THE ICE FORMATION METHOD: A NATURAL APPROACH TO OPTIMISE TURBOMACHINERY COMPONENTS

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ABSTRACT
Ice formation phenomena can be observed in many natural and technical processes. The formation of an ice layer is mainly dependent on the surrounding thermal and flow characteristics such as surface temperatures and natural or forced convection in fluids. Those, however, are greatly influenced by the geometric constraints of the considered system. A naturally grown ice layer aspires in steady state to a minimum of energy dissipation. Driven by this goal, this phenomenon can be used as a natural idea creator for a further numerical optimisation of complex geometries. This paper presents in detail the methodology of an approach combining a natural optimisation represented by the naturally grown ice layer together with a conventional numerical optimisation method. Exemplarily, this approach is applied to optimise the shape of the separating web in a 180-degree bend in terms of minimised pressure drop as it is used in internal gas turbine cooling channels. For this, the separating web is cooled and the different ice layers are classified according to Reynolds numbers and wall to fluid temperature ratios. In a second step, the experimentally best performing ice contour is taken as starting geometry for a subsequent applied numerical optimisation using evolutionary algorithms.
Keywords: 180-degree bend, evolutionary strategy, ice formation method, optimisation, turbine blade cooling channel.

1 INTRODUCTION

1.1 Ice formation and solidification as a natural optimisation tool

Processes of solidification and accumulation, either manipulative or spontaneous, occur in several technical and natural applications. Exemplary mentioned are for instance the freeze-shut of pipelines or the casting of metals. In these cases, melting and freezing need to be controlled in order to minimise losses or even prevent destruction of those systems. Several analogies can also be found in nature, such as atherosclerosis in the human blood circuit or sedimentation in rivers, governed by the transport equation for the sediments rather than by the energy equation.

Ice formation and solidification studies have been performed over the past 40 years with two major goals. On the one hand, solidification phenomena have been investigated for a better understanding of basic questions concerning the formation of ice layers and the freezing especially in internal flows. On the other hand, extensive research has been done to improve and optimise the performance of technically relevant geometries for internal as well as external flows using these phenomena.

Early studies of the liquid solidification in tubes have been performed by Zerkle and Sunderland [1] and Özişik and Mulligan [2]. The former investigators dealt with the comparison of their theoretical and experimental data at steady-state conditions upon laminar flow. The latter studied the transient freezing behaviour in forced flow analytically. An analysis of solidification in parallel-plate channels with turbulent flow was followed by Shibani and Özişik [3] theoretically and by Seki et al. [4] in an experimental manner. Weigand and Beer [5, 6] provided several experimental, theoretical, and numerical investigations of wavy ice layers occurring inside a cooled parallel-plate channel. As a result, a good agreement was found by comparing their numerical predicted ice layers with their measurements. Further research has been done by Neumann [7] who studied the formation of ice layers in a planar diffuser both experimentally and numerically. Weigand et al. [8] performed ice layer formation experiments

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in a planar nozzle with different convergence angles in order to find and approve the criteria for the occurrence of wavy ice layers.

Additionally to the above mentioned basic investigations of the solidification phenomena, the formation of ice layers has been used to optimise technical configurations in terms of drag reduction, pressure loss, or heat transfer. This method has been applied for both internal and external flows. Using the ice formation as a natural optimisation method, Carlson [9] investigated the solidification of water on a cooled circular cylinder in cross flow and its effect on the drag coefficient both theoretically and experimentally. However, due to large error bars in the test environment, his results were inconclusive. Similar studies were followed by Okada et al. [10] and Cheng et al. [11] who primarily paid attention on the ice layer distribution and the heat transfer along the cylinder surface.

In a further step, LaFleur and Langston [12, 13] used the ice formation design method to reduce the drag of a junction between a cylinder and a flat endwall. Ice layer contours determined by ice formation experiments were analysed in a high-speed wind tunnel at high Reynolds numbers. The wind tunnel tests resulted in an average of 18% lower drag of the ice-form-contoured endwall and, therefore, clearly reduced secondary losses caused by the cylinder/endwall junction. Applying this knowledge to a real turbine configuration, LaFleur et al. [14] could determine an averaged heat transfer reduction by 24% compared to a rationally symmetric geometry of a second vane endwall.

Concurrently, a simple model for an internal flow was introduced by LaFleur [15]. He established a theoretical basis for selecting control parameters of a Couette ice form model. Applied to a diffuser, LaFleur's [16] results showed different designs with higher pressure recovery performance than conical diffusers. Besides, several studies by LaFleur [17, 18] supplied an introduction to the general ice formation design optimisation theory. A large number of study cases and governing equations were characterised and discussed in the mentioned published work.

In the present study, the ice formation method and its application to an internal gas turbine blade cooling channel is described in detail. It is used to improve the geometry of the separating web naturally by means of experiments. The resulting contours give an idea of well-performing shapes, i.e. 'good candidates' of novel geometries are found. Combining the experimental approach with a numerical optimisation, the ice-shaped web contour is taken as the starting geometry for a further numerical optimisation procedure. The contour is optimised with the objective of minimised pressure drop across the channel applying evolutionary algorithms as classical numerical optimisation tools. Hereby, the focus lies on the presentation of the method, its basic ideas, and their applicability to known technical problems.

