Advances in Modelling and Analysis C
Vol. 73, No. 3, September, 2018, pp. 102-111
Journal homepage: http://iieta.org/Journals/AMA/AMA_C

Fuzzy soft set based decision approach for financial trading

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http//doi.org/10.18280/ama_c.730305

Received: 14 May 2018
Accepted: 3 August 2018

Keywords:
portfolio optimization, fuzzy set, soft set, decision making problem, butterfly particle swarm optimization (BFPSO), particle swarm optimization (PSO)

ABSTRACT

According to the basic idea of financial market an investor used a decision making approach for the maximum return with respect to minimum risk. We test a novel decision making process to determine the optimal assets for making a portfolio and compare our method to Analytical Hierarchy Process. Based on measure of performance of two decision making process i.e. Fuzzy Soft Set and Analytical Hierarchy Process, the outcome is more reliable through fuzzy soft set from multi-input data set. The optimal portfolio is constructed using fuzzy soft set method. The aim of this paper is to select the optimal ratio of portfolio, in which multi objective portfolio optimization solved by the help of Butterfly Particle swarm optimization. This problem is formulated in mathematical programming in such a way that it has two main objectives, minimum risk and maximum return. In this paper the effectiveness of fuzzy soft set in financial problems is illustrated with example.

1. INTRODUCTION

Optimal portfolio of the assets has two fundamental characteristics i.e. return and risk. The main goal of investors is to gain more return on acceptable risk level that's why forecasting the future returns of investment plays a significant role for investors. The investors want to gain the maximum benefit with the least possible risk. Portfolio optimization is vital in addition to investments in the stocks. Portfolio theory states that: assets and investments should be invested in diversifying portfolio. As there are two methods discussed for decision making viz. Fuzzy soft set (FSS) and Analytical Hierarchy process (AHP), Portfolio selection formulations have been benefitted greatly by Fuzzy soft set theory in terms of integrating quantitative and qualitative information subjective preference of investor and knowledge of expert. Although the use of FSS, AHP and BFPSO for portfolio optimization is a new established research area, this field remains interesting because of its important financial aspects.

2. LITERATURE REVIEW

2.1 Related Works on MCDM

Researchers can choose the parameters they require, that very well defines the decision-making process. It also makes the process more efficient in the absence of partial information. [1] defined first time soft set by means of decision making approach. [2] addressed an application of soft sets theory in which identification of the final object is based on the set of inputs from different investors. Cagman et al. [3-4] introduced soft set based decision making methods. Naim Cagman [4] presented some new consequences based on Molodtsov's soft sets to make them more functional through operations which is known is uni-int decision making approach and [3] introduced a soft max–min decision making (SMmDM) approach. This method selects optimum alternatives from the set of the alternatives by an algorithm for solving many practical problems using soft max-min decision functions. By combining the interval-valued fuzzy sets and soft set models, Feng et al. [5] presented an adjustable approach to FSS based decision making by means of level soft sets. Kong et al. [6] described concept of choice values designed for crisp soft sets is not fit to solve decision making problems involving FSS. [7] A generalized fuzzy soft set introduced by trapezoidal fuzzy soft set on the concept of soft set as well as other basic operations was defined on trapezoidal fuzzy soft set as like AND, OR, Distribution and De Morgan's law. Some feasible property and operations of multi expert group decision making situation by intuitionistic fuzzy soft matrix (IFSM) [8]. GS [9] introduced FSS based traffic accident alert model in which accidental places was predicted. The higher discrimination and strong determined solution are the approaches of the problems of the fuzzy soft set decision making among multi observer input data sets, Jose Carlos et al. [10]. The AHP defines the decision making approach as the means to prioritize the alternatives among the proposed multi criteria decision making (MCDM) methods; [11]. Here, the problem of decision making is solved in the form of hierarchy or different set of levels likes goal, the alternatives and the criteria. The main advantage of AHP is to obtain ratios of the alternatives with the help of pairwise comparison. [12] introduced extensively AHP. It is very popular and has been applied in wide variety of MCDM in last 20 years. AHP has been applied in huge variety of application in different fields like medical science, management science, research and development, marketing, finance, social studies and other areas where choice, prioritization and forecasting are required in the decision.

2.2 Related works on multi objective programming by BFPSO

Kennedy and Eberhart [13] proposed Particle Swarm

Optimization (PSO) which is used for solving continuous optimization problems and it is also used in various applications of science and engineering field like industry, finance, engineering design, Management Science, portfolio Selection, automobile engineering, aircraft design etc. It's more popular optimization method since it is based on population search. Eberhart and Kennedy [14-15] have also reported nonlinear functions. In PSO, the probable solutions, called particles, are flown through the problem space by learning (following) from the current optimal particle and its memory. PSO has been also applied here for getting optimal position in distribution [16]. Butterfly Particle swarm optimization (BFPSO) [17], Hybrid Butterfly Based PSO [18], Mean Particle swarm optimization (MPSO) [19], Exponential particle swarm optimization (EPSO) [20], Centre particle swarm optimization [21], Particle Swarm Optimization algorithm for multi-objective with stripes [22] (ST- MOPSO) etc. are simplified pattern of PSO.

2.3 Multi-objective problem solving in portfolio optimization

A multi-objective non-linear programming model is presented by P Jana et al. [23] where fuzzy non-linear programming technique is used for rebalancing multi-objective programming (MOP) for any potential return and risk. Leon et al. [24] represented portfolio optimization using fuzzy decision theory. MCDM approach used to solve portfolio selection problem by Ehrgott et al. [25]. Ramaswamy [26] proposed a bound portfolio optimization model using fuzzy decision theory. Pankajgupta et al. [27] Studies a hybrid approach to select the assets of portfolio with the help of AHP and fuzzy multi-objective linear programming.

