

3.2. Similarity of Properties

The similarity between properties is made based on methods proposed in (Guarino and Welty, 2000; Altamiranda *et al.*, 2015). These methods compare the distance between the set of properties of two classes. The objective is to identify if each property p_n of the set of properties P of a class C coincides with another property p'_m of the set of properties of P 'of another class C'. For this comparison, a measure of lexical similarity is used, specifically, the Levenshtein Distance[1]. Thus, the similarity of properties of classes C and C' is:

$$Sim\_P(C, C') = \frac{|P \cap P'|}{|P \cup P'|} \tag{1}$$

Where:

– P y P' are the sets of properties of the classes C and C'

– $|P \cap P'|$ is the intersection between these sets.

– $|P \cup P'|$ is the union between these sets.

The macro-algorithm for calculating the "Similarity of properties" (Sim_P) is called "Algorithm 1". It obtains the list of properties of the two elements to compare (steps 3 and 4), to calculate the similarity between its properties (step 7). The similar properties are counted, considering those that have a lexical similarity greater than a given threshold (step 11). After obtaining the number of similar properties, an

1. Levenshtein's distance, is a distance between words, which is the minimum number of operations required to transform one string into the other. It is widely used in information theory and computer science (Source: Wikipedia).

average is determined considering the number of common and non-common properties (step 12). In this case, we define:

– |PA ∩ PB|: is the set of the common properties between concepts A and B.
– |PA ∪ PB|: is the set of all the properties of the concepts A and B.

Algorithm 1. Macro-algorithm to calculate the similarity between properties

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Input: Class A, Class B
Procedure:
1: **Similarity between Properties** (Class A, Class B)
2: {
3: **Get** Property List (PA) from A. Get_Properties ()
4: **Get** Property List (PB) from B. Get_Properties ()
5: **For** i from 1 to PA.length()
6: **For** j from 1 to PB.length(){
7: Get Lexical similarity
8: SL[i][j]=Distance(PA[i].name, PB[j].name) }
9: //Calculate number of similarities between properties (common properties), using a threshold
10: //Calculate the total of common and non-common properties
11: |PA ∩ PB| = Number of common properties
12: |PA ∪ PB| = Number of common and non-common properties
13: //Calculate the similarity S (Equation 1)
14: $S = \frac{|PA \cap PB|}{|PA \cup PB|}$
15: Return S
16: }
Output: S

3.3. Generation of meta-ontologies

For the generation of a domain meta-ontology, the meta-concepts must be determined. These meta-concepts represent generic classes, which have inheritable properties. Specifically, during the tour of the ants in a search space, a collective learning process is carried out, where for each alignment of a given concept of the O ontology, important information is obtained for the establishment of the generic classes based on the common properties among the aligned concepts. Also, the quality of the solutions used to update the pheromone considers the common properties of the aligned concepts. In this way, all the learning that the ants obtains in their different routes can be used to form a generalized ontology of the domain, that is, a meta-ontology. Particularly, the collective knowledge generated by the ants is the set of common properties between the aligned concepts, which are the properties of a higher or general class that define the meta-concepts.

In order to register and use this collective learning, when the ants build their solutions then they update the “Collective Learning Matrix of Properties” (MACP, for its acronym in Spanish). The ants place in MACP the common properties between the concepts involved in the visited alignments, or the new concepts obtained in the enrichment of the O ontology (see Table 1).

The process for the generation of meta-ontologies is detailed in the "Algorithm 2" macro-algorithm. In the MACP matrix, in the columns are the concepts of the source ontology of the multiple combination process, and in the rows, first, the aligned concepts of the ontologies involved in the multiple combination process, and then, the new concepts obtained through the source ontology during its enrichment. These new concepts are considered important for the generation of the meta-ontology, since they are neighboring concepts (children, siblings or parents) with some similarity.

Algorithm 2. Macro-algorithm for generation of meta-ontology

Inputs: Common properties (PC) between concepts.
Procedure:
1: To register and to use the PCs, through the "Collective Learning Matrix of Properties" (MACP, for its acronym in Spanish). Table 1 shows the structure of the MACP, which is a matrix where the common properties of the concepts are recorded. In each cell, the "Common Properties List" (LPC, for its acronym in Spanish) is saved, with the names of the common properties between two concepts: if there are two concepts C and C ', it would be the set of properties of $| P \cap P' |$, obtained using the Macro-algorithm of Similarity of Properties.
2: After the generation of MACP, the groupment process begins with the concepts that have common properties. In this case, a "Table of Concepts with Common Properties" (TCPC, for its acronym in Spanish) is generated (See Table 2).
3: The generation of meta-concepts will be done from the TCPC, applying the macro-algorithm for the generation of meta-concepts called "Algorithm 3", based on the definitions presented in section 3.3.
Output: Meta-Ontology.