1.2 Optimisation of geometries

Optimisation of complex geometries exposed to fluid flow is difficult since local separation and re-attachment phenomena induce local vortices and are often followed by highly three-dimensional flow characteristics. Each change of the geometry influences the flow characteristics often in an unpredictable way due to its non-linear character.

Besides, classical numerical optimisation methods are all characterised by the steps 'selection' and 'variation' [19] (see Fig. 1), meaning that well-performing candidates are selected and then varied in order to obtain an optimum. Their use bear the difficulty of the preliminary definition of restrictive parameters, such as the number of design variables, their position, and their solution space, forming the geometric contours that are to be investigated. This decision of the parameter definition is normally left to the engineering experience of the person in charge and is therefore restrictively humanly controlled. However, choosing self-evident approaches as good candidates for starting contours limits the amount of solutions to a pool of reproductions of this starting geometry and does not create innovative solutions with possibly better performance.

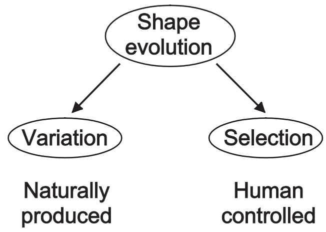

Figure 1: Shape evolution by variation and selection adapted from [15].

In order to overcome some of these restrictions, methods exist to create systematically parameter pools that define a geometric problem, such as the morphological box [20] or the dimensional analysis [21]. Interactions of the free parameters and their weighting and impact on the final solution can then be analysed using the Taguchi method [22], initially developed to analyse the weighting of single parameters in a production chain, but also applicable to numerical problems. For example, it was applied by Lutz et al. [23] to compare different parameterisation approaches and the parameter's interactions by means of good and fast convergence.

The here presented method, though, shows a completely different approach to create a pool of defining parameters. Without submitting the process to a human-controlled goal function, the ice contour adapts continuously to the flow according to the given thermal and geometric constraints. It belongs, according to the classification scheme of LaFleur [16], to the harnessed natural methods (see Fig. 2). Influencing parameters are only the test conditions, mainly the flow velocity and the temperature ratio between the chilled wall and the flowing liquid.

Due to the fact that the growing of an ice layer is a natural process and any impact caused by human control is reduced to a minimum, the ice formation method can help to provide innovative geometries and create larger manifolds in the pool of possible solutions.

The obtained ice contours are 'optimised' by nature. With the proper parameter control, the ice layer even tends to a state of minimum energy dissipation as has been shown analytically by LaFleur for a Couette flow [15]. This means that the final (or optimum) pathway can represent a compromise of minimum energy consumption and minimum flow resistance. In this way, it is possible to get an idea of how a new and innovative geometry could look like. However, the method is not likely to be applied to find directly an optimum, such as it is represented in the *constructal law* of the generation of flow configuration by Bejan [24] but rather used as an idea creator.

Therefore, since other optimisation criteria such as minimised drag or minimised heat transfer rates are more often the desired goal functions in technical applications, in the present paper, the experimentally found ice contour is taken as starting geometry for a subsequently applied numerical optimisation with the goal function of minimum pressure drop across the channel. The naturally grown ice shape determines in that way the choice of parameterisation for the numerical optimisation, normally left to the engineer's experience and knowledge of the problem.

Applying this approach, the optimisation starts – with regard to Fig. 2 – being a natural one with the ice formation. A promising geometry is then chosen from the several experiments, i.e. a 'good candidate', which means that the optimisation continues to become humanly controlled. The process is switched from the right branch in Fig. 2 to the left branch. The chosen geometry is then submitted to a conventional numerical optimisation algorithm, so the process ends in being purely analytical. This combination of natural and numerical approach leads to a large new geometry range, which is impossible to be detected by standard methods.

