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H_∞ control oriented LFT modelling of linear dynamical system

Tamal Roy[1*], Ranjit Kumar Barai[2], Rajeeb Dey[3]

[1] Electrical Engineering Department, MCKV Institute of Engineering Liluah, Howrah 711204, India
[2] Electrical engineering Department, Jadavpur University, Kolkata 700032, India
[3] Electrical Engineering Department NIT, Silchar, Assam 788010, India

Corresponding Author Email: tamalroy77@gmail.com

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ABSTRACT

This paper presents a systematic formulation of control-oriented linear fractional transformation (LFT) modelling of the linear dynamical system, truly integrates the objective of control theory. A novel methodology has been introduced for modeling quality improvement to achieve certain performance specification considering the modeling uncertainties arising due to the difference between the mathematical model and the actual system and the presence of disturbance signal during the formulation of LFT framework. For the convenience of compact modelling, the generalized transfer function of the linear dynamical system has been represented into the LFT framework by incorporating the real parametric uncertainties enter rationally into the system modelling. The generalized LFT modeling algorithm is convenient to address the issues like identifiability and persistence of excitation for a huge class of system model structures can be accommodated because of its general nature. The proposed modelling algorithm has been applied to a benchmarked industrial mechatronics system, to verify the effectiveness of control theory.

1. INTRODUCTION

In the last few decades, there has been widespread interest in the development of the parametric representation of the system modeling characterized by the system variations. These system variations result from inadequate knowledge about the physical systems, changes in the system operating conditions and neglecting high-frequency dynamics, time delay, nonlinearities, etc. are described as model uncertainties. Uncertainties arise in the system model can invariably affect the stability and the control system performance. Therefore, from the perspective of the design of an appropriate control law, substantial research efforts have been directed towards the model quality improvement by incorporating the model uncertainties in the system modeling.

The objective of the system modelling is the formulation of an appropriate parametric model of a system, which is effective for predicting the nature of the system in different operating conditions. Therefore, any effort to formulate generalized state model for such real dynamical system results into a very huge and complex mathematical model. An extremely accurate model of a real system may be inconvenient due to the high quality of computational complexity. The resulting control law will also become complicated, and their implementation in the real-time platform will be difficult. However, from the perspective of the design of appropriate control law for the given system, it becomes necessary to select an appropriate mathematical model of the system in order to obtain optimum control performance. During the design stages, control laws impose certain mathematical structure in the system modelling otherwise the design becomes impractical or result in a very poor control law. Therefore, optimal control performance of a physical system is achieved by deriving the generalized modelling and identification, instead of deriving the complicated mathematical modelling. Moreover, implacability of the robust H_∞ control strategies and explicit set of state-space equations result into a compact and manageable control system which gain popularity in various applications. A robust H_∞ control strategy for a differential drive Tractor Trailer system has been derived for output feedback control using mu-synthesis, for nominal plant subjected to model uncertainties and output disturbances is discussed in [1]. A set membership H_∞ identification technique has been introduced to investigate the model perturbations for implementing robust control methodologies in [2]. The robust control law of uncertain LPV systems and identification of LPV systems has been represented in LFT framework discussed in [3]. A paradigm shift in the modelling of the dynamic systems has been occurred by introducing robust control theory where associated modelling leads to the system model in linear fractional transformation (LFT) framework [4]. This modelling structure has been considered to be consisting of a nominal model and optimal uncertainty set bound around a prefixed nominal model in [5]. Uncertainties are inherent in all system models and it adversely effects on the stability and the performance of the control system, as they are unknown during the analysis and design. The information of model uncertainties has been included in any form while modelling is a great challenge for controller synthesis, mainly based upon the uncertainty modelling. The physical systems are generally being modelled as unstructured uncertainty, dynamic structure uncertainty and parametric uncertainty where parametric uncertainty is a part of the structure uncertainty as discussed in [5]. Several approaches of uncertainty representation towards the characterization and quantization of uncertainty describe the perturbations of the model to be controlled during the system identification steps in [6]. An emerging uncertainty modelling technique has been successfully implemented to the robust control performance in a feedback-like modelling structure in linear fractional transformation (LFT) framework, where each of the uncertainty relates to physically meaningful parameters of the actual system discussed in [7]. A general descriptor type LFT model consisting of rational parametric matrices in a generalized form in terms of arbitrary rationally dependent multivariate function has been proposed in [8]. This paper proposes a novel generalized H_∞ control oriented LFT modelling algorithm of a generic n^{th} order linear dynamical system even in the face of the plant uncertainty and disturbances to address the robust controller design problem. A systematic approach for casting the generic n^{th} order transfer function representation of a dynamical system in terms of LFT framework not only makes it convenient for the application of modern robust control technique like μ-synthesis-based H_∞ controller design but also a compact and manageable modelling representation considering the parametric uncertainties. The proposed generalized uncertainty-modelling algorithm has been motivated by the fact that for any linear physical system model structure of any order can be accommodated in the LFT framework due to its generic modelling structure has never been addressed in the literature. Finally, this paper successfully implements the proposed uncertainty-modelling algorithm on a benchmark industrial platform, namely Brushless Direct Current (BLDC) motor and validates the performance of the H_∞ control law in real time environments. Moreover, the effectiveness of LFT modelling of the BLDC motor has been verified by realizing the robust stability and performance.

