



A Novel Approach to Control the Sidelobe Levels in Orthogonal Frequency Division Multiplexing Radar Waveform Design Using Broyden-Fletcher-Goldfarb-Shanno Algorithm

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ABSTRACT

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Employing orthogonal frequency division multiplexing (OFDM) for radar applications has attracted many researchers in recent days. In OFDM systems the reduction of out-of-band (OOB) radiations is one of the most discussed and researched topics. To successfully design an OFDM based overlay system, it is necessary to reduce the sidelobe levels in OFDM signals. In this paper a novel technique for reducing the sidelobe in OFDM radar signals is projected and examined. Subcarrier weighting technique is the method used to scale down the sidelobe heights by multiplying real valued weighting coefficients with the used subcarriers. In order to obtain optimal subcarrier weights a numerical optimization technique called Broyden-Fletcher-Goldfarb-Shanno (BFGS) is utilized. The proposed scheme applies BFGS method to enhance the performance of OFDM radar signal. The reduction in sidelobe levels thus obtained from the proposed method shows the superiority in functioning (small sidelobe crest and good resolution) shown with extensive simulation results.

1. INTRODUCTION

In current day scenario there is a huge surge in demand for more data due the rapid growth of multimedia radio products, all points towards the need of a novel technology which aide very high-speed data transmission rates. Orthogonal Frequency Division Multiplexing (OFDM) can be utilized to meet such demands [1]. Similarly, in the field of radar communications there is a dire need for such new techniques which balances the contrarian relationship between range and resolution detection capabilities. The range of radar is defined as the maximum distance at which the radar is capable of detecting the targets, whereas the resolution of radar is the ability to distinguish between targets that are at close proximity [2]. We use the term “contrarian relationship” because larger pulse width of a signal favours range detection whereas smaller pulse width signal enhances resolution detection capabilities of a radar system, a trade-off is thus present between the two. In addition, attaining high frequency slope and adequate linearity will be difficult in linear frequency modulation (LFM) with single carrier signal which led to the growth of multi-carrier schemes.

In a bid to contribute towards the above mentioned necessities we need to either create additional spectral resources which satisfies the given requirements or make effective utilisation of the existing resources to achieve the same. Since creation of additional spectral resources is money mongering and time consuming, we have our focus tilted towards making better utilisation of existing resources. A step in that direction is to make use of multi-carrier phase coded signals (MCPC). The term “Multi-carrier” is coined because the channels used are able to transmit a large number of carriers simultaneously unlike a traditional single carrier signal [3]. Though Multi-carrier signals seems to be the perfect

solution for our problems, they have a few shortcomings namely presence of out of band radiation, also called sidelobes and high peak to mean envelope power ratio (PMEPR). Multi-carrier signals naturally will have a high value of PMEPR [4]. The PMEPR of a MCPC signal denotes the presence of fluctuation in the combined subcarrier. Due to these high variations in amplitude, the transmitting amplifier should be exceptionally keen on these transitions.

Sidelobes are the unwanted radiations in any transmission signal, they are omni-directional and have no real significance. It is difficult to completely eliminate the sidelobes but it can be reduced to maximum extent possible by making use of certain reduction algorithms or any other reduction techniques. We are mainly focused on reducing the sidelobes, since sidelobes are the primary cause for the increased noise in the multi-carrier signals. The paper is ordered as follows. Section 2 summarizes the related work on multi-carrier. The concept of MCPC is presented in Section 3. Subcarrier weighting in multi-carrier is discussed in section 4. In section 5, MCPC implementation based on BFGS are presented. Results and conclusions are drawn which highlights the ability of proposed technique in section 6 and 7 respectively.

2. RELATED WORK

Currently radars are applied for automotive applications using multicarrier techniques [5]. The prominent features in radar signal processing are range and resolution. There is a need of novel approaches which balances the tradeoff between range and resolution detection capabilities and in the literature [6] mentioned about high-resolution radars in real time signal processing. Range of the radar signal can be improved by increasing the pulse width, but this reduces the resolution of