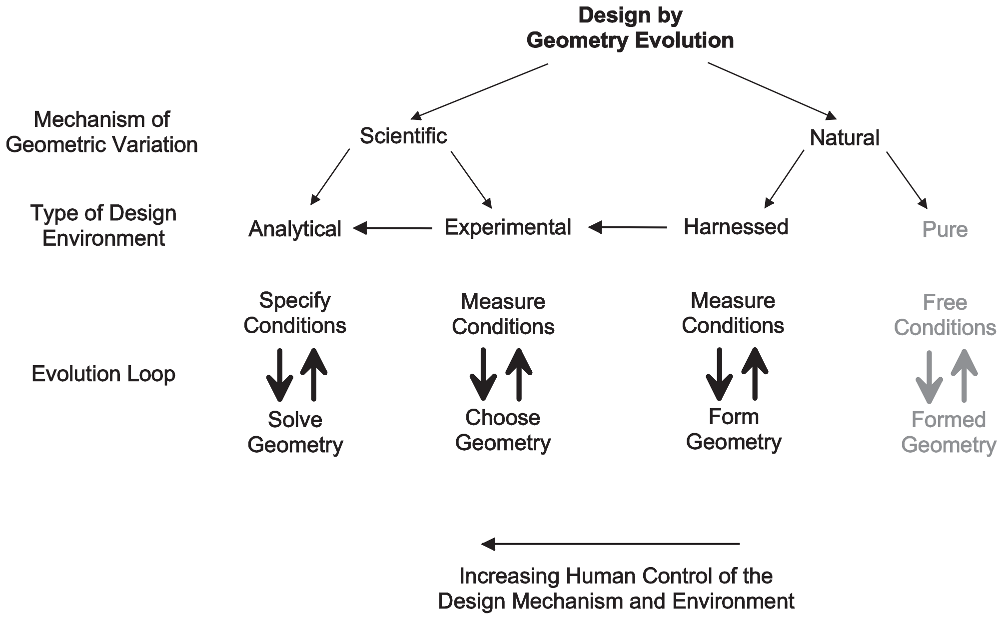

Figure 2: Design classification tree adapted from [16].

2 THE ICE FORMATION METHOD AND ITS APPLICATION TO AN INTERNAL COOLING CHANNEL CONFIGURATION

The optimisation of technically applied geometries and configurations in turbomachinery always follows certain goal functions which depend on the range of application. Apart from design limits, heat transfer and aerodynamics or hydrodynamics play an important role especially when designing electronic and high-temperature devices. The desired increase of efficiency requires an optimisation of both heat transfer and aerodynamic performance. However, the variation of these factors is often limited by economic and ecologic reasons.

An example for a high-temperature combustion device, mentioned above, is a simple aircraft turbine engine. In order to increase the thermal efficiency of such a gas turbine, one can increase, for example, the turbine inlet temperature. This, however, leads to the necessity of a higher cooling air mass flow which has to be taken from the compressor. The reduced inlet mass flow of the turbine thus leads to a lower degree of thermal efficiency of the whole engine. Therefore, an optimal balance between heat transfer and pressure loss must be found in order to reach the optimal operating point.

One of the most heavily loaded components in turbomachines is a turbine blade of the first high-pressure stage since those are directly exposed to the hot gas from the combustor. The turbine inlet temperature clearly exceeds the maximum allowable temperature of the blade's material. Therefore, a very complex system of cooling channels and blowing holes is required to provide an active cooling of the blade's metal surface, as it is shown exemplarily in Fig. 3. Higher temperatures for a higher thermal efficiency would either require a higher cooling air mass flow or an optimised geometry of the cooling channels. The latter is the objective of engineers around the globe with the goal to make aircraft engines more efficient and produce lower emissions without loosing performance.

The blade's internal multi-pass cooling system, shown in Fig. 3, is designed containing some 180-degree bends for deflecting the flow. The flow field of such a 180-degree bend shows some loss producing sources, e.g. separation zones and vortices, and a very non-uniform heat transfer distribution. The

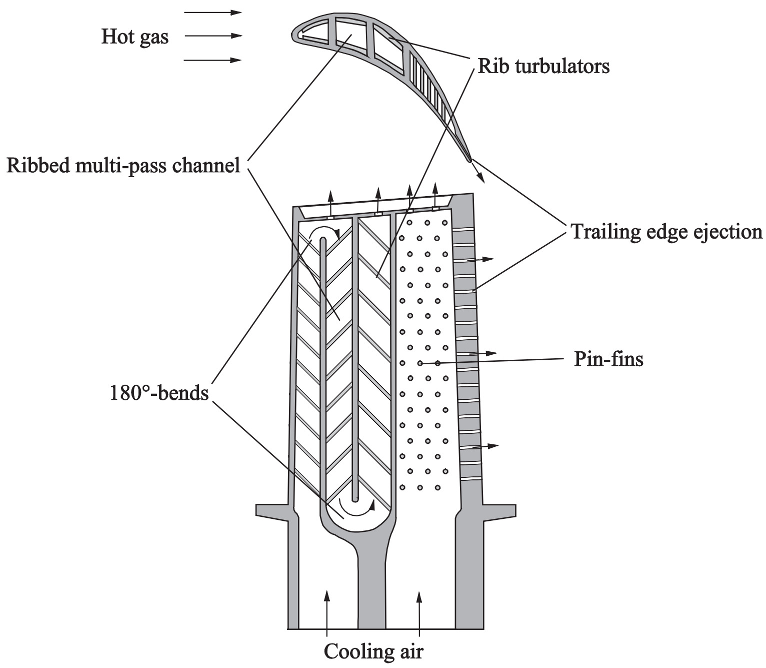

Figure 3: Turbine blade with internal multi-pass cooling channel system.

major vortex formations can be seen in the two corners and at the recirculation zone downstream of the bend as shown in Fig. 4. Local velocity maxima and impingement flows usually result in locally enhanced heat transfer rates. Those regions with high local changes in the heat transfer coefficients are preferably avoided since they induce high thermal stresses in the blade wall which can cause early material failure.