Table 1. Collective learning of properties matrix (MACP)

		Concepts of the source ontology			
		C_1	**C_2**	**…**	**C_N**
Aligned concepts	**C'_1**	$PC_{1,} \ldots PC_N$	…	…	…
	C'_2	…			
	…				
	C'_N	…			…
New acquired concepts	**CN_1**	…			…
	CN_2	…			…
	…				
	CN_N	…			…

The generation of meta-concepts will be carried out from the TCPC shown in Table 2, applying the “Algorithm 3” macro-algorithm for the generation of meta-concepts, based on the definitions that are presented in Section 3.3.

Table 2. Table of common properties among concepts (TCPC)

Common Property	**Concepts**
PC_1	C_1, C_2 , $C_{3....}$, C_N
PC_2	C_1 , C_2 , $C_{3....}$, C_N
…	**…**
PC_N	C_1 , C_2 , $C_{3....}$, C_N

3.4. Definitions

In this section are presented the definitions based on the category theory, which formally define the process of meta-ontology generation.

Definition 1. A ***Context*** X=(C, P, M, R, S) is a combination of a set of ontologies in RDF format *(O)*, where C is a set of concepts, P is a set of properties, M is a sub-set of the Cartesian product CxC, R a set of relationships of incidence between properties of concepts, and S a set of relationships between parent-child concepts.

Definition 2. M is composed by the Cartesian product CxC, of the concepts that participate in the properties, and they correspond to the subjects and objects that participate in the RDF triple. The first term of each pair CxC will be the subject, and the second term will be the object.

Definition 3. The incidence relation R can be represented as: P→M or P→CxC. It is given by the properties that define a connection between a pair of concepts, or rather, subjects and objects. Corresponds to the RDF triple, or RDF ontological structure:

$$R:\ S \xrightarrow{\quad P \quad} O$$

Definition 4. The ***Domain*** of an incidence relation R is represented by the subject that participates in the property, and corresponds to the first term of the pair CxC. It corresponds to the domain of a morphism in a category. In an RDF structure, it corresponds to the domain of a property, that is, the "rdfs: domain" of the "owl: ObjectProperty".

Definition 5. The ***Range*** of an incidence relation R is represented by the object that participates in the property (morphism), and corresponds to the second term of the pair CxC (co-domain) of a morphism in a category. The range of a property with an RDF structure is “rdfs: range” of the “owl: ObjectProperty”.

Definition 6. In order to establish a relation between a **FATHER** concept, or a base class, and a **SON** concept, or a derived class, the relation "Sub-Class" is used, and in this case, it is represented by:

$$\textbf{FATHER} \leftarrow \textbf{SON}$$

The RDF term to describe that class1 is a subclass of class2 is:

Class1 "rdfs:subClassOf" Class2

Definition 7. A **Category** is a collection of concepts that have one or more properties in common. Based on Category theory, we have that in a context X, a category Cat1 will be defined for a structure (C, P), where:

– C is a set of Concepts (Objects in the Category Theory).

– P is a set of Properties (Morphisms in the Category Theory), such that if:

$$f \in P, \text{ then } \quad f : A \rightarrow B \text{ , for } \quad A,B \in C$$

Where A is the domain and B is the range of the property.

Definition 8. The ***Scope*** of a category is represented by the set of domain concepts that are used for its properties (morphism).

Definition 9. The ***Precision*** of a category is represented by the set of properties of the category.

Definition 10. A Cat1 category is a ***Sub-Category*** of Cat2 (Cat1 $\subseteq$ Cat2) if C1 $\subseteq$ C2 and P1 $\subseteq$ P2, where C1, C2 are the set of concepts, and P1, P2 are the set of properties of Cat1 and Cat2, respectively.

Definition 11. An ontological category ***Cat-O*** is a category made up of all concepts and properties involved in the context.

Definition 12. A generic sub-category ***Sub-Cat-G*** is one with a range greater than a threshold (this threshold will be a minimum defined according to the universe of concepts. If it is not defined, then it is used 1).

Definition 13. A specific sub-category ***Sub-Cat-E*** is one with scope equal to a threshold (this threshold will be a minimum defined according to the universe of concepts. If it is not defined, then it is used 1)

Definition 14. A list of Ordered Sub-Categories (LSO, for its acronym in Spanish) for a context X is defined as:

$$\text{LSO}=\{\text{Sub-Cat}_1,\ \text{Sub-Cat}_{2,\ldots,}\text{Sub-Cat}_N \mid \forall\ \text{Sub-Cat}_i \subseteq \text{Cat-O y} \\ \text{Sub-Cat}_{i+1} \subseteq \text{Sub-Cat}_i\ \}$$

Where sub-categories are sorted from the most general (greater scope), to the most specific (lower scope).

3.5. Macro-Algorithm for generating meta-concepts

The macro-algorithm for the generation of meta-concepts in the context X = (C, P, M, R, S) is called Algorithm 3.

Algorithm 3. Macro-algorithm for the generation of meta-concepts

Inputs: A context X=(C,P,M,R,S)

Process:

1: The Cat-O ontological category is established, which will consist of all the concepts and properties involved in the context.

2: From the Cat-O global category, the possible sub-categories that can be defined from this global ontology are defined, following the definitions of sub-categories (see definitions 12 and 13).

3: The LSO is created for the context X. The LSO is created based on previous defined sub-categories, which have been ordered.