The rest of the paper is organized as follows: Section 2 represents the stepwise derivation of the uncertainty-modeling algorithm for a generalized n^{th} order linear dynamical system. Section 3 describes the H_∞ control oriented LFT modeling of BLDC motor. Section 4 describes H_∞ controller design specification and section 5 represent the H_∞ controller design. Section 6 describes the performance analysis of the proposed modeling algorithm on BLDC motor with a simulation study. Finally, section 7 draws the conclusions.

2. H_∞ CONTROL ORIENTED MODELLING OF NTH ORDER LINEAR DYNAMICAL SYSTEM

Linear dynamical systems represented by the mathematical operational method in order to express in terms of the transfer function to determine the performance of the system. A conventional way of representation studies the effect of the different components and predicts the system behavior. The transfer function can be derived by simple mathematical manipulation of the differential equations that describes the real physical systems. The system transfer function is the mathematical representation in between system input and output, i.e., it indicates the input signal that faces the dynamic elements before appearing on the system outputs.

Consider a single-input-single-output linear system represented by the n^{th} – order differential equation

$$\frac{d^n y(t)}{dt^n}+a_1\frac{d^{n-1}y(t)}{dt^{n-1}}+\cdots+a_{n-1}\frac{dy(t)}{dt}+a_n y(t) = \beta_1\frac{d^{n-1}u(t)}{dt^{n-1}}+\beta_2\frac{d^{n-2}u(t)}{dt^{n-2}}+\cdots+\beta_{n-1}\frac{du(t)}{dt}+\beta_n u \quad (1)$$

where y is the output and u is the input of the system and β_i and a_i for $i = 1,2,\dots\dots,n$

Derivation of the LFT of modeling of n^{th} order system:

An n^{th} order differential equation is considered to represent the linear dynamical system to formulate the control-oriented LFT modeling. The formulation of the H_∞ control oriented LFT modeling of n^{th} order transfer function can be derived by the following steps.

Step 1: The transfer function representation of the linear dynamical system is obtained from the differential equation in (1).

If the initial conditions are zero, its complex counterpart is obtained simply by a substitution of $\frac{d^i}{dt^i}$ by s^i and $y(t)\rightarrow Y(s)$ and $u(t)\rightarrow U(s)$. The generalized transfer function of the system is

$$G(s)=\frac{Y(s)}{U(s)}=\frac{\beta_1 s^{n-1}+\beta_2 s^{n-2}+\cdots+\beta_{n-1}s+\beta_n}{s^n+a_1 s^{n-1}+\cdots+a_{n-1}s+a_n} \quad (2)$$

The state space representation of the transfer function can be expressed as

$$\dot{x}_n=-a_n x_1-a_{n-1}x_2-\cdots\cdots-a_2 x_{n-1}-a_1 x_n+u \quad (3)$$

$$y=\beta_1 x_n+\beta_2 x_{n-1}+\cdots\cdots+\beta_{n-1}x_2+\beta_n x_1 \quad (4)$$

The block diagram representation of the linear time-variant system can be drawn from the equation (3) and (4) is shown in Fig. 1.

Step 2: The uncertain parameters a_i and β_i characterize the model uncertainties and lead to the variation in the system nature. Therefore, it is assumed that the parameter values are lying within a known interval. Now system parameters a_i and β_i expressed as

$$a_i=\bar{a}_i(1+p_{a_i}\delta_{a_i}) \text{ for } i=1,2,\cdots\cdots,n \quad (5)$$

and

$$\beta_i=\bar{\beta}_i(1+p_{\beta_i}\delta_{\beta_i}) \text{ for } i=1,2,\cdots\cdots,n \quad (6)$$

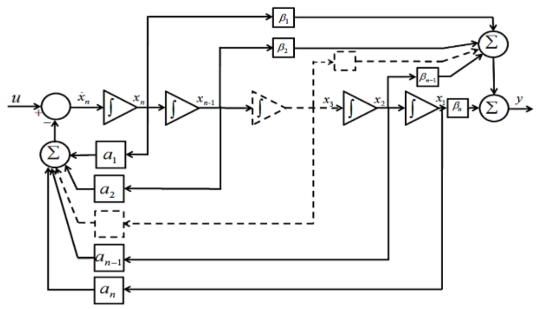

Figure 1. Block diagram representation of n^{th} order linear dynamical system

where $\bar{a}_i$ and $\bar{\beta}_i$ are the nominal value of the parameters, a_{β_i} and p_{β_i} are to represent the possible perturbations of the parameters respectively and δ_{a_i} and

δ_{β_i} are assumed to be unknown and lie within the interval $[-1,1]$.