Over the past years in gas turbine development, optimisation was driven mostly by the experience and the know-how of the engineers. In order to reduce the aerodynamic losses, recirculation and vortex zones were ‘filled up’ with blade material and therefore suppressed. Simultaneously, the heat transfer distribution was straightened. A solution for this approach is shown in Fig. 5. Additionally, guide vanes of different types could be inserted in order to ‘guide’ the flow through the bend and prevent separation.

Since the optimisation of these internal gas turbine cooling channels is of high technical relevance and over the past years a lot of experience could have been gained, the ice formation method is here applied as an idea creator for novel web shapes and the presented combination of natural and conventional (numerical) optimisation is shown in this case exemplarily.

The formation of an ice layer on a supercooled surface, called the parent surface, is a naturally occurring process which provides a free moving water–ice interface. The ice layer growth is based on a combination of heat transfer and momentum transfer and their interaction with the wall, the latter consisting of the frozen ice layer. Or, in other words, the formation of this interface ice layer directly corresponds to the present local fluid dynamic and thermodynamic conditions. It means that there is a direct relation and dependence between the shape of the water–ice interface and the surrounding flow field. The final shape of the interface at steady-state conditions is determined by a local heat transfer equilibrium.

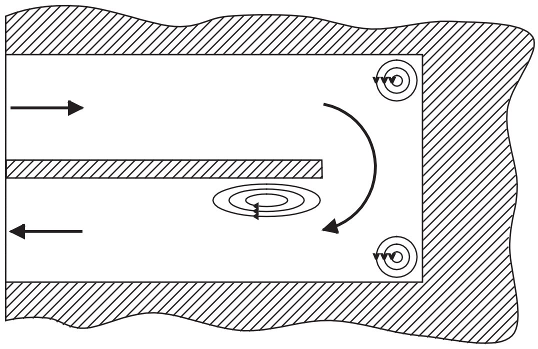

Figure 4: Sharp-edged 180-degree bend with major vortex and recirculation formations.

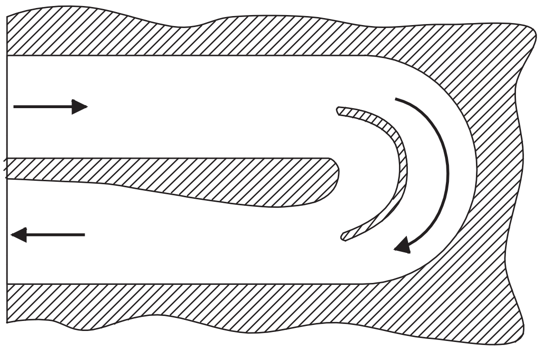

Figure 5: 180-Degree bend with pressure loss optimised geometry, as presented in [25].

This naturally found new shape includes then a compromise between a minimum of aerodynamic (hydrodynamic) drag and a minimum of heat transfer. In few words, a natural occurring flow will always follow the ‘path of least resistance’ within the limits of existing constraints. The ice will form according to the chosen geometric constraints, i.e. the a priori chosen channel geometry, the flow velocity, and the cooling temperature, in such a way that for this specific channel configuration a natural optimum is found.

In order to attain the technical goal of minimal pressure drop across the channel with this web shape, the experimentally ice-contoured web geometry that showed the least pressure drop for the channel flow in the experiments is taken as starting configuration for a subsequent numerical optimisation run, which will be shown later.

3 SET-UP AND RESULTS OF THE EXPERIMENTAL INVESTIGATION FOR THE INTERNAL COOLING CHANNEL WEB GEOMETRY

In the experimental part of the investigation, the separating web geometry, taken from a simple smooth two-pass internal cooling configuration of a turbine blade, including the connecting 180-degree bend (see Fig. 6), was examined. Here, the separating web of the cooling channel acts as the parent surface. The test section consists of an inlet and outlet duct, each 2 m in length, with the same quadratic cross-section ($W = H = 0.1$ m). A calming section installed ahead amounts to 20 hydraulic diameters

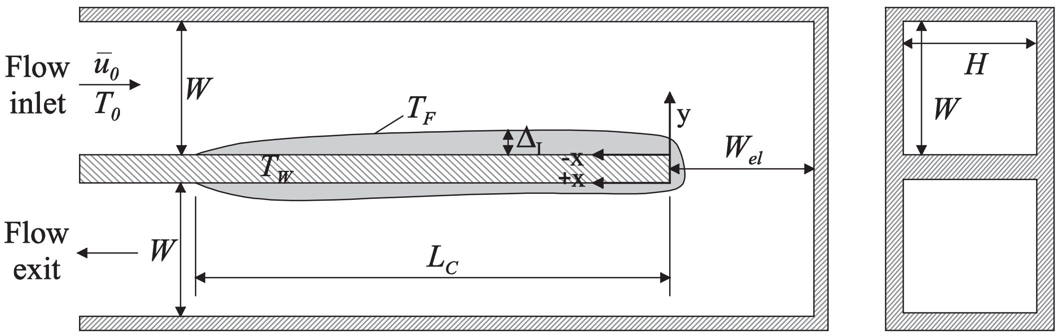

Figure 6: Test section with measured quantities and exemplarily sketched ice layer (grey).

to provide a fully developed velocity profile. All side walls are made of Plexiglas™ in order to provide unlimited optical access. The dividing web consists of segments made of copper. In order to cool the segments to a homogenous temperature, serpentine milled cooling channels are designed inside. The coolant is provided by a refrigeration unit at a minimum constant temperature of −20°C. All temperatures were determined by K-type thermocouples. The fluid flow rate was measured due to the pressure difference at a calibrated Venturi nozzle. The flow rate of the water, the inlet water temperature, and the temperature of the chilled separating web and the coolant temperature were controlled permanently and held constant during the entire test.