4: The sub-categories are classified as Sub-CAT-G and Sub-Cat-E.

5: The Sub-CAT-G are established as candidates for a meta-concept.

6: To structure the meta-ontology for the context, based on the Sub-Cat-G, the relationships of "Sub-Class" between these sub-categories are established. Specifically for this case, a Cat1 is "Sub-Class" of a Cat2, if Cat1 is "Sub-Category" of Cat2. Based on this, the scope of the parent class must be greater than the scope of the child class.

7: Note: The name to identify the meta-concepts (sub-categories) of the meta-ontology will be given by the properties (precision of the subcategory) and the range that define each subcategory.

Output: The Meta-Ontology for the context X = (C, P, M, R, S))

3.6. Quality Metrics of a Meta-Ontology

The obtained meta-concepts represent generic classes, which come to be super-classes of the concepts present in the ontologies. One concept can be a sub-class of a meta-concept, if it can inherit all its properties:

$$\mathrm{MC} \leftarrow \mathrm{C} \iff \mathrm{P}_{\mathrm{MC}} \subset \mathrm{P}_{\mathrm{C}} \tag{2}$$

Where:

– P_C: Set of Properties of Concept C.

– P_MC: Set of properties of meta-concept MC.

To determine which meta-concept is associated with the concept, we consider first the leaf meta-concepts in the meta-ontology. If there is not leaf meta-concept from which all its properties can be inherited, the meta-concepts of the next higher level are considered, and so on, until reaching the root of the meta-ontology.

To determine the quality or validity of a meta-ontology, we will consider 3 properties defined in (Guizzardi, 2007): *Robustness*, *Completeness and Precision*.

These properties or measures of quality, will determine the validity of a meta-ontology with respect to an ontology.

Robustness: A meta-ontology MO is Robust (R) with respect to an ontology O, if each meta-concept in MO represents at least one concept (or perhaps several) in the ontology O. The degree of robustness can be defined by the following way:

$$R(MO, O) = \frac{|MC\_R|}{|MC|} \tag{3}$$

Where:

– MC_R: Set of Meta-Concepts that meeting the criterion of robustness:

$$\forall\ \mathrm{MC} \in \mathrm{MO}, \exists\ \mathrm{C} \in \mathrm{O} \mid \mathrm{MC} \leftarrow \mathrm{C}$$

– MC: Set of all Meta-Concepts.

Completeness: A meta-ontology MO is Complete (C) with respect to an ontology O, if each concept in O is represented by at least one meta-concept in OM. The degree of completeness can be defined as follows:

$$C(MO, O) = \frac{|C\_MC\_C|}{|C|} \tag{4}$$

Where:

– C_MC_C : Set of concepts of O that are defined for some meta-concept:

$$\forall\ \mathrm{C} \in \mathrm{O}, \exists\ \mathrm{MC} \in \mathrm{MO} \mid \mathrm{MC} \leftarrow \mathrm{C}$$

– C: Set of all concepts of O.

Precision: A meta-ontology MO is Precise (P) with respect to an ontology O, if each concept is associated maximum to a meta-concept (or in any case none) in MO. The degree of accuracy can be defined as follows:

$$\mathrm{P}(\mathrm{MO}, \mathrm{O}) = \frac{|\mathrm{C\_MC\_P}|}{|\mathrm{C}|} \tag{5}$$

Where:

– C_MC_P: Set of O concepts that are defined for only one meta-concept:

$$\forall\ \mathrm{C} \in \mathrm{O}, \exists!\ \mathrm{MC} \in \mathrm{MO} \mid \mathrm{MC} \leftarrow \mathrm{C}$$

– C: Set of all concepts of O.

4. Case Study

To visualize the process of generation of a meta-ontology, the ontology shown in Figure 2 is considered, which is part of the resulting ontology of the combination process of multiple ontologies proposed in (Mendonca *et al.*, 2015), for the transport area. In that work, two ontologies (O1 and O2) are proposed to enrich the source ontology (see Figure 1).

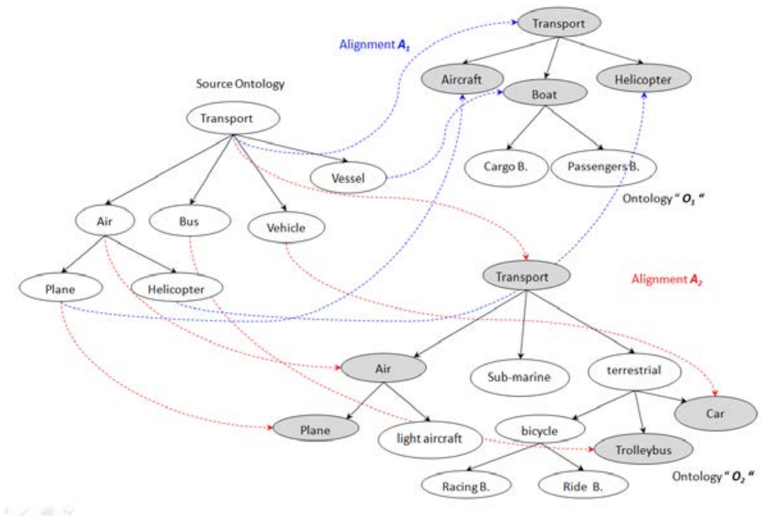

Figure 1. Participating ontologies in the multiple combination process (Mendonca et al.*, 2015)*