The upper LFT representation of the parameter a_i can be expressed as

$$a_i = F_U(M_{a_i}, \delta_{a_i}) \text{ for } i = 1, 2, \cdots\cdots, n \tag{7}$$

where Note that $M_{a_i} \in$ □$^{2\times 2}$

Similarly, β_i can be expressed as an upper LFT representation

$$\beta_i = F_U(M_{\beta_i}, \delta_{\beta_i}) \text{ for } i = 1, 2, \cdots\cdots, n \tag{8}$$

where note that $M_{\beta_i} \in$ □$^{2n\times 2}$. The upper LFT representations of the system parameters are described individually and system states are represented by x_j where $j = n - i + 1$ and $i = 1, 2, \cdots\cdots, n$ are shown in Fig. 2 and Fig. 3 respectively.

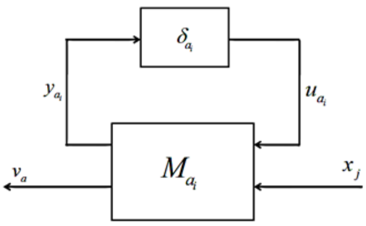

Figure 2. Representation of uncertain parameter δ_{β_i} in upper LFT framework

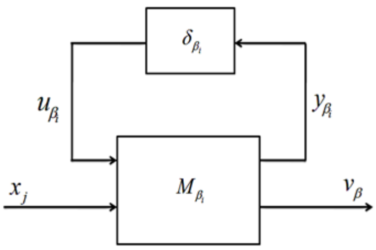

Figure 3. Representation of uncertain parameter δ_{β_i} in upper LFT framework

Step 3: The block diagram representation of the n[th] order linear dynamical system is shown in the Fig. 1 can be redrawn with uncertain parameters δ_{a_i} and δ_{β_i} in the upper LFT framework as shown in the Fig. 4. Treating u_{a_i} and u_{β_i} are the outputs of the uncertainty blocks δ_{a_i} and δ_{β_i} are fed as an input to M_{a_i} and M_{β_i} respectively.

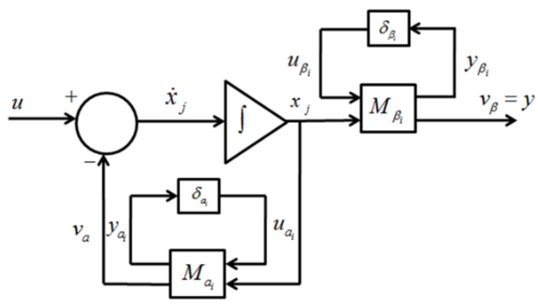

Figure 4. Block diagram representation with uncertain parameters

Similarly, y_{a_i} and y_{β_i} are the outputs of M_{a_i} and M_{β_i} that are fed to the uncertain blocks δ_{a_i} and δ_{β_i} respectively.

The output expressions of augmented vectors of the upper LFT structures from the Fig. 4

$$\begin{bmatrix} y_{a_i} \\ v_a \end{bmatrix} = \begin{bmatrix} 0 & \bar{a}_i \\ p_{a_i} & \bar{a}_i \end{bmatrix} \begin{bmatrix} u_{a_i} \\ x_j \end{bmatrix} \tag{9}$$

Note that $y_{a_i} \in$ □$^{n\times 1}$ and $v_a \in$ □$^{n\times 1}$

$$\begin{bmatrix} y_{\beta_i} \\ y \end{bmatrix} = \begin{bmatrix} 0 & \bar{\beta}_i \\ p_{\beta_i} & \bar{\beta} \end{bmatrix} \begin{bmatrix} u_{\beta_i} \\ x_j \end{bmatrix} \tag{10}$$

Note that $y_{\beta_i} \in$ □$^{n\times 1}$ and $y \in$ □$^{n\times 1}$

The mathematical expression of the system states $\dot{x}_j$ and the output y can be expressed from the Fig.4.

$$\dot{x}_j = -\bar{a}_i x_j u - p_{a_i} u_{a_i} + u \tag{11}$$

$$y = p_{\beta_i} u_{\beta_i} + \bar{\beta}_i x_j \tag{12}$$

The upper LFT representation of the linear dynamical system

$$\begin{bmatrix} \dot{x}_j \\ y_{a_i} \\ y_{\beta_i} \\ y \end{bmatrix}_{4n\times 1} = \begin{bmatrix} -\bar{a}_i & -\bar{p}_{a_i} & 0 & I \\ \bar{a}_i & 0 & 0 & 0 \\ \bar{\beta}_i & 0 & 0 & 0 \\ \bar{\beta}_i & 0 & p_{\beta_i} & 0 \end{bmatrix}_{4n\times 4} \begin{bmatrix} x_j \\ u_{a_i} \\ u_{\beta_i} \\ u \end{bmatrix}_{4n\times 1} \tag{13}$$