All temperatures measured were determined with an accuracy of 0.3 K, which results in a maximum uncertainty up to 10% of the dimensionless temperature difference Θ_C. The Reynolds number was determined with a calibrated Venturi nozzle and an accuracy of at least 2.5%. Due to the very small values of the pressure drop in the smooth bend which ranges from only 120 to 300 Pa, the estimated deviation achieved almost 25% for very low Reynolds numbers. The semi-automatic evaluation of the ice layer thickness resulted in an accuracy of the order less than 0.3 mm.

3.1 Morphology of the ice layers

The goal of the present work is to investigate the formation of an ice layer along a chilled separating web section of a 180-degree bend as mentioned above. In order to achieve a maximum improvement of the geometry, the bend and the separating web in particular were chosen to be as simple as possible (see Fig. 7). For the experiments, a number of parameter combinations of Reynolds number and dimensionless temperature ratio were investigated. Here, the Reynolds number Re is based on the hydraulic diameter of the squared cooling channel without ice layer, the mean inlet velocity of the test section, and the temperature dependent kinematic viscosity of water. The dimensionless temperature ratio represents the ratio of the chilled wall temperature T_W and the water flow temperature T_0 based on the water freezing temperature T_F.

The general shapes of the ice layer contours were surprising but evident. Figure 7 shows such an ice layer contour for a Reynolds number of 15,000 and a temperature ratio of 6. The formation of this naturally grown web contour can be described by analysing the local flow pattern. The shape of the ice layer contour in the inlet region is governed by the solidification of the boundary layer in turbulent flow that occurs between the cooled web wall and the channel flow. Depending on the Reynolds number and the dimensionless temperature difference, smooth and wavy ice layers (with one wave), respectively (see Fig. 8, Re = 20,000/30,000), can be seen in the inlet channel of the test section.

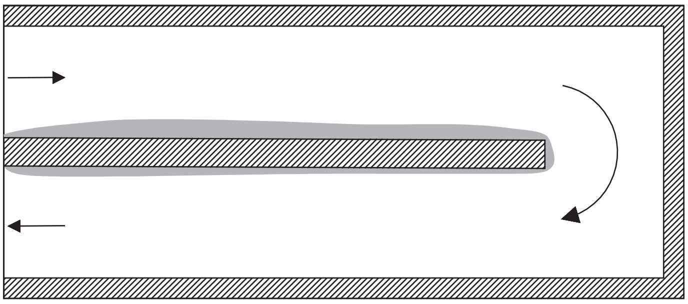

Figure 7: Schematic drawing of the test section with an experimentally determined ice layer contour.

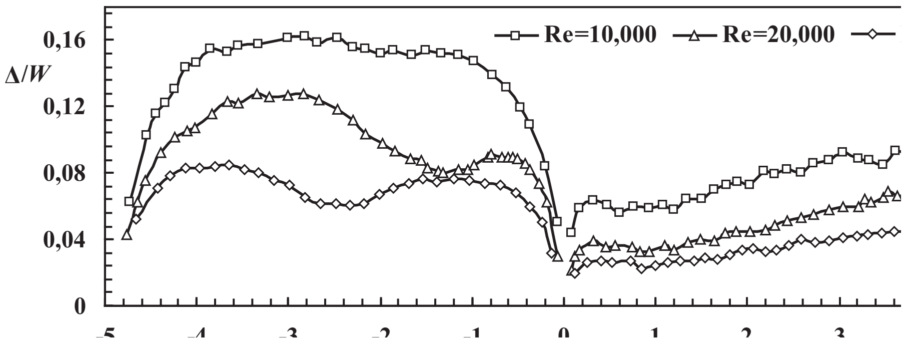

Figure 8: Plotted ice layer distribution over the chilled web length.

As wavy ice layers we denote contours that show at least one maximum and one minimum point. Those wavy ice layers occur for certain combinations of Reynolds number and temperature difference when the main flow is strongly accelerated and therefore laminarised due to the increasing thickness of the ice layer near the entrance of the chilled section of the bend.