The application of ACO in a problem implies the definition of the space of solutions that will visit the ant colony in search of the optimal solution (see section 3.1). In this case, the problem is defined as the combination of an ontology, called "source ontology", with other N ontologies, in this case 2 ontologies, which have already been aligned with it. In the case of the Figure 1, for the enrichment of the O ontology using the ontologies O1 and O2, already exists a set of alignments A1 and A2 for each one with O, and the idea is to select for each concept C belonging to O, the best possible alignment from the N alignments already defined, based on the highest similarity and enrichment that may obtain the O ontology. These possible combinations of the concepts Cs belonging to O with the aligned concepts with them, define the space of solutions that will visit the ants.

On the other hand, among each pair of aligned concepts, a similarity measure is defined, which refers to the grade of similarity between them. In (Mendonca *et al.*, 2015), during this multiple combination process were used the lexical, property and structural similarity metrics. They use the property similarity metric to compare the different concepts aligned among the ontologies, to determine the common properties. This is the necessary input to define the generic concepts that will be candidates to meta-concepts of multiple ontologies. Table 3 shows the properties of some concepts of the ontologies involved in the process, and Table 4 the similarity values between the concepts, using these similarity metrics.

Table 3. Properties of the concepts

Ontology O	***Properties***
Transport	*Move_Person*
Air	*Travels_through_Air*
Bus	*Travels_through_Land, Has_wheel*
Vehicle	*Travels_through_Land, Has_wheel*
Plane	*Travels_through_Air, Has_wings*
Helicopter	*Travels_through_Air, Has_Rotor*
Vessel	*Travels_through_Water*
Ontology O1	***Properties***
Transport	*Move_Person*
Aircraft	*Travels_through_Air, Has_wings*
Boat	*Travels_through_Water*
Helicopter	*Travels_through_Air, Has_Rotor*
Ontology O2	***Properties***
Transport	*Move_Person*
Air	*Travels_through_Air*
Sub-Marine	*Travels_through_Water*
Terrestrial	*Travels_through_Land*
Plane	*Travels_through_Air, Has_wings*
Light Aircraft	*Travels_through_Air, Has_wings*
Bicycle	*Travels_through_Land, Has_wheel*
Trolleybus	*Travels_through_Land, Has_wheel*
Car	*Travels_through_Land, Has_wheel*

Finally, to determine the quality of the selected alignment is considered the "Grade of Enrichment" (GE) criterion, which is an indicator of the amount of new concepts obtained by the source ontology after selecting an alignment for a concept. This criterion, the similarity metrics, and in general, the multiple combination process, are explained in detail in (Mendonca *et al.*, 2015).

The resulting ontology from the multiple combination process proposed in (Mendonca *et al.*, 2015), is shown in Figure 2. There, the new concepts acquired by the ontology are highlighted. These new concepts will be used for the generation of meta-concepts. In specific, for the generation of the meta-ontology, the common properties between the concepts of the origin ontology and the new concepts acquired that enrich the ontology are considered.

Table 4. Similarities values among concepts

Ontology O	***Ontology O1***	***Sim (C,C´)***
Transport	*Transport*	*1*
Air	-	-
Bus	-	-
Vehicle	-	-
Plane	*Aircraft*	*0.8*
Helicopter	*Helicopter*	*1*
Vessel	*Boat*	*0.9*
Ontology O	***Ontology O2***	***Sim(C,C´)***
Transport	*Transport*	*1*
Air	*Air*	*1*
Bus	*Trolleybus*	*0.5*
Vehicle	*Car*	*0.8*
Plane	*Plane*	*1*
Helicopter	-	-
Vessel	-	-

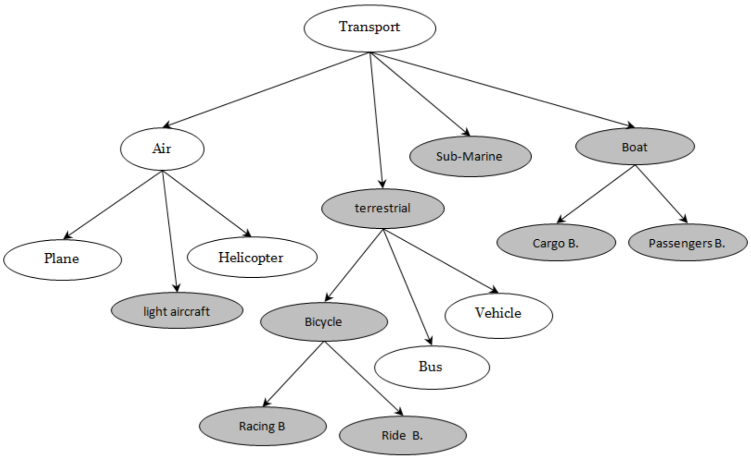

Figure 2. Resulting Ontology after the combination process of multiple ontologies (Mendonca et al., *2015)*

To show the application of our approach, a part of this resulting ontology will be taken, which is shown in the Figure 3.