The input output relationship of the open loop linear dynamical system

$$\begin{bmatrix} \dot{x}_j \\ \hline y_z \\ \hline y \end{bmatrix} = G_{sys} \begin{bmatrix} x_j \\ \hline u_w \\ \hline u \end{bmatrix} \tag{14}$$

where

$$G_{sys} = \left[\begin{array}{c|cc|c} -\bar{a}_i & -\bar{p}_{a_i} & 0 & I \\ \hline \bar{a}_i & 0 & 0 & 0 \\ \bar{\beta}_i & 0 & 0 & 0 \\ \hline \bar{\beta}_i & 0 & p_{\beta_i} & 0 \end{array}\right], \; y_z = \begin{bmatrix} y_{a_i} & y_{\beta_i} \end{bmatrix}^T \text{ and } u_w = \begin{bmatrix} u_{a_i} & u_{\beta_i} \end{bmatrix}^T$$

The input outputs representation of the uncertainty block

$$\begin{bmatrix} u_{a_i} \\ u_{\beta_i} \end{bmatrix} = \Delta \begin{bmatrix} y_{a_i} \\ y_{\beta_i} \end{bmatrix} \tag{15}$$

where,

$$\Delta = \begin{bmatrix} \delta_{a_i} & 0 \\ 0 & \delta_{\beta_i} \end{bmatrix}_{2n\times 2}$$

The upper LFT representation of perturbed linear dynamical system is described by

$$y = F_U(G_{sys}, \Delta)u \tag{16}$$

The state space representation of the LFT modeling is expressed as

$$\begin{bmatrix} \dot{x}_j \\ y_z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{12} & D_{22} \end{array}\right] \begin{bmatrix} x_j \\ u_w \\ u \end{bmatrix} \tag{17}$$

where,

$$A = [-\bar{a}_i],\ B_1 = [-\bar{p}_{a_i} \quad 0],\quad B_2 = [I]$$

$$C_1 = \begin{bmatrix} \bar{a}_i \\ \bar{\beta}_i \end{bmatrix},\ D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\quad D_{12} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$C_2 = [\bar{\beta}_i],\ D_{12} = [0 \quad \bar{p}_{\beta_i}] \text{ and } D_{22} = [0]$$

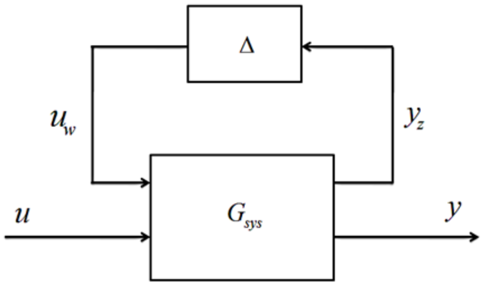

Figure 5. Upper LFT representation of the linear dynamical system

3. REAL TIME IMPLEMENTATION OF THE PROPOSED MODELLING ALGORITHM

The effectiveness of proposed modeling algorithm has been verified on a Brushless DC motor. The BLDC motor is a very useful test platform to validate the feasibility of different industrial applications, especially in the areas of aeronautics, industrials automation, production, electrical vehicles, computer peripherals, etc. Generally, the BLDC motor is a permanent magnet synchronous motor driven by dc voltage, usually applied in many high-performance industrials applications. In order to design precise and stable control, substantial research efforts have been devoted to improving the model quality by incorporating the all unmodeled dynamics occurred due to imperfect or incomplete knowledge about the physical systems [9].

3.1 A mathematical model of a BLDC motor

The mathematical representation of the BLDC motor based on the parameters is illustrated below [10]

$$G(s) = \frac{\frac{1}{K_e}}{\tau_m.\tau_e.s^2 + \tau_m.s + 1} \tag{18}$$

where

$K_e = \dfrac{3.R_\phi.J}{\tau_m.K_t}$ = Phase value of the EMF (voltage) constant

$\tau_m = \dfrac{3.R_\phi.J}{K_e.K_t}$ = Mechanical Constant and

$\tau_e = \dfrac{L}{3.R}$ = Electrical Constant

The mathematical modeling of BLDC motor express in (18) can be represented as

$$\frac{\omega_m(s)}{V_s(s)} = \frac{\frac{1}{K_e\tau_m\tau_e}}{s^2 + \frac{1}{\tau_e}s + \frac{1}{\tau_m.\tau_e}} \tag{19}$$

where $\omega_m(s)$ is the angular velocity and $V_s(s)$ is source voltage.