In a distance away from the entrance of the chilled region, the ice layer tends to a constant thickness and the flow acceleration ceases. This leads to a re-transition of the flow to a fully turbulent state which results in a higher local heat flux and therefore in a decreasing ice layer thickness. Weigand and Beer [6] found a criterion for the occurrence of wavy ice layers in an asymmetrically cooled parallel plate channel, which is similar to the current configuration of the inlet channel of this test section, to be

$$\Theta_C > 0.34 + \frac{\mathrm{Re}_{2W}}{5657}, \tag{1}$$

where $2W$ is the hydraulic diameter for a channel with a large width to height ratio. It has to be mentioned that the Reynolds number of Weigand and Beer is based on twice the distance of the parallel plates which denotes the channel width W in our test configuration. More detailed information on the criteria and the formation of wavy ice layers is given by Weigand and Beer [5].

At the beginning of the deflection, the channel flow is accelerated due to the streamline curvature and detaches from the web. For all parameters, a small dent was found at the detaching point of the flow in the bend. The decrease in thickness of the ice layer at that point is caused by the increased heat flux from the fluid to the solid ice layer. Further downstream, the flow shapes a radius which grows with decreasing Reynolds numbers. In the outlet channel, the enhanced turbulence level and the recirculation zone result in a thinner ice layer close to the bend. The minimum ice layer thickness can be found in the re-attachment region where the maximum heat transfer occurs. Further downstream, the flow is re-developing to parallel flow pattern and hence the ice layer grows in thickness.

Summarising, we can see that the free surface – the ice layer – tries to adapt to the surrounding flow field and tries to minimize the heat transfer and pressure drop simultaneously.

3.2 Influence of the ice layer contour on the pressure drop in the channel

The normalized pressure drop coefficient for the ice-formed web tip geometry is measured and calculated according to

$$\Delta p^* = \frac{\Delta p}{\rho/2\, \bar{u}_0^2}, \tag{2}$$

where Δp is defined as the difference of the static pressure at the inlet of the iced bend region and the outlet $\Delta p = p_{in} - p_{out}$, normalised by the dynamic pressure at inlet of the iced web.

For a Reynolds number of 20,000, the pressure drop coefficient Δp^* is plotted over the range of the dimensionless temperature difference Θ_C in Fig. 9. As references, the measured pressure drop for the squared web tip geometry and the measured data shown by Idelchik [26] are taken.

Comparing the data of the two basic web geometries, the water channel measurements at Re = 20,000 predict an 8% higher pressure drop than the one stated by Idelchik [26]. However, this reference data were determined with an air flow and is basically valid for Re > 40,000. Furthermore, the pressure data were obtained far upstream and downstream of the bend where no ice layer was initialised.

The current experimental results clearly reveal a reduced pressure drop of the bend configurations with an iced web tip geometry up to 15% compared to the basic bend. An optimum was found for $\Theta_C = 5$. This is precisely where the pressure drop coefficient amounts to a value of $\Delta p^* = 2.14$. It can be clearly stated that for all parameters depicted in Fig. 9, the pressure loss coefficients of the ice-contoured bend geometries show lower values compared to the initial starting contour in the experiments. The small variations at the different temperature ratios are caused by the different shapes and thickness of the ice layers. The formation of single waves and the growth and decay of the ice result in an alteration of the channel pressure drop.

4 NUMERICAL OPTIMISATION OF THE WEB GEOMETRY

4.1 Optimisation procedure

Since the experimentally found improved ice-contoured web geometry does not represent the optimum in terms of minimum pressure drop but rather of minimum energy dissipation, the geometry was taken as a starting configuration and optimised numerically using evolutionary algorithms in order to find the optimum for a technical application, which would be in the present case a minimised pressure drop. For this, the experimentally determined ice contour was parameterised and analysed numerically. The applied method using a Bezier curve approach for the parameterisation of the geometry is described in detail in [27].

Once mapped, the channel was submitted to a numerical optimisation algorithm and implemented in the process chain depicted in Fig. 10. Each set of design parameters – representing one individual in a generation – resulted in a curve that forms the bend's geometry. It was classified with regards to its 'fitness', meaning the optimisation objective, i.e. the minimised pressure drop across the channel. The