In this paper, approach for decision making through FSS and AHP methods involving multi inputs data sets, has been considered. A comparison has been done between FSS and AHP methods through performance measure for making a portfolio of stocks, and it is concluded that better outcome is through FSS. Further a MOP is solved by BFPSO which is used to get the rationale proportion of the stocks. The main use of the BFPSO technique in coping with Portfolio Selection problems is the most important applications of PSO is to predict the proportion of the stocks that have maximum profit with minimum risk, using some common indicators that give advice of trade-off. BFPSO algorithm is used to test on financial data.

The organization of paper is as follows: The organization of paper is as follows: In Section 2, literature on MCDM approach is reviewed. Section 3 focuses on basic notion of FSS. In Section 4, description of decision making approaches of FSS. Section 5 describes the decision making approach of AHP. Section 6 represents measure of performance. Section 7 introduces BFPSO method. Section 8 focuses on Portfolio objectives in terms of the assets selection. In Section 9, problem formulation is presented. In section 10, result and discussion are proposed. Section 11 presents some conclusion from the result.

3. PRELIMINARIES

The possibilistic distributions methodology for the possibility theory, posed by Zadeh [28], has played an important role in the development of fuzzy set theory. The theories of rough set, vague set, fuzzy set etc. have their inherent difficulties as pointed out in 1999 by Molodtsov [29]. Molodtsov proposed the soft set as a completely generic mathematical tool for modelling uncertainty.

3.1. Soft set

Consider E is the soft set of parameterized family of subset of universal set U, where $A \subset E$ and F is given by F: A → P (U) [29]. Example: Let U be the set of four assets given by U = {S_1, S_2, S_3, S_4}. Let parameter E is given by {Return, High Return, Risk, Low Risk, Liquidity, Medium Liquidity}. Let A = {High Return, Return, Low Risk, Medium Liquidity} = {P_1, P_2, P_3, P_4}. Now suppose that, F is a mapping given by, F (P_1) = {S_2, S_4}, F (P_2) = {S_1, S_3}, F (P_3) = {S_2, S_3}, F (P_4) = {S_1, S_4}. Then the Soft Set is (F, A) = {F (P_1), F (P_2), F(P_3), F (P_4)}. For tabular representation of soft set (F, A), S_{ij} = 1when $S_i \in F$ (P_i) otherwise S_{ij}=0; where S_{ij} are entries in Table 1.

Table 1. Tabular form of soft set (F, A)

Assets	P_1	P_2	P_3	P_4
S_1	0	1	0	1
S_2	1	0	1	0
S_3	0	1	1	0
S_4	1	0	0	1

3.2. Fuzzy soft set

Consider U is a universal set and E is a set of parameters. A pair (F, A) is called an FSS over U [30], where F is a mapping given by, F: A → P (U); where P (U) denotes the set of all fuzzy sets of U and A ⊂ E. Example: Let U = {S_1, S_2, S_3, S_4, S_5, S_6} be the six selected assets i.e. Gail, Lupin, Ntpc, Sbin, Bankbaroda and Sail. Let E = {P_1, P_2, P_3, P_4} be the parameters of assets where P_1, P_2, P_3 and P_4 are return, risk, dividend and liquidity respectively. Let $A \subset E$ and fuzzy soft set is (F, A) = {F (P_1), F (P_2), F (P_3)}: Where P_1 stands for return, P_2 stands for risk and P_3 stands for dividend in Table 2.

Table 2. Tabular form of fuzzy soft set

Assets	P_1	P_2	P_3
S_1	0.0547	0.0127	0.0207
S_2	0.1262	0.0000	0.0069
S_3	0.0000	0.0029	0.0222
S_4	0.1235	0.0151	0.0073
S_5	0.0641	0.0154	0.0031
S_6	0.0201	0.0145	0.0149

3.3. Weight judgment method on FSS

On the FSS (F, A), the membership value of fuzzy variable C_{ij} (i = 0, 1......n: j = 0, 1 ...m) is given by

$$F(C_{ij}) = C_{ij} + \lambda; \lambda \geq \max\{|x|, |y|, ...|z|\} \tag{1}$$

where λ is any positive real number. Obviously, a measure of how close is the actual the forecast quantity [31]. Moreover representation of FSS (F, A) in a tabular form is shown in Table 4.

4. FUZZY SOFT SET DECISION MAKING APPROACHES

Two technical superiorities of this method.

1. This algorithm avoids the information loss i.e. imposed by AND operator, in input data of originating from multi-source input parameter data set.

2. Since comparison table construction in a novel way thereby allows to generate the new scores after compelling computation. The basis is relative difference instead of absolute difference. In this way assessment is done in a better way of natural among all existing alternatives, with respect to each attribute.

4.1 Product operator 'AND' used in parameters

In multi valued criteria, T-norm fuzzy logics are a family of non-classical logics, informally delimited by having a semantics that takes the real unit interval [0, 1] for the system of truth values and functions called t-norms for permissible interpretations of conjunction. T-norm generalizes conjunction having prominent example of product and minimum t- norms. Roy et al [2] Introduced a tabular representation of the resultant FSS that performs product operation represented by either "AND" or "$\wedge$" where "AND" is the minimum operator for decision making approach. Due to loss of objective's information, Jose Carlos R Alcantud [10] used a new aggregation approach in which product operator used in place of min operator. When the original data originate in multi-source input parameter data set, this algorithm avoids the loss of information that min operator imposes. The recourse of the product as the AND operator at the information fusion stage guarantees a more faithful assessment of the combined parameters than in earlier solution, since it incorporates all the original constituents.