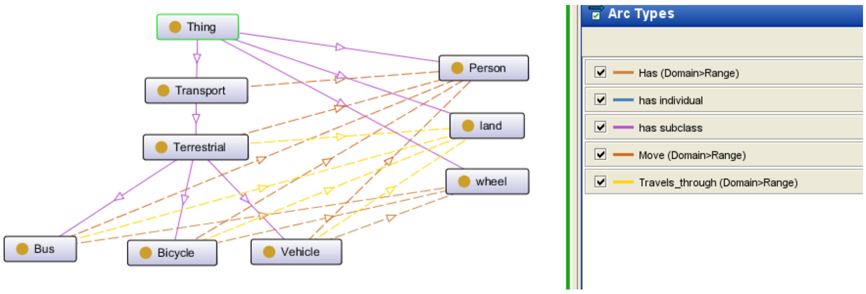

Figure 3. Part of the resulting ontology after the combination process (Mendonca et al., *2015).*

The main steps for the generation of a meta-ontology are shown in the "Algorithm 2" macro-algorithm. We show the execution of this macro-algorithm in the ontology of the Figure 3. First, it generates the MACP matrix for generating meta-concepts that will be candidates to form the meta-domain ontology. For this, the common properties between the concepts of the source ontology and the aligned ontologies are considered, as well as the new acquired concepts that enrich the source ontology. These properties are recorded in the Table 5 (it is the resulting MACP).

At the moment that the ants in each iteration of the algorithm compare each concept of the origin ontology with the aligned concepts, and determine the similarity of properties, select the common properties between both concepts and register them in MACP. Then, the same process is carried out between the concepts of the origin ontology and the new concepts that constitute the enrichment. In this case, the properties of the origin concept are compared with the properties of the new neighboring concepts (sons, brothers and parents).

Thus, this matrix is the result of the collective learning that is generated via ACO algorithm. The common properties were determined by calculating the similarity between the properties of the involved concepts according to Equation (1), with a threshold of 0.8 in order to consider that two properties are similar or nearly equal. If the similarity is 1, then it means that they are the same and the name of the property is taken as it is. If it is less than 1, then they are similar and one of the two names is taken indistinctly to identify the common property.

It is important to emphasize that the considered concepts are those that participate as "domain" in the property being considered, and the name of the property must contain the "range" referenced.

After of the generation of the MACP matrix, the concepts that have common properties are grouped to generate the TCPC matrix (see Table 6). In this table, the information is integrated, relating the properties that are common between the concepts that act as a domain in the property.

Table 5. MACP for the case study

	Concepts of the Source Ontology							
	Concept	**Transport**	**Air**	**Bus**	**Vehicle**	**Plane**	**Helicopter**	**Vessel**
Aligned concepts	Transport (O1)	Move_Person						
	Transport (O2)	Move_Person						
	Air (O2)		Move_Person Travels_through_Air					
	Trolleybus (O2)			Move_Person, Travels_through_Land, Has_Wheel				
	Car (O2)				Move_Person, Travels_through_Land, Has_Wheel			
	Boat(O1)							Move_Person Travels_through_Water
	Aircraft (O1)					Move_Person Travels_through_Air, Has_wings		
	Plane (O2)					Move_Person Travels_through_Air Has_wings		
	Helicopter(						Move_Person	

	Concepts of the Source Ontology							
	O1)						Travels_through_Air, Has_Rotor	
New acquired concepts	Terrestrial			Move_Person, Travels_through_Land	Move_Person, Travels_through_Land			
	Sub-Marine							Move_Person, Travels_through_Water
	Light Aircraft		Move_Person, Travels_through_Air			Move_Person, Travels_through_Air, Has_wings	Move_Person Travels_through_Air	
	Bicycle			Move_Person, Travels_through_Land, Has_Wheel	Move_Person Travels_through_Land, Has_Wheel			
	Cargo B.							Move_Person Travels_through_Water
	Passengers B.							Move_Person Travels_through_Water
	Racing B.				Move_Person, Travels_through_Land, Has_Wheel			
	Ride B.				Move_Person, Travels_through_Land, Has_Wheel			

Table 6. TCPC for the case study

Common Property	**Concepts**
Move_Person	Transport, Terrestrial, Air, Bus, Car, Vehicle, Bus, Trolleybus, Bicycle, Racing B., Ride B., Plane, Aircraft, Light Aircraft, Helicopter Boat, Vessel, Sub-Marine. Cargo B., Passengers B.
Travels_through_Land	Bus, Car, Vehicle, Terrestrial, Bus, Trolleybus, Bicycle, Racing B., Ride B.
Travels_through_Air	Air, Plane, Aircraft, Light Aircraft, Helicopter
Travels_through_Water	Boat, Vessel, Sub-Marine. Cargo B., Passengers B.
Has_Wheel	Bus, Car, Vehicle, Terrestrial, Bus, Trolleybus, Bicycle, Racing B., Ride B.
Has_wings	Air, Plane, Aircraft, Light Aircraft,
Has_Rotor	Helicopter

Starting from this last table (only the concepts and properties shown in Figure 3 will be used to show the process), the macro-algorithm is executed for the generation of the meta-concepts (see Algorithm 3), applying the corresponding definitions. First, the elements in the context must be established, according to definition 1:

– Set of ontologies O: O1, O2 y O_source (See Figure 1).