3.2 LFT modelling of BLDC motor

A systematic formulation for the uncertainty modeling of BLDC motor represents the mathematical modeling of the system that results into a control oriented linear fractional transformation (LFT) modeling framework and characterized the system uncertainties in terms of maximum possible relative error around the nominal value can be lumped into a perturbation block Δ.

Now consider the state variables,

$$\omega_m = x_1$$
$$\dot{\omega}_m = x_2$$

Therefore,

$$y = x_1 = \omega_m$$
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{1}{\tau_m\tau_e}x_1 - \frac{1}{\tau_e}x_2 + \frac{1}{K_e\tau_m\tau_e}V_s \tag{20}$$

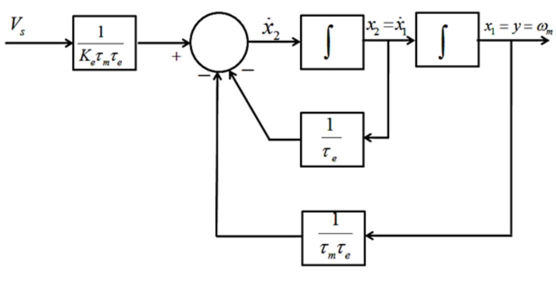

Figure 6. The block diagram representation of BLDC motor

The block diagram representation of the BLDC motor is shown in Fig.6.

Based on the practical realization, parameters of the BLDC motor $\tau_1 = \tau_e = \frac{L}{3R_\emptyset}$ and $\tau_2 = \tau_m \tau_e = \frac{JL}{K_e K_t}$ are considered as uncertain parameters. However, it is assumed, τ_1 and τ_2 are constants with a possible relative error of 10% around the nominal values [11].

The actual system parameter τ_1 of BLDC motor can be expressed as

$$\frac{1}{\tau_1} = \frac{1}{\bar{\tau}_1(1+p_{\tau_1}\delta_{\tau_1})} = \frac{1}{\bar{\tau}_1} - \frac{p_{\tau_1}}{\bar{\tau}_1}\delta_{\tau_1}(1+p_{\tau_1}\delta_{\tau_1})^{-1} \tag{21}$$

where, $\bar{\tau}_1$ is the nominal value of the parameter, $p_{\tau_1} = 0.1$ is the maximum relative uncertainty in the parameter and $-1 \le \delta_{\tau_1} \le 1$.

Therefore, the parameter $\frac{1}{\tau_1}$ of BLDC motor can be represented in the upper LFT framework

$$\frac{1}{\tau_1} = F_U(M_{\tau_1}, \delta_{\tau_1}) \tag{22}$$

where, Note that $M_{\tau_2} \in \Re^{2\times 2}$

$$\frac{1}{\tau_2} = \frac{1}{\bar{\tau}_2(1+p_{\tau_2}\delta_{\tau_2})} = \frac{1}{\bar{\tau}_2} - \frac{p_{\tau_2}}{\bar{\tau}_2}\delta_{\tau_2}(1+p_{\tau_2}\delta_{\tau_2})^{-1} \tag{23}$$

where, $\bar{\tau}_2$ is the nominal value of the parameter, $p_{\tau_2} = 0.1$ is the maximum relative uncertainty in the parameter and can represent as an upper LFT framework

$$\frac{1}{\tau_2} = F_U(M_{\tau_2}, \delta_{\tau_2}) \tag{24}$$

where, Note that $M_{\tau_2} \in \Re^{2\times 2}$

Using the above mathematical expression of τ_1^{-1} *and* τ_2^{-1} in (23) and (25) the block diagram in Fig. 6 can be redrawn in Fig.7. Treating u_{τ_1} and u_{τ_2} to be the output of the uncertain block δ_{τ_1} and δ_{τ_2}, which are fed as input to M_{τ_1} and M_{τ_2} block respectively.

Similarly, y_{τ_1} and y_{τ_2} are the output of M_{τ_1} and M_{τ_2}which are fed as an input to the δ_{τ_1} and δ_{τ_2} block respectively.

Now, augmenting the outputs vectors of upper LFT framework of M_{τ_1} and M_{τ_2} can be express as,

$$\begin{bmatrix} y_{\tau_1} \\ v_{\tau_1} \end{bmatrix} = \begin{bmatrix} -p_{\tau_1} & \frac{1}{\bar{\tau}_1} \\ -p_{\tau_1} & \frac{1}{\bar{\tau}_1} \end{bmatrix} \begin{bmatrix} u_{\tau_1} \\ x_2 \end{bmatrix} \tag{25}$$

Note that $y_{\tau_2} \in \Re^{1\times 1}$ and $v_{\tau_1} \in \Re^{1\times 1}$ and

$$\begin{bmatrix} y_{\tau_2} \\ v_{\tau_2} \end{bmatrix} = \begin{bmatrix} -p_{\tau_2} & \frac{1}{\bar{\tau}_2} \\ -p_{\tau_2} & \frac{1}{\bar{\tau}_2} \end{bmatrix} \begin{bmatrix} u_{\tau_2} \\ x_1 \end{bmatrix} \tag{26}$$

Note that $y_{\tau_2} \in \Re^{1\times 1}$ and $v_{\tau_2} \in \Re^{1\times 1}$.