radar. On the other hand, to improve the resolution, pulse width has to be reduced. Therefore, to balance both, it is suggested to apply the pulse compression technique [7]. Pulse compression takes advantage of the best features of both long and short duration pulses and allows the use of long waveforms to gain good energy and also matches the resolution of a short pulse by inner modulation of the long pulse. By employing this concept bandwidth can be utilized efficiently in multicarrier radar. Levanon [8] defined the concept of MCPC signal. Digital phase coding as internal modulation was used to develop MCPC signal. By suitably performing phase coding on multi-carrier, pulse compression can be achieved which results in enhanced range resolution for radar applications. The MCPC signal developed by Levanon was using polyphase coded schemes, namely P3, P4 methods, where these phase sequences developed will form a cyclically shifted complementary set. Multi-carrier signals naturally will have high levels of PMEPR. The P4 based MCPC signal had a limitation of envelope fluctuation. This was addressed by changing the sequence ordering of subcarrier and phase chips. By using the sequence order 3 5 2 1 4, PMEPR of 4.39 was attained. Similarly, with the sequence ordering 3 4 5 1 2, PMEPR was reduced to 1.73, but this led to increase in the sidelobe levels, which is not appreciated.

Several authors have investigated and conducted several experiments to curtail the impact of PMEPR on OFDM signals and majority works were focused on data transmission applications. Ma et al. [9] have proposed adaptive tone reservation (ATR) method for reducing envelope variation in MIMO OFDM. The ATR scheme proposed will iteratively carry out tone reservation on the antenna with an improved PAPR. The suggested method provides reduction in PAPR levels and drop in the level of computation, which is better when compared with regular tone reservation scheme. In the literature [10], improved tone reservation technique is followed which is developed with least squares approximation, this scheme generates the optimal peak-canceling signals with fast convergence. To improve further convergence rate and reduce several peaks of OFDM, multiple scaling SCR method [MS-SCR] is followed [11]. In the literature [12], a method to reduce the Inter Carrier Interference in MIMO OFDM is projected which controls low complexity in the channels which is of same frequency. Few researchers have explored to decline the PMEPR values in multicarrier signals for radar applications. Mozeson et al. [13] has proposed mutually orthogonal complementary sets (MOCS) MCPC where PMEPR of the MCPC pulse is more but in acceptable range. In the paper [14], it is shown that reducing variation of envelopes will lead to reduction in PMEPR and monotonically reduces the weight by employing iterative algorithm. Zhang et al. [15] has proposed waveform design for a dual function in radar communication system which is based on constant envelope orthogonal frequency division multiplexing phase

modulation (CE-OFDM-PM). In the literature [16], reduction of PMEPR in MCPC is demonstrated by using constant envelope (CE) modulation and filtering operations.

In condensing the sidelobes few authors have experimented. In the paper [17], a new concept called constellation adjustment is followed for suppressing the sidelobe in noncontiguous OFDM applications. Raghavendra et al. [18] reduced the PMEPR in MCPC radar signal using random phase update algorithm. This technique is applied for higher order of MCPC signal. Zhao et al. [19] made an analysis using OFDM linear frequency modulation (LFM) signals which resulted in more grating sidelobes. Later, a combined optimization approach, sequential quadratic programming and genetic algorithm were employed to reduce the sidelobe in LFM OFDM which balances both orthogonality and sidelobe levels simultaneously. Shi et al. [20] has attempted to minimize the power in OFDM radar waveform design. There are several shortcomings in OFDM radar signals, PMEPR and sidelobes are the major which should be focused to address. This paper focuses on controlling the sidelobe levels in MCPC radar waveform.

3. MULTI-CARREIR PHASE CODED SIGNALS

In the field of radar systems, the concept of multi-carrier signals was first introduced by Jankiraman et al. [21]. On the basis of OFDM concept, MCPC scheme was introduced for radar applications and it was first developed by Levanon [8]. MCPC pulse comprises of N number of sequences, each are N bits long. Each of these N sequences modulates N carriers that are transmitted concurrently and the carriers are separated such that they are equidistant. The separation between each carrier equals the inverse of bit duration which forms an OFDM signal. The MCPC radar signal is developed by phase modulating the orthogonal subcarrier with polyphase codes, namely P3 and P4. The schematic block diagram is shown in the Figure 1 for developing the MCPC radar signal.

The MCPC signal is developed which is built on the consecutively cyclic shifts of P4 sequences. The phases for P4 digital phase modulation scheme with N phases are described in the Eq. (1).

$$\phi_n = \frac{\pi}{N}(n-1)(n-N-1) \text{ where } n = 1, 2, 3, \dots, N \quad (1)$$

The P4 phase coding method obtained has lesser phase levels which exhibits the ideal periodic thumbtack autocorrelation function with peaky mainlobe and zero sidelobes. By assuming N = 5 and by repeatedly differing the values of N from 1, 2, ..., 5. Then several phase values are attained and it is enumerated in Table 1.