– Set of concepts C:

C={ Transport (Tra),Terrestrial (Ter), Bicycle (Bic), Bus (Bus), Vehicle (Veh), Land (Lan), Wheel (Whe),Person(Per)}

– Set of morphisms or properties P:

P={Move_Person(Mov_Per),Travels_through_Land(Tra_Lan), Has_Wheel (Has_Whe) }

– According to Definition 2, set M of cartesian product CxC:

M={<Tra,Per>,<Ter, Per>, <Bic, Per>, <Bus, Per>,<Veh, Per>, <Ter, Lan>, <Bic, Lan>, <Bus, Lan>,<Veh, Lan>, <Bic, Whe>, <Bus, Whe>,<Veh, Whe>}

– According to definition 3, set R of incidences between properties and objects:

R={ Mov_Per → <Tra, Per>, Mov_Per → <Ter, Per >, Mov_Per → <Bic, Per>, Mov_Per → <Bus, Per >, Mov_Per → <Veh, Per >, Tra_Lan → <Ter, Lan>, Tra_Lan → <Bic, Lan>, Tra_Lan → <Bus, Lan>, Tra_Lan → <Veh, Lan>, Has_Whe → <Bic, Whe>, Has_Whe → <Bus, Whe>, Has_Whe → <Veh, Whe>}

– According to definitions 5 and 6, the concepts that belong to the Domain and Range are the following:

Domain={Tra, Ter, Bic, Bus, Veh}

Range={Lan, Whe, Per}

In terms of a RDF ontology, an example of the domain and range specification is as follows:

```
<owl:ObjectProperty rdf:about="http://.../Ontologia_Transport#Has_Wheels">
<rdfs:domain rdf:resource="http://.../Ontologia_Transport#Bicycle"/>
<rdfs:range rdf:resource="http://.../Ontologia_Transport#Wheels"/>
</owl:ObjectProperty>

<owl:ObjectProperty rdf:about="http://.../Ontologia_Transport#Travels_through_Land">
<rdfs:domain rdf:resource="http://.../Ontologia_Transport#Bus"/>
<rdfs:range rdf:resource="http://.../Ontologia_Transport#Land"/>
</owl:ObjectProperty>
```

– According to definition 6, set S of parent-child relationships:

$$S=\{Tra \leftarrow Ter,\ Ter \leftarrow Bic,\ Ter \leftarrow Bus,\ Ter \leftarrow Veh\}$$

In terms of a RDF ontology, and OWL language, it would be :

```
<owl:Class rdf:about="http://.../Ontologia_Transport#Bus">
<rdfs:subClassOf rdf:resource="http://.../Ontologia_Transport#Terrestrial"/>
</owl:Class>
```

Following the steps of the macro-algorithm for the generation of meta-concepts (see Algorithm 3):

1. Cat-O (based on definitions 7 and 11) is defined as the ontological category of the context, which contains all concepts and morphisms involved in the ontology:

   Cat-O=({ Tra, Ter, Bic, Bus, Veh, Lan, Whe, Per},
   {Mov_Per, Tra_Lan, Has_Whe})

2. Starting from the global category Cat-O, and according to definition 10, some sub-categories that can be defined from this global ontology are:

   Sub-Cat1=({Tra,Ter,Bic,Bus,Veh,Per,Lan,Whe}, {Mov_Per,Tra_Lan,Has_Whe}) Scope= 5
   Sub-Cat2=({Ter,Bic,Bus,Veh,Lan,Whe}, {Tra_Tie,Has_Whe}) Scope=4
   Sub-Cat3=({Bic, Bus, Veh, Whe}, {Has_Whe}) Scope= 3

3. The list of ordered sub-categories LSO is created for the previous context X, according to definition 14:

   LSO={Sub-Cat1, Sub-Cat2, Sub-Cat3}

4. Sub-CAT-G and Sub-CAT-E are defined based on definitions 12 and 13. Sub-CAT-G would be: Sub-Cat1, Sub-Cat2, Sub-Cat3. In this case, there are no specific sub-categories.

5. Sub-CAT-G's are established as candidates for meta-concepts: The candidates for meta-concepts are Sub-Cat1, Sub-Cat2 and Sub-Cat3.

6. To structure the meta-ontology for this context, based on Sub-Cat-G, the relationships of "Sub-Class" between sub-categories are established (see Figure 4): Sub-Cat3 ← Sub-Cat2 ← Sub-Cat1

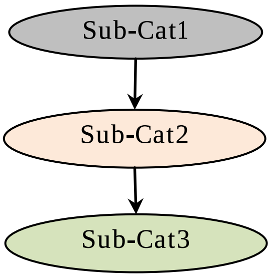

Figure 4. Candidate sub-categories for meta-concepts

The name to identify the meta-concepts of a meta-ontology, will be given by the properties and the range that define each subcategory. The resulting meta-concepts are shown in Figure 5. Specifically, in Figure 5, on the left side is part of the resulting ontology from the alignment of the ontologies proposed in (Mendonca *et al.*, 2015), which was previously shown in Figures 1 and 2, and on the right side are the Category Candidates for meta-concepts.