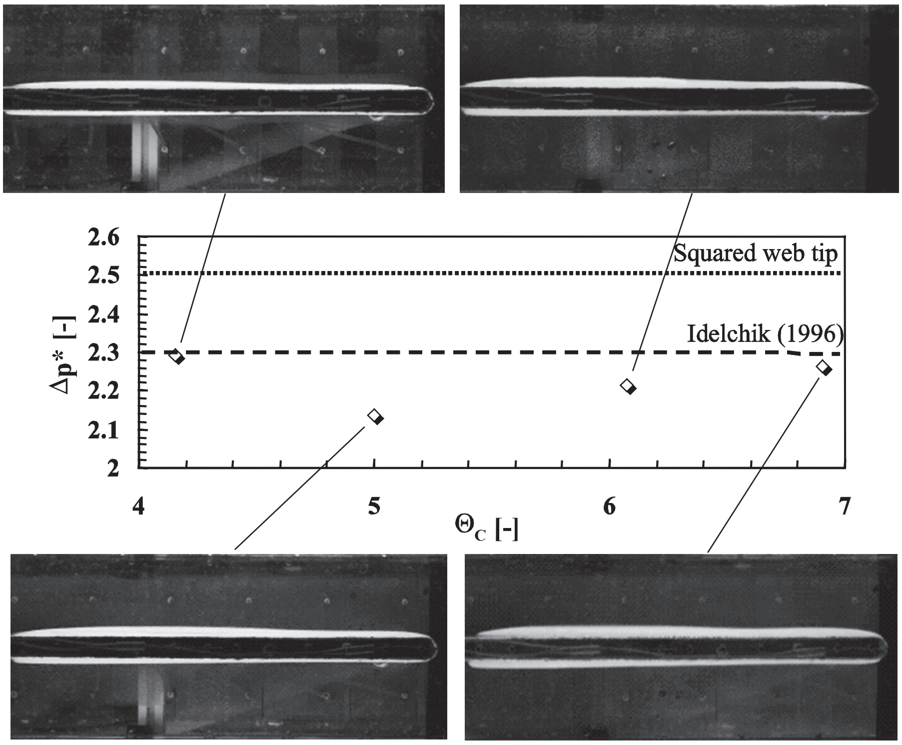

Figure 9: Pressure drop of the contoured web bend for Re = 20,000 and various Θ_C.

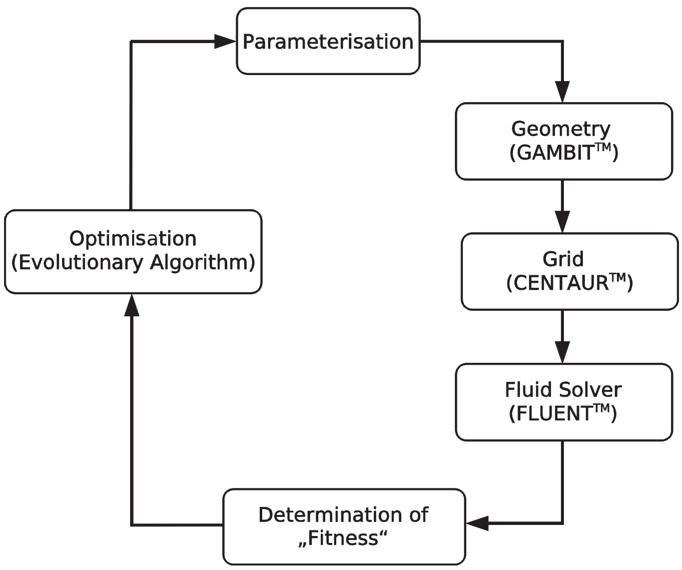

Figure 10: Process chain for the optimisation.

optimiser then created a new set of parameters that was again classified. One run in the process chain represented one iteration and the process was stopped when the pressure drop converged to a minimum.

The channel geometry was created using GAMBIT™ 2.3.16 [28], a FLUENT™ tool. At the outlet, a pressure outlet boundary condition $p_{out} = 0$ is chosen, which means that the difference to the operating pressure is zero. At the inlet of the channel, a fully hydrodynamically developed velocity field is imposed. A hybrid grid with roughly 800,000 cells is generated using CENTAUR™ [29]. It consists of 12 prism layers at the boundary, which are filled up with tetrahedra.

For solving the Reynolds-averaged Navier–Stokes, energy, and turbulence model equations, second-order accuracy was used. They were computed until the residual values converged to levels lower than 10^{-5}. Momentum and energy equations were solved using FLUENT™ 6.3.26 [30] with a standard k-ε turbulence model using a two-layer approach for the wall treatment (called enhanced wall treatment). The fluid under consideration was air described by the ideal gas law.

As optimisation algorithms, a combination of a genetic algorithm from Deb [31] and an evolutionary strategy from CAone™ [32] were used. Both algorithms belong to the family of evolutionary algorithms that are based on the imitation of organic evolution processes as rules for optimum seeking procedures. They both implement the principles of 'population', 'mutation', and 'recombination'. For the presented problem, these algorithms are the appropriate choice since their stochastic search in the design space allows them to find the global optimum in problems with various local optima if the objective function cannot be described by an analytic function. A detailed description of their combined application and their parameter settings for the presented problem can be found in [26].

4.2 Optimisation results

Figure 11 shows the streamlines of the fluid flow for the ice-contoured starting web geometry and the optimised web contour after a converged optimisation run (after 624 iterations) for a Reynolds number of Re = 50,000. It can be clearly seen that the separation bubble directly behind the bend has almost vanished due to the modified web geometry. Further more, the absolute pressure level at the inlet