The state vector of BLDC motor can be defined as

$$\mathbf{X} \triangleq [x_1 \quad x_2]^T \tag{27}$$

where $x_1 \triangleq \omega$ and $x_2 \triangleq \dot{\omega}$

The output expression of the BLDC motor

$$y = x_1 = \omega_m \tag{28}$$

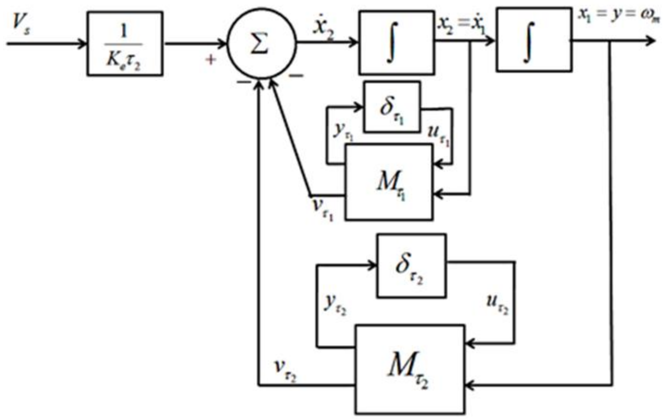

Figure 7. Block diagram of BLDC motor with uncertain parameters

The upper LFT representation of BLDC motor can be expressed as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \hline y_{\tau_1} \\ y_{\tau_2} \\ \hline y \end{bmatrix} = G_{bldc} \begin{bmatrix} x_1 \\ x_2 \\ \hline u_{\tau_1} \\ u_{\tau_2} \\ \hline V_s \end{bmatrix} \tag{29}$$

where,

$$G_{bldc} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{\bar{\tau}_1} & -\frac{1}{\bar{\tau}_1} & p_{\tau_1} & p_{\tau_2} & \frac{1}{K_e \tau_2} \\ 0 & \frac{1}{\bar{\tau}_1} & -p_{\tau_1} & 0 & 0 \\ \frac{1}{\bar{\tau}_1} & 0 & 0 & -p_{\tau_2} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}_{5\times 5}$$

Now, upper LFT representation of BLDC motor further can be expressed as

$$\begin{bmatrix} \dot{x} \\ y_\tau \\ y \end{bmatrix} = G_{bldc} \begin{bmatrix} x \\ u_\tau \\ V_s \end{bmatrix}$$

where,

$$y_\tau = \begin{bmatrix} y_{\tau_1} & y_{\tau_2} \end{bmatrix}^T \text{ and } u_\tau = \begin{bmatrix} u_{\tau_1} & u_{\tau_2} \end{bmatrix}^T$$

Input-output expression of the uncertainty blocks can be represented as

$$\begin{bmatrix} u_{\tau_1} \\ u_{\tau_2} \end{bmatrix} = \Delta \begin{bmatrix} y_{\tau_1} \\ y_{\tau_2} \end{bmatrix}$$

where, $\Delta = \begin{bmatrix} \delta_{\tau_1} & 0 \\ 0 & \delta_{\tau_2} \end{bmatrix}$

The upper LFT representation of perturbed BLDC motor is described by

$$y = F_U(G_{bldc}, \Delta)V_s \tag{30}$$

State space representation of BLDC motor is

$$G_{bldc} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{31}$$

where,

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\tau_1} & -\frac{1}{\tau_2} \end{bmatrix}_{2\times2}, B_1 = \begin{bmatrix} 0 & 0 \\ p_{\tau_1} & p_{\tau_2} \end{bmatrix}_{2\times2}, B_2 = \begin{bmatrix} 0 \\ \frac{1}{K_3\tau_2} \end{bmatrix}_{2\times1}, C_1 = \begin{bmatrix} 0 & \frac{1}{\tau_1} \\ \frac{1}{\tau_1} & 0 \end{bmatrix}_{2\times2}$$

$$C_2 = [1 \quad 0]_{1\times2}, D_{11} = \begin{bmatrix} -p_{\tau_1} & 0 \\ 0 & -p_{\tau_2} \end{bmatrix}_{2\times2}, D_{12} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2\times1}, D_{21} = [0 \quad 0]_{1\times2} \text{ and } D_{22} = [0]_{1\times1}$$