4.2 Comparison table

The entries C_{ij}, i, j = 1, 2,…….., n, are the sum of non-negative values [10] given by

$$C_{ij} = \frac{f_{i1} - f_{j1}}{M_1} + \frac{f_{i2} - f_{j2}}{M_2} + \frac{f_{i3} - f_{j3}}{M_3} + \ldots\ldots\ldots\ldots + \frac{f_{ik} - f_{jk}}{M_k} \quad (2)$$

where M_k is the maximum membership value of the any object i.e. for each q= 1, 2,……., n, $M_q = \max_{i=1,2,\ldots\ldots\ldots n} P_{ij}$: P_{ij} is the membership value of cell (i, j). Here i = 1, 2..... n; j = 1 ,2........ n; k = 1, 2.... m; n is number of stocks, m = the number of parameters. Clearly $C_{ij} = 0$ for i = j and C_{ij} represents numerical measure. Jose Carlos R Alcantud proposed the row sum represented by

$$R_i = \sum_{j=1}^{n} C_{ij} .$$

Here i and j are varying from 1 to number of stocks. P K Maji et al [2] proposed the column sum in the comparison matrix represented by the formula

$$D_i = \sum_{i=1}^{m} C_{ij} \text{ ; m = 1, 2 ...k,}$$

where k is a parameter and i is varying from 1 to number of stocks. The score of an object is T_i may be given as $T_i = R_i - D_i$. The problem here is to choose an object from the set of given objects with respect to a set of choice parameters P in [10]. Next point to consider is an algorithm for identification of an object, based on multi-observer input data.

4.3 Algorithm

An algorithm for prediction of an object based on multi input data characterised by Return, Risk, Dividend, and Liquidity is proposed. The inputs in algorithm are parameter of object. The output of proposed algorithm is optimal assets. Algorithm steps are

1. Input is the required number of assets.
2. The set of parameter $\{E_1, E_2, E_3, E_4\}$ is input as taken by the investor.
3. Construct the fuzzy soft set (F, A) using "AND" operator and place it in tabular form whose cell is denoted by (i, j).
4. For each parameter j, Let M_j be the maximum membership value of the assets i.e. $M_j = \max_{i = 1, 2, \ldots\ldots, k} P_{ij}$.
5. Now construct a K×K comparison matrix given by the equation:

$$\frac{P_{i1} - P_{j1}}{M_1} + \frac{P_{i2} - P_{j2}}{M_2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{P_{ik} - P_{jk}}{M_k} \quad (3)$$

6. Compute R_i as the sum of row i and T_i as the sum of element in column i then for each i compute the score $T_i = R_i - D_i$ of the assets.
7. Find decision value of any asset $\max_k T_K$.

5. DECISION MAKING APPROACH AHP

AHP is an approach for decision making problem. It structures the multiple choices of criteria into a set of hierarchy, assessing the relative importance of these criteria, comparing the alternatives for each criteria and determining an overall ranking of alternatives. The algorithm of AHP Saaty [32]:

Step 1: It provides paired comparisons $a_{ij}$of two alternatives i and j then preference matrix is calculated by

$$a_{ij} = a_j / a_i \quad (4)$$

where element of row (i) is to relative to element of column (j) for each criteria.

Step 2: For negative criteria, risk can be calculated by $a_{ij} = a_j / a_i$; element of row (j) relative to element of column (i).

Step 3: After normalizing the preference matrix, the relative scores calculated from preference matrix by weight of element

$$(i) = a_i / \sum_{i=1}^{n} a_i \quad (5)$$

Step 4: The weight (w_i , i=1, 2,………n) is obtained as the average value in the relative score matrix foreach criteria calculated from preference matrix.

Step 5: The overall weights ($w_{ij}^{'}$, i=1, 2,………n) of all alternatives from relative score for each criteria.

Step6: The value X_i for the alternatives is calculated by

$$\mathrm{X}_i = \sum_{i=1}^{n} w_i w_{ij}^{'} \quad (6)$$

Step 7: The alternatives with max X_i is optimal.

6. MEASURE OF PERFORMANCE

In this section, the FSS and AHP are compared by measure of performance [33] and measure of performance of method (L) is given by

$$\gamma_L = \frac{1}{\sum_{i=1}^{n}\sum_{i=1(i\neq j)}^{n}\left|\mu_{e_i}(o_p) - \mu_{e_j}(o_p)\right|} + \sum_{i=1}^{n}\mu_{e_i}(o_p) \quad (7)$$

where n represents the number of parameters and (o_p) be the membership value of the optimal object (o_p) for the parameter e_i .

7. OPTIMIZATION METHOD BASED ON BF-PSO

Butterfly finds an optimal solution for its ability to survive through potentiality of nectar (flower) and the aesthesia of the nectar. This includes the natural talent of a butterfly to sense the smell, colour and the chemical of the nectar as well as its own body action and the other butterflies. Butterfly can have the maximum flowers randomly, opt for the best and have the best further before commencement of next search (iteration). This process of selection goes on repeatedly with time consequently butterfly has the best locals (lbest) as per degree of nodes (flowers), causing selection of global best (gbest). They stay near such nectar where they can have the space for egg laying. But they don't have nest since their survivability depends on nectar to nectar in search of food. The more iteration (max no of flights) will be the algorithm termination with respective objective function fitness reformation. Since simplicity of having an antenna over mow of butterfly through it they sense the existence of flowers and get attracted to flowers; for optimal solution, very good relationship of butterfly and surrounding environment exists. After butterfly leaves the food, butterfly extract information through other butterflies and to detects new nectar in the new direction.