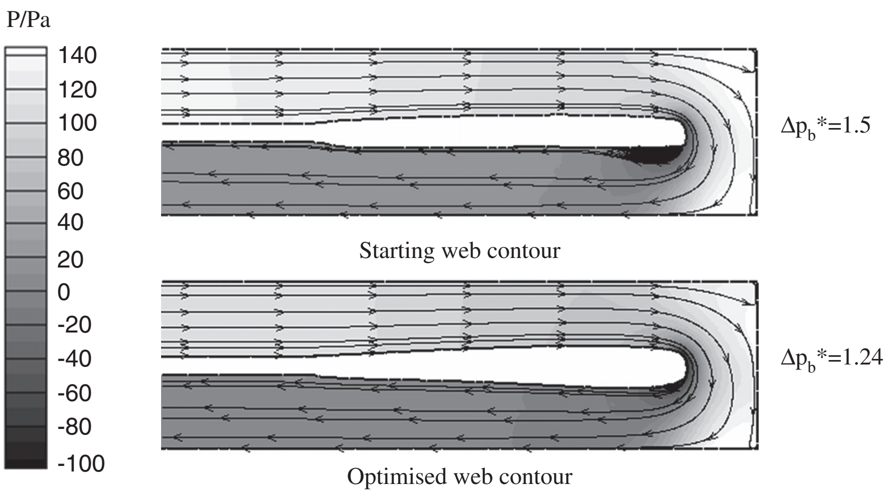

Figure 11: Pressure plot including streamlines in the midspan of the initial ice-shaped web and the optimised web for a computed 3D channel analysis.

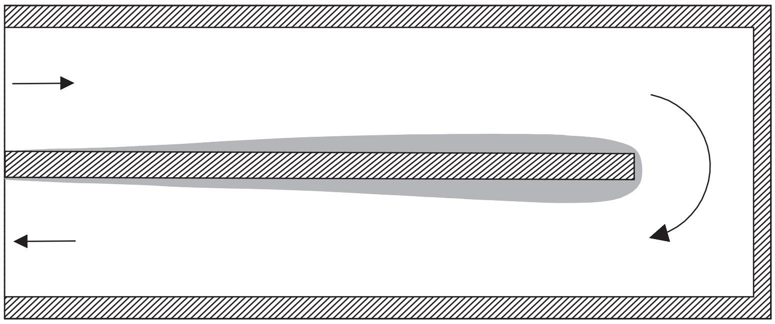

Figure 12: Final web contour shape at the end of the optimisation procedure.

is lower for the optimised contour and therefore the pressure drop across the channel is lower, too. The optimised shape results in a reduction in the pressure drop of 17% in the bend compared to the starting contour taken from the experiments.

This flow recirculation directly behind the bend is mainly responsible for the losses according to Metzger et al. [33]. As a consequence, the optimised contour has filled up the region of recirculation and produced a web geometry with a large radius and by this reduced the pressure drop. In detail, the final web shape is shown in Fig. 12. A comparison with the geometry used nowadays for gas turbine profiles (Fig. 5) points out a very similar web contour. The web is thickened on both sides, in the inlet as well as the outlet duct which is different to the industrial application. The radius at the web tip has about the same radius compared to the one in Fig. 5 which may cause a similar pressure loss. However, the occurring velocity peaks in the bend will be reduced due to the more equally narrowed cross section. It is interesting to notice that the profile shown in Fig. 5 has been developed by engineering experience over decades, whereas the profile depicted in Fig. 11 is obtained by the optimisation procedure outlined in the present paper.

5 CONCLUSIONS

The method which is presented here is a novel approach to optimise geometries exposed to a fluid flow with the help of nature's abilities to find an optimum. The naturally grown ice layer around a base geometry represents a far better solution in terms of minimum energy dissipation than the initial starting configuration. The reduction of the pressure loss coefficient and a smoothing of the internal bend flow emphasise this hypothesis. In combination with the subsequent numerical optimisation, the bend's pressure loss could be lowered to values comparable of those in high-performing industrial gas turbine blades used nowadays, the latter, however, having gone through a long-lasting developing process.

But further more, the ice contour shows ways to improve a geometry exposed to fluid flow by forming innovative surface geometries and provides a pool of good shape candidates. As it was shown in the present paper, those can be used as a starting point for a subsequent numerical optimisation while still the characteristic shape of the ice is maintained throughout the whole optimisation process. The method was shown to be applicable to an internal flow configuration by showing the generation of naturally shaped ice layers and their numerical optimisation with a combination of a genetic algorithm and an evolutionary strategy.

NOMENCLATURE

A	flow cross-section = HW (m^2)
D_h	hydraulic diameter = $4A/U$ (m)

H	channel height (m)
L_C	chilled web length (m]
L_C^*	dimensionless chilled web length = L_C/D_h (–)
Re	Reynolds number = $\bar{u}_0^2 D_h / \upsilon$ (–)
p	static pressure (Pa)
p^*	normalized static pressure = $\Delta p \left(0.5 \rho \bar{u}_0^2\right)^{-1}$ (–)
p_b^*	normalized static pressure in the bend (–)
T_0	channel inlet temperature of water (K)
T_F	freezing temperature of water (K)
T_W	wall temperature (K)
u	velocity in axial direction (m/s)
$\bar{u}_0$	mean inlet velocity in axial direction (m/s)
U	channel perimeter = $2H + 2W$ (m)
v	velocity magnitude (m/s)
W	channel width (m)
W_{el}	web to tip-wall distance (m)
Δ	ice layer thickness (m)
Θ_C	dimensionless temperature ratio = $(T_F - T_W)/(T_0 - T_F)$
ν	kinematic viscosity (m²/s)
ρ	density (kg/m³)

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