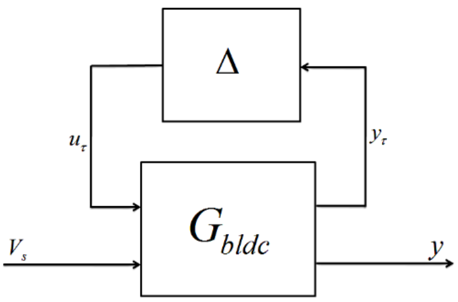

Figure 8. Upper LFT representation of BLDC motor

Table 1. Nominal values of the parameters of BLDC motor

Sl. No.	BLDC Motor Parameter	Symbol	Value	Unit
1.	Terminal resistance phase to phase	$R_\emptyset$	1.2	Ω
2.	Terminal inductance phase to phase	L	0.560	mH
3.	Torque Constant	K_t	25.5	mNm/A
4.	Rotor Inertia	J	92.5	gcm^2
5.	Mechanical Time Constant	τ_m	17.1	ms
6.	Electrical Time Constant	τ_e	155.56×10^{-6}	
7.	Nominal input Voltage	V	12	Volts

The singular value plot of the perturbed BLDC motor is shown in Fig.9

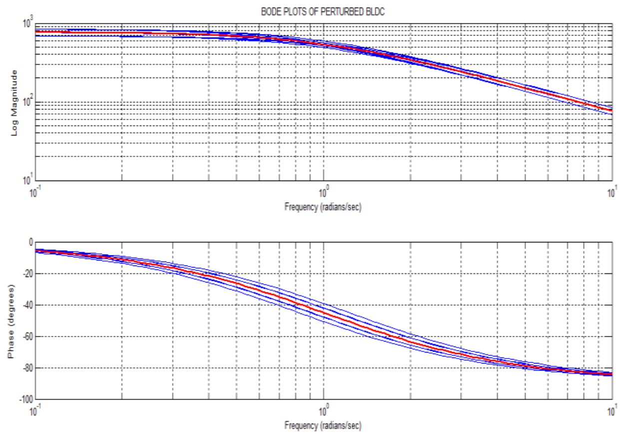

Figure 9. Singular value plot of BLDC motor

4. FREQUENCY DOMAIN VALIDATION OF LFT MODELLING

Frequency domain validation of LFT modeling has been investigated in the context of robust control theory. The objective of the uncertainty modeling validation in LFT framework has been performed by designing H∞ controller to achieve certain performance specification even in the presence of all possible uncertainties.

4.1 H∞ controller design specification

H∞ controlled BLDC motor in LFT framework is obtained the best possible performance even in the presences of outside disturbance and plant uncertainty. The H∞ controller of the closed-loop system makes the system internally stable. The closed loop structure of the BLDC motor with controller K is shown in figure 10, where $G = F_U(G_{bldc}, \Delta)$ in upper LFT framework.

Further, the closed loop BLDC motor achieves certain nominal performance criteria if it satisfies the given performance objective for the nominal model G_{bldc}.

$$\left\| \begin{bmatrix} W_p S(G_{bldc}) \\ W_u KS(G_{bldc}) \end{bmatrix} \right\|_\infty < 1 \tag{32}$$

where W_p and W_u are selected as weighting functions. The selection of the weighting function is such that the H∞ controller will not track high-frequency disturbance. The performance weighting function for BLDC motor is selected as $w_p = 0.095\frac{10s+100}{100s+10}$ and control weighting function is a scalar function can be considered as $w_u = 10^{-2}$.

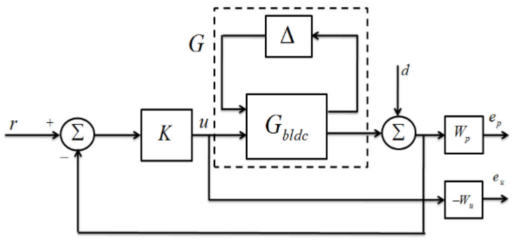

Figure 10. Closed loop structure of BLDC motor

Again, closed loop BLDC achieves robust stability with all possible perturbation and maintain robust performance with the performance objective

$$\left\| \begin{bmatrix} W_p S(G) \\ W_u KS(G) \end{bmatrix} \right\|_\infty < 1 \qquad (33)$$

where $S(G)$ is the sensitivity function of $G = F_U(G_{bldc}, \Delta)$.