BFPSO Technique [14]

BFPSO algorithm depends upon nectar probability factor and communication through sound sensibility means sensitivity. BF-PSO comprises the intelligence of butterfly to get the maximum quantity of nectar. In the standard PSO made some variations in optimization parameters by considering the effect of sensitivity and nectar probability. The sound sensitivity is nearer to the nectar source, and the minimum sensitivity is required to search the next nectar sources, so the range of the sensitivity and probability from 0.1 to 1. Then modified equations are the basic equations of BF-PSO technique given below for the velocity:

$$\begin{aligned} v^{(i+1)} &= Z(w^{(i)}v^{(i)} + s^{(i)}(1-p^{(i)})rand_1 C_1 \\ &\quad (lbest - x^{(i)}) + (p^{(i)})rand_2 C_2(gbest - x^{(i)})) \end{aligned} \quad (8)$$

Position update $x^{(i+1)} = x^{(i)} + \alpha^{(i)} v^{(i+1)}$ (9)

where, N-swarm size, i- number of iteration, w- range of inertia weight lies between 0.8 to 1.2, r_1 and r_2 are random numbers lies between [0, 1], C_1 and C_2 - velocity coefficients which are positive constants s.t. C_1 + C_2 = 4 and some functions are evaluated by the following formulas

1. Constriction facto

$$\phi = C_1 + C_2 \quad (10)$$

$$Z = (1) - \phi - \sqrt{(\phi^2 - 4\phi)} \quad (11)$$

2. Cognitive constant

C_1 =(((2/3) – 2)(iter /max iter))+2 (12)

3. Social constant

C_2 =2(iter /max iter) (13)

4. Inertia weight

w (t) =0.2 + ((max iter - iter)/max iter) (14)

5. The sensitivity function for BFPSO

s (t) = exp(-(max itera -itera)/ max itera) (15)

6. The probability of nectar amount is the important factor at particular nectar source. The probability range considered from 0.1 to 0.99.

The probability function for BFPSO

p (t) = Fgbest / Σ(Flbest+ Fgbest) (16)

where Flbest =Fitness of local best solutions, Fgbest = Fitness of global best solutions.

If it is the computational results means the proposed models are effective.

8. FINANCIAL MEASURES IN PORTFOLIO SELECTION

The development of modern portfolio theory is proposed by Harry Markowitz [34]. It explores how to reduce the risk for the investors by construct an optimal portfolio, assets taking into consideration the trade-off between expected returns and risk, in which semi variance is used as risk measure. It is more difficult to solve large scale problem with a dense covariance matrix. To overcome the problem of risk measure as variance, some researchers have started to get other risk measures to qualify the risk of portfolio, as like Markowitz [35], Sharpe [36], Konno and Yamazaki 1991[37], Speranza [38] and Simaan [39] etc.

8.1 Expected return

Let's assume that R_i be a random variable representing one month as i^{th} period of thirty assets. In particular, for 30 asset the historical return r_{it} of random the variable R_i during period t (t = 12). The expected value of the random variable can be approximated by the average derived from the past data, i.e.

$$r_i = E[R_i] = \frac{1}{T}\sum_{i=1}^{n} r_{it} \tag{17}$$

8.2 Risk

The semi-absolute deviation of return of portfolio of the 30 stocks during the period t, t = 12 is expressed by

$$w_t(x) = \left|\min\left\{0, \sum_{i=1}^{n}(r_{it} - r_i)x_i\right\}\right| = \frac{1}{\mathrm{T}}\sum_{t=1}^{\mathrm{T}} \frac{\left|\sum_{i=1}^{n}(r_{it} - r_i)\right| + \sum_{i=1}^{n}(r_i - r_{it})}{2}$$

$$w(x) = \frac{1}{\mathrm{T}}\sum_{t=1}^{\mathrm{T}} w_t(x) \tag{18}$$

Here $w(x)$ is used to measure the portfolio risk.

8.3 Dividend

The annual dividend of the i^{th} asset is given as

$$D_i = \sum_{t=1}^{\mathrm{T}} d_t \tag{19}$$

8.4 Liquidity

Liquidity is an important part of the investor. It is measured the degree of possibility concerned with the convert of investment into cash without a significant loss in worth [40]. Most of investors want to more liquidity because returns on stock with high liquidity tend to increase with time. A trapezoidal fuzzy number $\tilde{L} = (la, lb, \alpha, \beta)$ with tolerance interval a, b, left fuzziness $\alpha > 0$ and right fuzziness $\beta > 0$, denoted by turnover rate if its membership function retains the following form

$$\mu(t) = \begin{cases} 1 - \frac{a-t}{\alpha} & if\ a-\alpha \le t \le a, \\ 1 & if\ a \le t \le b, \\ 1 - \frac{t-b}{\beta} & if\ a \le t \le b+\beta, \\ 0 & otherwise \end{cases} \tag{20}$$

Using the fuzzy extension principle [41], the crisp possiblistic mean value of the turnover rate can be expressed by

$$L = \left(\frac{1a + 1b}{2} + \frac{\beta - \alpha}{6}\right) \tag{21}$$

9. PROBLEM FORMULATIONS

The multi objective programming is formulated for portfolio selection problem.

$$\begin{aligned} & Max \sum_{i=1}^{n} r_i x_i \\ & Min \sum_{i=1}^{T} \frac{\left|\sum_{i=1}^{n}(r_{it} - r_i)x_i\right| + \sum_{i=1}^{n}(r_i - r_{it})x_i}{2T} \\ & Max \sum_{i=1}^{n} d_i x_i \\ & S.t. \sum_{i=1}^{n}\left(\frac{La_i + Lb_i}{2} + \frac{\beta_i - \alpha_i}{6}\right)x_i \ge L, \\ & \sum_{i=1}^{n} x_i = 1, \\ & l_i \le x_i \le u_i, i = 1,2,...,n \\ & x_i \ge 0, i = 1,2,...,n \\ & l_i \in \{0,1\}, i = 1,2,...,n \\ & u_i \in \{0,1\}, i = 1,2,...,n. \end{aligned} \tag{22}$$

where L is constant which is given by investor, x_i represents the stocks. We transform it into the following form to eliminate the absolute valued function in Eq (22).