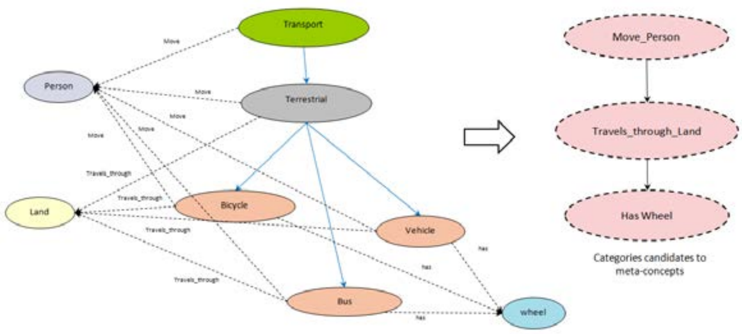

Figure 5. Categories for meta-concepts

The categories selected for this case in transport area are: "Move_Persons", "Travels_through_Land", "Has_Wheels", which through the "Sub-class" relationship are structured to create a hierarchy of classes, in order to form a meta-ontology for the domain. Each one will be a meta-concept of the domain, that will be associated through the property to the corresponding rank:

Move
"Move_Persons" ---------------------> "Persons"
Travels
"Travels_through_Land" ---------------------> "Land"
Has
"Has_Wheels" ---------------------> "Wheel"

This process must be done considering all concepts of the participating ontologies in the multiple combination process (see Figure 1). The resulting meta-ontology for the transport domain is shown in Figure 6.

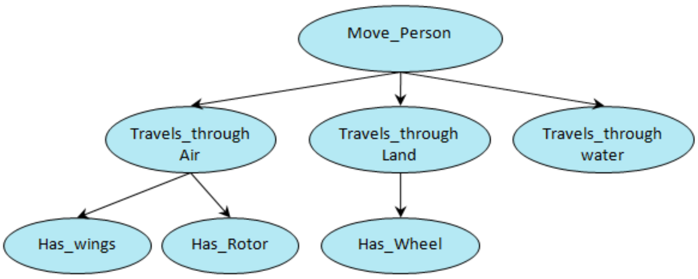

Figure 6. Resulting meta-ontology for the transport domain

To determine the quality of the resulting meta-ontology, the criteria presented in section 3.5 are applied. In this case, the quality of the resulting meta-ontology (MO) can be determined, with respect to the set of ontologies that participated in the multiple combination process (O1, O2 and O_Source). For this, the criteria of *Robustness, Completeness and Precision* will be considered.

The degree of robustness of MO with respect to O_Source, according to Equation (3), is:

$$R(MO,O) = \frac{|MC\_C\_R|}{|MC|} = \frac{7}{7} = 1$$

Where:

– MC_C_R = {Move_Person, Travels_through_Air, Travels_through_Land, Travels_through_Water, Has_Wings, Has_Rotor, Has_Wheels }

– MC = {Move_Person, Travels_through_Air, Travels_through_Land, Travels_through_Water, Has_Wings, Has_Rotor, Has_Wheels }

The degree of robustness of MO with respect to O1 is:

$$R(MO, O_1) = \frac{|MC\_C\_R|}{|MC|} = \frac{5}{7} = 0.70$$

Where:

– MC = {Move_Person, Travels_through_Air, Travels_through_water, Has_Wings, Has_Rotor }

– MC = {Move_Person, Travels_through_Air, Travels_through_Land, Travels_through_water, Has_Wings, Has_Rotor, Has_Wheels }

The degree of robustness of MO with respect to O2 is:

$$R(MO, O_2) = \frac{|MC\_C\_R|}{|MC|} = \frac{6}{7} = 0.85$$

Where:

– MC_C_R = {Move_Person, Travels_through_Air, Travels_through_Land, Travels_through_water, Has_Wings, Has_Wheels }

– MC = {Move_Person, Travels_through_Air, Travels_through_Land, Travels_through_water, Has_Wings, Has_Rotor, Has_Wheels }

On average, the degree of robustness of the meta-ontology is: 0.83.