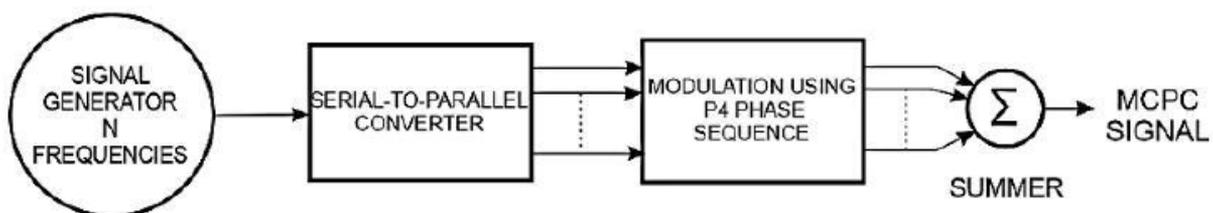


Figure 1. Schematic diagram of OFDM signal generator

Table 1. Periodically shifted complimentary codes with length of N=5

Sequence No	P4 Phase Elements				
1	0°	-144°	-216°	-216°	-144°
2	-144°	-216°	-216°	-144°	0°
3	-216°	-216°	-144°	0°	-144°
4	-216°	-144°	0°	-144°	-216°
5	-144°	0°	-144°	-216°	-216°

The MCPC-OFDM signal envelope is given by Eq. (2)

$$s(t) = \sum_{v=1}^N \left[\sum_{u=1}^N \exp(j\phi_{u,v}) \exp \left\{ j \left[2\pi f_s t \left(\frac{N+1}{2} - u \right) \right] \right\} \right] x[t - (v-1)t_c] \quad (2)$$

$$\text{where } x(t) = \begin{cases} 1 & \text{if } 0 \leq t \leq t_c \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

There are N subcarriers and N phase modulation chips with t_c as the duration of each chip. The term $\exp\{j[2\pi f_s t(N+1/2-u)]\}$ in the Eq. (2) corresponds to the subcarrier u . The term $\exp(j\phi_{u,v})$ represents the v^{th} element of the u^{th} sequence modulating the subcarrier u . The term f_s denotes the frequency separation between any two adjacent subcarriers.

4. SUBCARRIER WEIGHTING

In the paper [22], a novel method called subcarrier weighting scheme is used to reduce sidelobe levels in OFDM signals. In this concept the individual subcarriers are multiplied with suitable scaling coefficients. The subcarrier coefficients are obtained in such a way that the sidelobes of the transmitted signal are minimized according to an optimization algorithm. The coefficients are chosen appropriately such that they satisfy few constraints defined and also must interfere destructively reducing the sidelobe level.

Let us consider a MCPC signal represented as $s(n)$. If discrete auto-correlation function (ACF) is applied to MCPC signal and corresponding equation is defined as:

$$R(m) = \sum_{n=0}^{N-1} s(n) * s(n-m) \quad (4)$$

where, $s(n)$ is the MCPC signal.

In order to enhance the capability of radar signal to distinguish between neighbouring targets, i.e., to improve the target resolution, then the sidelobes of the ACF should be suppressed. The integrated sidelobe (ISL), which indicates the magnitude of ACF is defined as

$$ISL = \sum_m |R(m)|^2 \quad (5)$$

To reduce the ACF, the ISL is minimized iteratively. The objective function of our problem can be thus represented as:

$$f(w_k) = \min_{w_k} ISL \quad (6)$$

where, $f(w_k)$ is the objective function to be minimized, w_k is the weight vector $[w_0, w_1, w_2, \dots, w_k]$. Substituting $R(m)$ from Eqns. (4) in (5), we obtain:

$$ISL = \sum_{n=1}^{N-1} \left| \sum_{m=k+1}^N s(n) * s(n-m) \right|^2 \quad (7)$$

The above equation belongs to a class of quadratic optimization problem. In the literature [23] quadratic optimization is mentioned and Broyden-Fletcher-Goldfarb-Shanno (BFGS) is one such numerical optimization technique which is commonly used in solving energy minimization problems. Here BFGS method is used to minimize the objective function defined in Eq. (6).