$$\begin{aligned} & Max \sum_{i=1}^{n} r_i x_i \\ & Min \frac{1}{\mathrm{T}} \sum_{t=1}^{\mathrm{T}} \mathrm{P}_t \\ & Max \sum_{i=1}^{n} d_i x_i \\ & S.t. \sum_{i=1}^{n}\left(\frac{La_i + Lb_i}{2} + \frac{\beta_i - \alpha_i}{6}\right)x_i \ge L, \\ & \mathrm{P}_t + \sum_{i=1}^{n}(r_{it} - r_i)x_i \ge 0, t = 1,2,...,\mathrm{T}, \\ & \sum_{i=1}^{n} x_i = 1, \\ & \mathrm{P}_t \ge 0, \forall t = 1,2,...,\mathrm{T}, \\ & l_i \le x_i \le u_i, i = 1,2,...,n \\ & x_i \ge 0, i = 1,2,...,n \\ & l_i \in \{0,1\}, i = 1,2,...,n \\ & u_i \in \{0,1\}, i = 1,2,...,n. \end{aligned} \tag{23}$$

Here upper bound (u_i) and lower bound (l_i) as constraint are included on the investment to avoid the large number of very small investment (lower bound) and at the same time to insure a sufficient diversification of investment (upper bound). The upper bound and lower bound were chosen attentively so that the feasible solution will exist. The above problem is multi objective linear programming problem. We can use several methods to find feasible solutions.

10. NUMERICAL ILLUSTRATION

We took the prices of thirty stocks shown in Table 3 from 1 January 2014 to 31 December 2014. The daily closing prices of thirty stocks are picked keeping in mind that portfolio covers diversify areas. Table 3– Table 6 provide an insight into the data characteristics. The exchange codes of thirty stocks are given in Table 3. The expected rates of returns of the stocks are listed in Table 4 which contains dividend and risk. Liquidity calculated in form of trapezoidal fuzzy number [42-43] in Table 5.

Table 3. Stock ID of 30 stocks

St. ID	Name	St. ID	Name	St. ID	Name
C_1	GAIL	C_{11}	IDEA	C_{21}	BANKBARODA
C_2	ONGC	C_{12}	MTNL	C_{22}	SAIL
C_3	IOC	C_{13}	TATMOTORS	C_{23}	TATA STEEL
C_4	BPCL	C_{14}	MARUTI	C_{24}	WIPRO
C_5	CIPLA	C_{15}	BAJAJ-AUTO	C_{25}	TCS
C_6	LUPIN	C_{16}	HEROMOTOCO	C_{26}	MINDTREE
C_7	AJANTPHARM	C_{17}	M&M	C_{27}	INFY
C_8	BHARTIARTL	C_{18}	HDFCBANK	C_{28}	BHEL
C_9	INFRATEL	C_{19}	ICICIBANK	C_{29}	NBCC
C_{10}	TATACOMM	C_{20}	SBIN	C_{30}	LT

In portfolio management, the investor can acquire values of these parameters by using the Delphi Method [44] . For explanation purpose, we represent the method to calculate liquidity for the stock C_1 in detail. First, we calculate the frequency of turnover ratios by historical data. We find that most of the historical turnover ratios fall in the intervals [0.00003-0.00004], [0.00004-0.00005], [0.00005-0.00006], [0.00006-0.00007], [0.00007-0.00008]. Figure 1 Present the frequency distribution of historical turnover ratio for stock C_1. We take the average of the intervals [0.00003-0.00004] and [0.00007-0.00008] as the left and the right end points of the tolerance interval, respectively. Thus, the tolerance interval of the fuzzy turnover ratio is [0.00004, 0.00008]. By observing all the historical data, we use 0.00004 and 0.00008 as the minimum possible value and the maximum possible value of uncertain turnover ratios in the future, respectively. Thus, the left width is 0.00004 and the right width is 0.00008. The fuzzy turnover rate of Stock C_1, is therefore, [0.00004, 0.00008, 0.00003, 0.00008] in Table 5. Similarly, we obtain the fuzzy turnover rates of all thirty stocks.

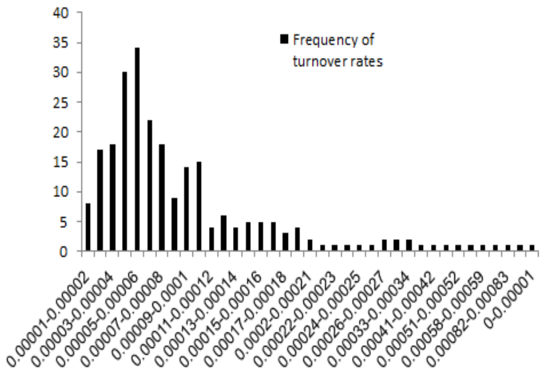

Figure 1. Frequency of turnover ratios for stock C_1

10.1 FSS decision making approach

Let U be the set of objects $\{C_1, C_2, \ldots\ldots\ldots\ldots, C_{30}\}$. The parameter E is the set of parameter {Return, Growth Rate of Net Profit, Dividend, Liquidity Ratio, Dividend Ratio of Working Capital and Liability, Fixed Stocks Turnover Ratio, Liability of stocks, Risk}. Let A be the subset $\{S_1, S_2, S_3, S_4\}$ of the set of parameter E in which the return, dividend, liquidity and risk are calculated by 'AND' operator for the decision making approach because objective is to avoid potential dramatic losses of information. Let's assume that a set of selection parameter is S_1=Re∧Ri∧D, S_2=Re∧Ri∧L, S_3= D∧L∧Re, S_4 =D∧L∧Ri, where Re used in place of return, Ri for risk, D for dividend and L for liquidity.

A predicate name F (S_1) for S_1 and $\{C_1, C_2, \ldots\ldots\ldots\ldots, C_{30}\}$ is an approximate value set. Consider fuzzy soft set (F, A) = {F (S_1), F (S_2), F (S_3), F (S_4)}.