The degree of completeness of MO with respect to O_Source, according to Equation (4), is:

$$C(MO, O) = \frac{|C\_MC\_C|}{|C|} = \frac{7}{7} = 1$$

Where:

– C_MC_C = {Transport, Air, Bus, Vehicle, Plane, Helicopter,Vessel}

– C = {Transport, Air, Bus, Vehicle, Plane, Helicopter,Vessel}

The degree of completeness of MO with respect to O1 is:

$$C(MO, O_1) = \frac{|C\_MC\_C|}{|C|} = \frac{6}{6} \; 1$$

Where:

– C_MC_C = {Transport, Aircraft, Helicopter, Boat , Cargo B., Passengers B.}

– C = {Transport, Aircraft, Helicopter, Boat, Cargo B., Passengers B.}

The degree of completeness of MO with respect to O2 is:

$$C(MO, O_2) = \frac{|C\_MC\_C|}{|C|} = \frac{11}{11} = 1$$

Where:

– C_MC_C ={Transport, Air, Terrestrial, Plane, Light aircraft, Sub-marine, Bicycle, Trolleybus, Car, Racing B., Ride B.}

– C = {Transport, Air, Terrestrial, Plane, Light aircraft, Sub-marine, Bicycle, Trolleybus, Car, Racing B., Ride B.}

On average, the degree of Completeness of the meta-ontology is 1.

The degree of precision of MO with respect to O_Source, according to Equation (5), is:

$$P(MO, O) = \frac{|C\_MC\_P|}{|C|} = \frac{7}{7} = 1$$

Where:

– C_MC_P = {Transport, Air, Bus, Vehicle, Plane, Helicopter, Vessel}

– C = {Transport, Air, Bus, Vehicle, Plane, Helicopter, Vessel}

The degree of precision of MO with respect to O1 is:

$$P(MO, O_1) = \frac{|C\_MC\_P|}{|C|} = \frac{6}{6} = 1$$

Where:

– C_MC_P = {Transport, Aircraft , Helicopter, Boat , Cargo B., Passengers B.}

– C = {Transport, Aircraft , Helicopter, Boat , Cargo B., Passengers B. }

The degree of accuracy of MO with respect to O2 is:

$$P(MO, O_2) = \frac{|C\_MC\_P|}{|C|} = \frac{11}{11} = 1$$

Where:

– C_MC_P ={Transport, Air, Terrestrial, Plane, Light aircraft, Sub-marine, Bicycle, Trolleybus, Car. Racing B., Ride B.}

– C = {Transport, Air, Terrestrial, Plane, Light aircraft, Sub-marine, Bicycle, Trolleybus, Car. Racing B., Ride B.}

On average, the degree of precision of the meta-ontology is 1.

According to the obtained results, the average degree of robustness of the meta-ontology is 0.83, the average precision degree is 1, and the degree of completeness is 1, which means that the meta-ontology describes all concepts of the domain (Complete), with the highest precision, and also, the concepts in the meta-ontology correspond quite well to the concepts described in each ontology in the domain (Robustness). Getting a value of 1 in robustness is very difficult, and would correspond to a perfect alignment between all the ontologies involved in the integration process.

5. Conclusions and future work

In this work, the category theory was applied to formally present the process of generation of a domain meta-ontology, based on the collective learning generated during the process of multiple combination of ontologies proposed in (Mendonca *et* *al.*, 2015). Our approach exploits the collective learning generated by ants, in order to build meta-concepts. The measures of similarity based on the properties of the concepts, identify the common properties in order to establish the categories (meta-concepts) that form the meta-ontology for a specific domain.

In this process of formalization, the definition of similarity based on the properties of the concepts is determinant, since it provides the inputs to identify the common properties and establish the categories, which allow defining the meta-concepts. Based on the category theory, the ACO algorithm allows grouping concepts with common properties to define generic concepts that will be the meta-concepts.

In particular, the category theory defines the "category" and "subcategory" concepts, which can be applied within the ontologies to establish the categories associated with a specific domain, based on concepts that share the same properties, which correspond to the objects and morphisms established in this theory. In the case study is shown the grouping of concepts with common properties, and the structuration of the categories, until the generation of a meta-ontology, following our approach.

It is important to highlight that no previous works were found where the category theory is used in the creation of meta-ontologies. The category theory has been used for the alignment of ontologies (Zimmermann *et al.*, 2006) or the formalization of ontological operations (Aliyu *et al.*, 2015). Additionally, the previous works about the generation of meta-ontologies, such as (Kokla and Kavouras, 2001) and (Pinto *et al.*, 1999), do not present any formal process that can be applied in some domain with defined ontologies; therefore, no strategies and definitions have been established in the process of generation of meta-ontologies. Thus, this proposal is considered an important contribution in this area, which starts from a set of ontologies in a given domain and from an automatic collective learning process (see Tables 2 and 5), to achieve the construction of a meta-ontology for that domain.

As future work, it is important to emphasize that some definitions must still be established to complement those presented in this paper, to establish categories and meta-concepts in an ontological context without common alignments and properties among the ontologies to be integrated. Our proposal starts from those two aspects: there are alignments and common properties between the ontologies to integrate. On the other hand, the formalization presented here can be complemented and applied in an ontological emergence process where meta-ontologies play a fundamental role. Therefore, it is interesting to incorporate this formalization of meta-ontology generation into a global schema, like the proposition in (Mendonca *et al.*, 2016), where this approach would provide a way to integrate the ontologies that are being incorporated into the ontological framework of the system, including those that emerge, as inputs for the generation of new meta concepts. It is also important to apply the proposals to much more complex and real case studies, for a real validation of the proposal and thus corroborate the quality of the resulting meta-ontologies.

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