5. MCPC SIGNAL BASED ON BFGS ALGORITHM

The numerical optimization BFGS algorithm is an iterative method for solving unconstrained nonlinear optimization problems [24]. It is one of the most computationally efficient quasi-Newton iterative algorithms, in which the Hessian matrix (2nd order partial derivative) is not directly computed, rather approximated using updates specified by gradient evaluations, to guarantee the convergence rate [25-27].

The outline of the algorithm is as follows:

- Initialize the weight vector matrix $w = w_0, w_1, w_2, \dots, w_k$ using an existing or a random sequence.
- Compute the approximate initial gradient of the objective function $\nabla f(w_k)$ which is the objective function described in Eq. (6).
- Obtain a search direction p_k , by solving the equation:

$$B_k * p_k = -\nabla f(w_k) \quad (8)$$

where, B_k is an approximation of the Hessian matrix, which is a square matrix of the 2nd order partial derivative of f_k .

- Perform one-dimensional line-search in the direction found in previous step, to find step-size α_k so that,

$$\alpha_k = \text{argmin}_f(w_k + \alpha p_k) \quad (9)$$

- Set $s_k = \alpha_k * p_k$ and update $w_{k+1} = w_k + s_k$, which will be the new weight vector.
- Compute $x_k = \nabla f(w_{k+1}) - \nabla f(w_k)$, which is the difference between gradients of objective functions of successive iterations.
- Update the Hessian matrix of the next iteration as:

$$B_{k+1} = B_k + \frac{x_k x_k^T}{x_k^T s_k} - \frac{B_k s_k s_k^T B_k^T}{s_k^T B_k s_k} \quad (10)$$

using s_k, x_k computed in the previous steps.

- Repeat the above steps till the convergence is greater than a pre-defined tolerance value. The convergence at each iteration can be checked by computing the norm of the gradient

$$\|\nabla f(w_k)\| < \epsilon \quad (11)$$

where, ε is a pre-defined tolerance value.

After the termination of the algorithm we obtain the optimal weight vector w_k , which is then multiplied with the subcarriers of MCPC signal. The algorithm is summarized in the following flowchart shown in Figure 2.