So turnover ratio is solved for C_1 by the Eq. (21). Table 6 expressed the turnover rate of each stock.

Roy and Maji [2] shown that the object identification problem in which optimal decision is taken based on maximum score of the object. This method demands construction of comparison table from the resultant fuzzy soft set Table 7. Comparison Table 8 computed from FSS Table 7 which is the result of the Eq. (2).

Table 4. Return, dividend & risk

St. ID	Retur-n	Dividend	Risk	St. ID	Return	Dividend	Risk
C_1	0.02476	0.02498	0.01857	C_{16}	0.04037	0.03765	0.01780
C_2	0.02358	0.02597	0.03455	C_{17}	0.03243	0.01041	0.02101
C_3	0.04403	0.00230	0.03090	C_{18}	0.03297	0.01744	0.01483
C_4	0.06681	0.03067	0.02586	C_{19}	0.05006	0.01644	0.06324
C_5	0.04340	0.00417	0.02597	C_{20}	0.06372	0.01288	0.09271
C_6	0.04321	0.00530	0.02066	C_{21}	0.05609	0.02589	0.04156
C_7	0.09541	0.00685	0.03586	C_{22}	0.02269	0.02597	0.05073
C_8	0.01166	0.01963	0.02461	C_{23}	0.01255	0.02210	0.03769
C_9	0.06181	0.07245	0.02775	C_{24}	-.00013	0.01442	0.01354
C_{10}	0.03821	0.01284	0.02757	C_{25}	0.01110	0.03121	0.01542
C_{11}	-.00077	0.00269	0.02944	C_{26}	-.00762	0.01462	0.05193
C_{12}	0.06560	0.00000	0.07494	C_{27}	0.03131	0.02139	0.04830
C_{13}	0.02902	0.00442	0.01288	C_{28}	0.04486	0.01304	0.04017
C_{14}	0.06167	0.00485	0.01923	C_{29}	0.17598	0.01198	0.06475
C_{15}	0.02681	0.02294	0.02094	C_{30}	0.04372	0.02569	0.03690

Table 5. Fuzzy turnover rates of 30 stocks

St. ID	$\tilde{L}$	*St. ID*	$\tilde{L}$
C_1	0.00004 0.00008 0.00003 0.00008	C_{16}	0.00005 0.00013 0.00004 0.00014
C_2	0.00002 0.00001 0.00001 0.00006	C_{17}	0.00005 0.00013 0.00004 0.00014
C_3	0.00003 0.00007 0.00002 0.00007	C_{18}	0.00004 0.00008 0.00003 0.00008
C4	0.00009 0.00021 0.00007 0.00022	C19	0.00008 0.00016 0.00007 0.00017
C5	0.00010 0.00027 0.00008 0.00023	C20	0.00024 0.00040 0.00022 0.00042
C6	0.00006 0.00014 0.00005 0.00015	C21	0.00025 0.00045 0.00022 0.00047
C7	0.00010 0.00022 0.00008 0.00023	C22	0.00006 0.00014 0.00005 0.00015
C8	0.00003 0.00007 0.00002 0.00007	C23	0.00058 0.00094 0.00053 0.00098
C9	0.00002 0.00001 0.00001 0.00006	C24	0.00004 0.00001 0.00003 0.00008
C10	0.00012 0.00024 0.00010 0.00025	C25	0.00003 0.00001 0.00000 0.00007
C11	0.00005 0.00001 0.00004 0.00009	C26	0.00049 0.00008 0.00045 0.00080
C12	0.00028 0.00006 0.00024 0.00064	C27	0.00005 0.00001 0.00004 0.00009
C13	0.00001 0.00074 0.00009 0.00014	C28	0.00012 0.00024 0.00010 0.00025
C14	0.00007 0.00015 0.00006 0.00016	C29	0.00007 0.00035 0.00003 0.00038
C15	0.00004 0.00008 0.00003 0.00008	C30	0.00005 0.00013 0.00004 0.00014

Table 6. Liquidity for stock

Stock ID	$\tilde{L}$	*Stock ID*	$\tilde{L}$	*Stock ID*	$\tilde{L}$
C_1	0.00007	C_{11}	0.00004	C_{21}	0.00039
C_2	0.00002	C_{12}	0.00051	C_{22}	0.00012
C_3	0.00006	C_{13}	0.00043	C_{23}	0.00084
C_4	0.00018	C_{14}	0.00013	C_{24}	0.00003
C_5	0.00021	C_{15}	0.00007	C_{25}	0.00003
C_6	0.00012	C_{16}	0.00011	C_{26}	0.00034
C_7	0.00019	C_{17}	0.00011	C_{27}	0.00004
C_8	0.00006	C_{18}	0.00007	C_{28}	0.00021
C_9	0.00002	C_{19}	0.00014	C_{29}	0.00027
C_{10}	0.00021	C_{20}	0.00035	C_{30}	0.00022

Table 7. Fuzzy soft set table

Stock ID	S_1	S_2	S_3	S_4
C_1	0.0001570	0.0000004	0.0000001	0.0000001
C_2	0.0001263	0.0000001	0.0000000	0.0000000
C_3	0.0000148	0.0000004	0.0000000	0.0000000
C_4	0.0002600	0.0000015	0.0000007	0.0000004
-	-	-	-	-
-	-	-	-	-
C_{29}	0.0000791	0.0000018	0.0000008	0.0000001
C_{30}	0.0000147	0.0000013	0.0000006	0.0000003

From Table 10, the value of each stocks C_{23}, C_{21}, C_4 and C_{30} is different in which the closeness is in form $T_{23} > T_{21} > T_4 > T_{30}$. This situation comes as follows $C_{23} > C_{21} > C_4 > C_{30}$. By the results, Tatasteel, Bankbaroda, Bpcl and LT are selected as the optimal alternative according to the decision making approach.So C_4, C_{21}, C_{23} and C_{30} could be selected in the diversify area. The four stocks are selected as optimal alternatives by fuzzy soft sets. The investor's behaviour understanding by the survey [45]is the basis of portfolio construction of four stocks by which, investor's portfolio diversification is in narrow range 3-10 stocks. If two decision values are equal in R and D then Mean Potentiality Approach (MPA) [32] will be applied.

Table 8. Comparison table for FSS

	C_1	C_2	C_3	C_4		C_{30}
C_1	0.00000	0.31872	0.42500	0.00000	-	0.01448
C_2	0.00000	0.00000	0.19452	0.00000	-	0.00000
C_3	0.00000	0.08824	0.00000	0.00000	-	0.00000
C_4	1.26476	1.58348	1.68976	0.00000	-	0.42992
C_5	0.32353	0.58869	0.52488	0.00000	-	0.05882
C_6	0.14706	0.31222	0.26690	0.00000	-	0.00000
-	-	-	-	-	-	-
-	-	-	-	-	-	-
C_{29}	0.95023	1.21538	1.23933	0.16516	-	0.30090
C_{30}	0.84932	1.15356	1.25985	0.00000	-	0.00000

Table 9. Represents the value of R & D

Stock ID	*R*	*D*	*Stock ID*	*R*	*D*
C_1	5.888640	13.60267	C_{16}	22.49821	7.159950
C_2	2.718180	19.99372	C_{17}	4.007710	13.68040
C_3	0.677490	21.14163	C_{18}	5.020890	13.45933
C_4	33.85035	3.621550	C_{19}	5.405480	13.04919
C_5	7.874630	12.59247	C_{20}	8.461540	19.38354
C_6	3.525620	15.98280	C_{21}	48.58256	2.314740
C_7	11.01198	9.998310	C_{22}	5.956590	12.55139
C_8	3.368460	15.16790	C_{23}	72.06663	2.218600
C_9	27.23320	12.62127	C_{24}	0.867860	21.17376
C_{10}	12.29522	8.372780	C_{25}	6.105180	14.82501
C_{11}	0.064650	23.42184	C_{26}	8.079390	12.11506
C_{12}	3.823530	18.88580	C_{27}	0.176480	22.63346
C_{13}	24.05016	9.206590	C_{28}	9.202800	9.701790
C_{14}	5.206070	14.80684	C_{29}	23.13818	6.422520
C_{15}	5.354970	13.73892	C_{30}	22.61789	5.286680

Table 10. Score *T* of 30 stocks

Stock ID	T	*Stock ID*	*T*	*Stock ID*	T
C_1	-7.71402	C_{11}	-23.3572	C_{21}	46.26782
C_2	-17.2755	C_{12}	-15.0623	C_{22}	-6.5948
C_3	-20.4641	C_{13}	14.84358	C_{23}	69.84803
C_4	30.2288	C_{14}	-9.60077	C_{24}	-20.3059
C_5	-4.71784	C_{15}	-8.38395	C_{25}	-8.71983
C_6	-12.4572	C_{16}	15.33826	C_{26}	-4.03567
C_7	1.013663	C_{17}	-9.67269	C_{27}	-22.457
C_8	-11.7994	C_{18}	-8.43844	C_{28}	-0.49899
C_9	14.61194	C_{19}	-7.64371	C_{29}	16.71565
C_{10}	3.922439	C_{20}	-10.922	C_{30}	17.33121

10.2 AHP decision making approach

An advantage of the AHP is that it is designed to handle situations in which the subjective judgments of individuals constitute an important part of the decision process. The following important points in the AHP, (i) Object – selecting the stocks for making a portfolio, (ii) Criteria- return, dividend liquidity and risk are selected to criteria by Table 4 and Table 6 and (iii) Alternatives- there are thirty stocks C_1, C_2, ..., C_{30} for alternatives. Now calculated the preference matrix for the criteria return by Eq (4) which is showing the example of the result for ten alternatives.

Table 11. Preference matrix for return criteria

	C1	**C2**	**C3**	**C4**		**C30**
C1	1.00000	1.05004	0.56234	0.37060	-	0.56633
C2	0.95219	1.00000	0.53554	0.35294	-	0.53934
C3	1.77833	1.86742	1.00000	0.65903	-	1.00709
C4	2.69806	2.83322	1.51737	1.00000	-	1.52813
-	-	-	-	-	-	-
-	-	-	-	-	-	-
C29	7.10706	7.46309	3.99682	2.63404	-	4.02516
C30	1.76567	1.85412	0.65439	0.65439	-	1.00000

Table 12. Relative score for return criteria

	C1	C2	C3		C30	w'_{1j}
C1	0.02094	0.02094	0.02094	-	0.02094	0.02094
C2	0.01994	0.01994	0.01994	-	0.01994	0.01994
C3	0.03723	0.03723	0.03723	-	0.03723	0.03723
C4	0.05649	0.05649	0.05649	-	0.05649	0.05649
-	-	-	-	-	-	-
-		-		-	-	-
-	-	-	-	-	-	-
-	-	-	-	-	-	-
C9	0.05226	0.05226	0.05226	-	0.05226	0.05226
C10	0.03230	0.03230	0.03231	-	0.03231	0.03231