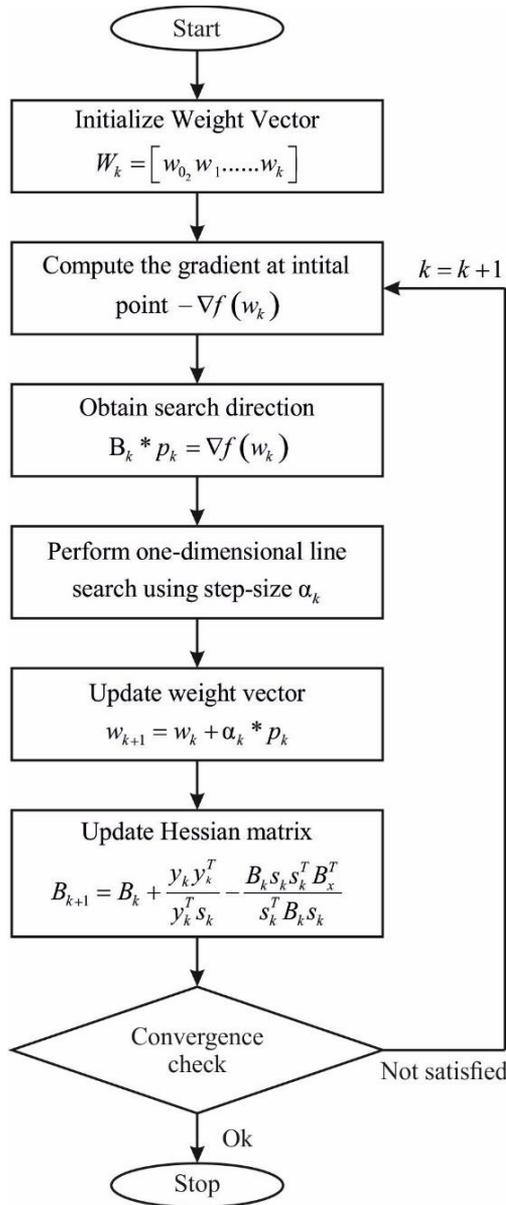


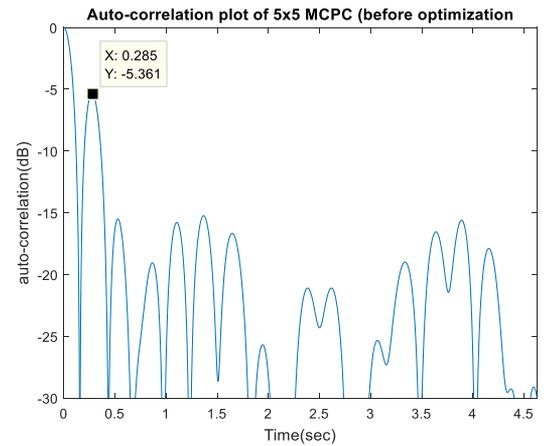
Figure 2. The outline of BGFS algorithm

6. RESULTS

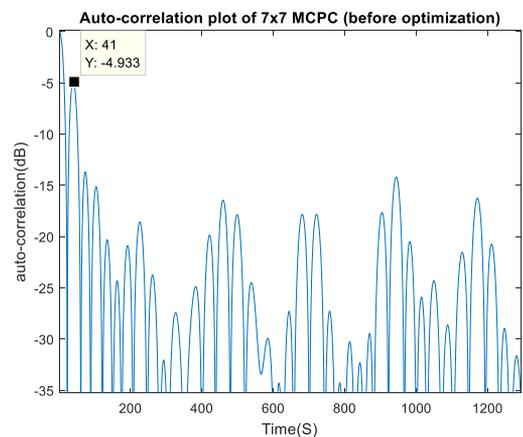
In this section, two kinds of models are preferred as initial sequences for correlation. The first is MCPC signal developed by modulating the subcarrier using polyphase code, P4 type and the other is subcarrier weighting based BGFS type. The BFGS algorithm-based subcarrier weighting technique has been tested for MCPC signals with subcarrier sequence lengths of 5, 7 and 9.

The autocorrelation plots of the MCPC signal which is developed using P4 type for 5x5, 7x7 and 9x9 carriers are shown in Figure 3 (a-c) respectively, which is called as without optimization. Similarly, the autocorrelation plots of MCPC

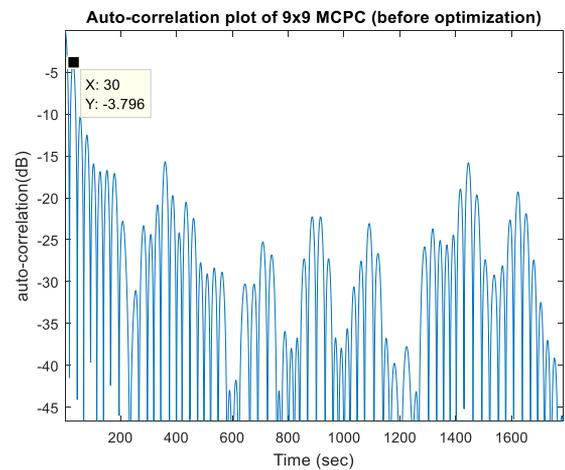
signal based on BGFS are shown in Figure 4 (a-c) respectively, are called as with optimization. Comparison are made for PMEPR, First sidelobe level (FSL), Peak sidelobe level (PSL) and Integrated side-lobe level (ISL). Reduction in FSL, PSL and ISL, which is the magnitude of sum of sidelobes has been observed for all the sequence lengths of BGFS based MCPC signal. Further, longer the sequence length, greater is the reduction in ISL is observed. Tabulation of the observed reduction across 3 sequence lengths is done in Table 2.