Table 13. Overall Weights (w'_{ij}) of 30 alternatives

Criteria	***Alternatives***							
	C1	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**
Return	0.02094	0.01994	0.03723	0.05649	0.03670	0.03654	0.08067	0.05226
Dividend	0.01786	0.03323	0.02972	0.02487	0.02493	0.01987	0.03449	0.02669
Liquidity	0.04616	0.04799	0.00425	0.05667	0.00771	0.00979	0.01266	0.13387
Risk	0.01252	0.00358	0.01073	0.03220	0.03757	0.02147	0.03399	0.00358
	C10	**C11**	**C12**	**C13**	**C14**	**C15**	**C16**	**C17**
Return	0.03231	-0.00065	0.05547	0.02454	0.05214	0.02267	0.03413	0.02742
Dividend	0.02652	0.02832	0.07208	0.01239	0.01850	0.01952	0.01712	0.02021
Liquidity	0.02373	0.00497	0.00000	0.00817	0.00896	0.04239	0.06957	0.01924
Risk	0.03757	0.00716	0.09123	0.07692	0.02326	0.01252	0.01968	0.01968
	C18	**C19**	**C20**	**C21**	**C21**	**C22**	**C23**	**C24**
Return	0.02788	0.04233	0.05388	0.04743	0.01918	0.01061	-0.00011	0.02788
Dividend	0.01427	0.06083	0.08918	0.03998	0.04880	0.03625	0.01302	0.01427
Liquidity	0.03223	0.03038	0.02380	0.04784	0.04799	0.04084	0.02664	0.03223
Risk	0.01252	0.02504	0.06261	0.06977	0.02147	0.15027	0.00537	0.01252
	C18	**C19**	**C20**	**C21**	**C21**	**C22**	**C23**	**C24**
Return	0.00939	-0.00644	-0.02647	0.03793	0.14880	0.03697	0.00939	-0.00644
Dividend	0.01483	0.04995	0.04646	0.03864	0.06228	0.03549	0.01483	0.04995
Liquidity	0.05767	0.02701	0.03952	0.02410	0.02214	0.04747	0.05767	0.02701
Risk	0.00537	0.06082	0.00716	0.03757	0.04830	0.03936	0.00537	0.06082

Table 14. Represents the value of X_j

Stock ID	X_i	Stock ID	X_i	Stock ID	X_i
C1	0.00304	C11	0.00088	C21	0.01100
C2	0.00382	C12	0.01660	C22	0.00551
C3	0.00240	C13	0.00674	C23	0.02568
C4	0.00806	C14	0.00368	C24	0.00091
C5	0.00344	C15	0.00285	C25	0.00366
C6	0.00229	C16	0.00669	C26	0.00697
C7	0.00901	C17	0.00192	C27	0.00447
C8	0.00209	C18	0.00218	C28	0.00492
C9	0.02138	C19	0.00704	C29	0.02884
C10	0.00372	C20	0.01534	C30	0.00643

The value of X_j calculated from Eq (6).

For normalizing Table 11, relative score is calculated by Eq (5) for return criteria. The weight (w'_{1j}) assigned to thirty alternatives for the return criteria will be taking as the average value in the last column in Table 12. Similarly for criteria dividend, liquidity and risk, weights (w'_{ij}) are evaluated to thirty alternatives. Table 13 presenting the overall weights (w'_{ij}) to all alternatives for each criteria.

From Table 14, maximum values of the stocks are X_{29}, X_{23}, X_9 and X_{12}. Now C_{29}, C_{23}, C_9 and C_{12} are selected as optimal alternatives by AHP approach.

10.3 Comparison of the two approaches

Performance of two approaches, FSS and AHP, are estimated by Eq (7) in Table 15.

From Table 15, $\gamma_{L1>}\gamma_{L2}$: Its mean that FSS decision making approach is better than AHP approach. At last, optimal object by FSS method is taken for making a portfolio. The next step is to obtain a proportion of portfolio optimization Eq (22) and Eq (23) by solving BFPSO MOP problem. MATLAB is used for solving the portfolio optimization problem of four stocks, considering that expected return, risk, dividend and liquidity are fuzzy number. The optimal proportions of the stock in the Table 16 eare 0.0700, 0.0800, 0.0400, 0.7100, 0.0200, 0.0100 and 0.0700.

The results confirm the efficiency of BFPSO tool of MATLAB for its fast convergence towards the better solution and its interesting computing time. The return is 0.0319, dividend is 0.0090 and risk is 0.0012 are the portfolio of seven stocks.

Table 15. The value of performance measures

Name of approaches	Measure of Performance	Optimal Stocks	Value of ($\boldsymbol{\gamma}$)
FSS decision making	γ_{L1}	C_{23},C_{21} C_{30}, C_4.	854.55545
AHP decision making	$\boldsymbol{\gamma_{L2}}$	C_{29}, C_{23}, C_9, C_{12}.	4.26990

11. CONCLUSION

Portfolio allocation by comparing the performance measures of approaches like FSS and AHP, resultantly found the FSS better for decision making approach. A portfolio of four assets based on FSS was prepared. In other existing methods, formation of portfolio is done by choosing random assets on parameter basis not on ranked but FSS is systematic, found ranked, well-known by compare the values of R and D. In this, portfolio optimization has been carried out by the proportion of stocks of portfolio obtained by BFPSO algorithm by considering the effect of sensitivity of butterfly and probability of nectar. BF-PSO shows good convergence rate and found with good accuracy as well good convergence. Our proposed algorithms in section 4 and section 5 are different from financial proposals in the literature. The approach developed here for stocks allocation in portfolio using FSS to study the portfolio selection problem is unique in its kind.

Table 16. Represents proportion of stocks

Stock ID	Proportion	Stock ID	Proportion	Stock ID	Proportion
C_1	0	C_{11}	0	C_{21}	0.1800
C_2	0	C_{12}	0	C_{22}	0
C_3	0	C_{13}	0	C_{23}	0.1200
C_4	0.0400	C_{14}	0	C_{24}	0
C_5	0	C_{15}	0	C_{25}	0
C_6	0	C_{16}	0	C_{26}	0
C_7	0	C_{17}	0	C_{27}	0
C_8	0	C_{18}	0	C_{28}	0
C_9	0	C_{19}	0	C_{29}	0
C_{10}	0	C_{20}	0	C_{30}	0.3000

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