(a) 5x5 MCPC



(b) 7x7 MCPC

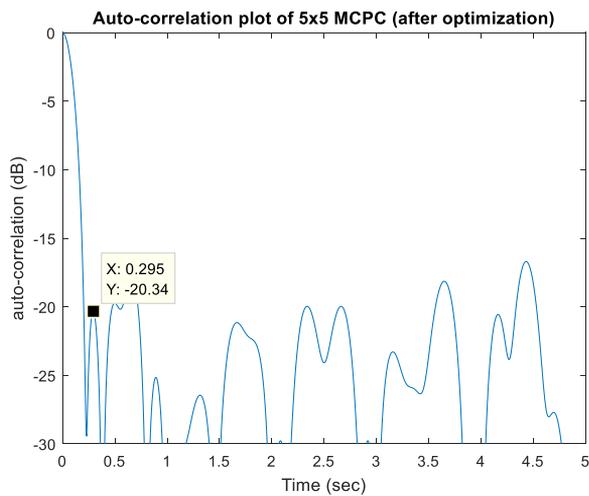


(c) 9x9 MCPC

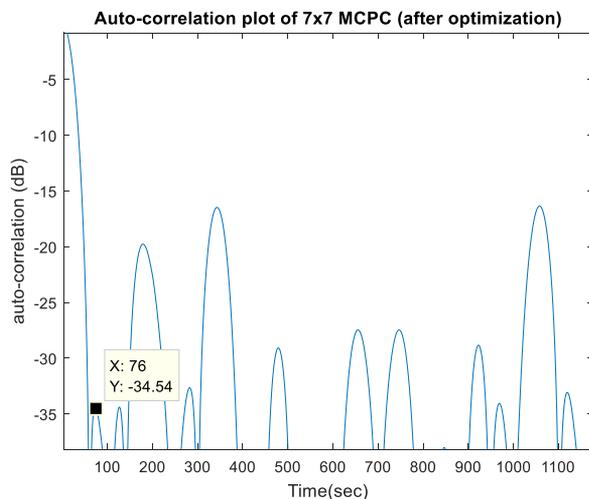
Figure 3. Autocorrelation of MCPC (without optimization)

Table 2. Comparison of sidelobe and PMEPR

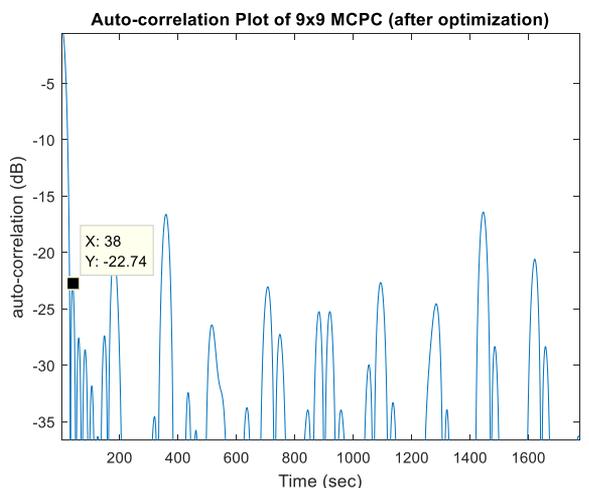
Sequence Length	Without Optimization				With Optimization			
	FSL (dB)	PSL (dB)	ISL	PMEPR	FSL (dB)	PSL (dB)	ISL	PMEPR
5	-15	-15	97	3.8	-28	-21	80	4.2
7	-5.23	-5.23	198	2.1	-14.24	-14.24	164	2.9
9	-3.168	-3.168	207	3.36	-12.44	-12.44	160	1.96



(a) 5x5 MCPC



(b) 7x7 MCPC



(c) 9x9 MCPC

Figure 4. Autocorrelation of MCPC (optimization)

Table 2 depicts the results of MCPC signal with and without optimization. The sequence ordering 3 2 4 1 5 is chosen for 5x5, 7 5 3 6 1 2 4 for 7x7 and for 9x9 sequence ordering 7 5 6 3 1 8 4 2 is selected. Same sequence ordering is selected for comparing MCPC with optimization. We can observe a reduction of 13dB in FSL for subcarrier length of 5, a reduction of 9dB for subcarrier length of 7 and a ~9dB reduction in the case of length 9. Significant reductions in ISL can also be observed, which highlights the efficiency of BFGS algorithm.

7. CONCLUSIONS

In this paper, an optimization algorithm is applied to MCPC signal. The ultimate outcome is to improve the radar range resolution and detection capabilities which are possible only if we combine both larger and smaller pulse width. As they both have a contrarian relation which is difficult to generate an ideal signal. The signal thus generated has few problems associated, namely out of band radiations or sidelobes. Consequently, efforts were made to reduce up to the maximum extent possible by using numerical optimization technique called BFGS. The simulation results show the reduction in the sidelobe levels by 13dB in FSL, 6dB in PSL and 17 in ISL for 5x5 subcarrier MCPC signal. Similar improvement was observed for 7x7 and 9x9 subcarrier. The reduction values alongside with computer simulations highlight the superiority of the proposed technique.

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