4.2 H∞ controller design

A control system is robust if it remains stable and achieves certain performance criteria in all possible uncertainties. The demand for robust stability means H∞ norm of the controller-less than 1 for the uncertainty input/output relationship. The weighting filters are attached to scale the outputs. The H∞ controller has been designed for the interconnected structure shown in Figure 10 to minimizes the $\|.\|_\infty$ norm over the stabilizing controllers K, where P is the transfer function matrix of the system. The $F_L(P, K)$ is the nominal closed-loop system transfer matrix from disturbance to the errors e as shown in the Fig. 11, where $e=\begin{bmatrix} e_p \\ e_u \end{bmatrix}$

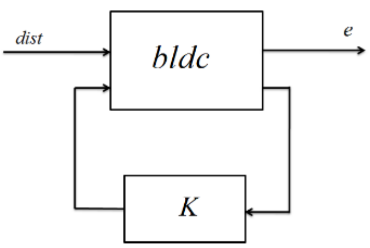

Figure 11. Close loop BLDC motor with H∞ controller

The H∞ controller design has been conducted by using MATLAB function *hinfsyn,* which determine the (sub) optimal H∞ control law. The design H∞ controller for the closed loop system achieves H∞ norms equal to 1.0000. The controller K has one input and one output with three states. The most desirable property of the designed controller is that it consists of all stable poles, which makes the system more favorable in practices.

Robust stability test of the BLDC motor is based on the μ-analysis conducted on the 2 × 2 diagonal block of the transfer matrix and computes upper and lower bounds for the structured singular value. The system is robust enough to provide stability over the uncertainty. Therefore, the frequency response of the closed-loop system with H∞ controller achieves the robust stability with a maximum value of μ is equal to 0.12031 in 1.7 rad/sec and less than 0.1174 for all frequencies is shown in the Fig. 12. The maximum value of μ shows the allowable structured perturbation norms less than $\frac{1}{0.12031}$ i.e., the stability maintains for $\|\Delta\|_\infty < \frac{1}{0.12031}$.

The robust performance of the closed-loop system with H∞ controller is also verified by μ - analysis. The closed-loop transfer function has one input one output. The robust performance of the designed system is achieved, if and only if $\mu_{\Delta_P}(.)$ is less than 1 in all frequencies. The closed-loop system achieved robust performance with H∞ controller since the maximum value of μ is equal to 0.10906 for low frequency and further decreases for higher ranges of frequency is shown in the Fig.13.

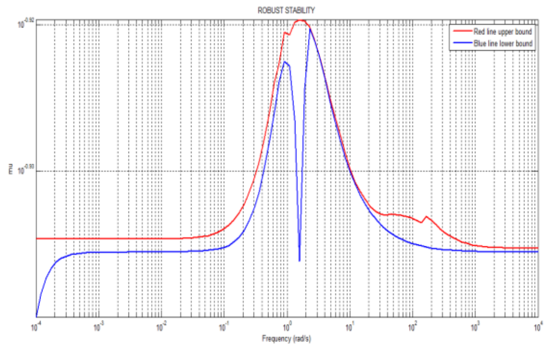

Figure 12. Robust stability test BLDC motor with of H∞ controller

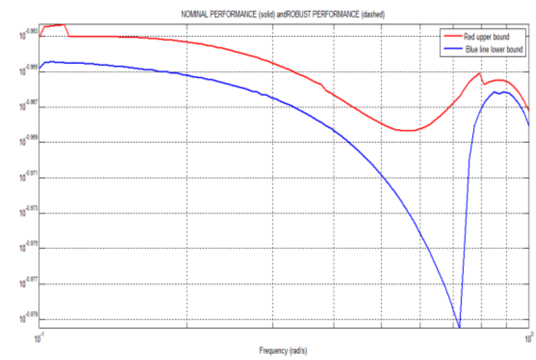

Figure 13. Robust performance of BLDC motor with H∞ controller

While performing the transient response standard input signal (input step signal with final value of 12) has been considered to verify the effectiveness of the proposed modelling structure in time domain. Open loop transient response of the perturbed BLDC motor is shown in the Fig 14. It is observed that the transient response reaches the steady-state value with a small settling time (approximately equal to .0.8 s) is shown in the Fig 15.

It is clearly evident from the outcomes of the simulation studies that the proposed control law yields satisfactory performance in the frequency domain as well as in the time domain. However, the abovementioned outcomes of the H∞ control law make the closed-loop structure of the candidate system robust enough than the other control-oriented modelling structure. Therefore, it should be remembered that proposed control-oriented LFT modeling approaches are capable of ensuring robust stability and performance, that, in turn, makes it more versatile and acceptable than any other application-oriented modeling approaches.

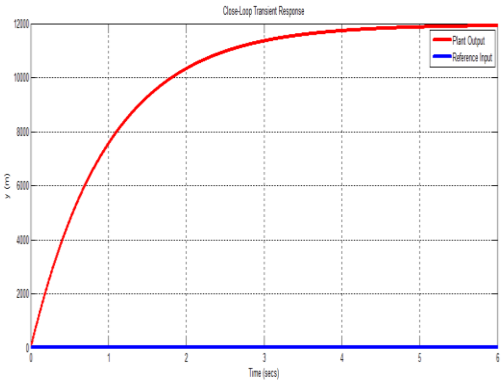

Figure 14. Open loop transient response of the perturbed BLDC motor

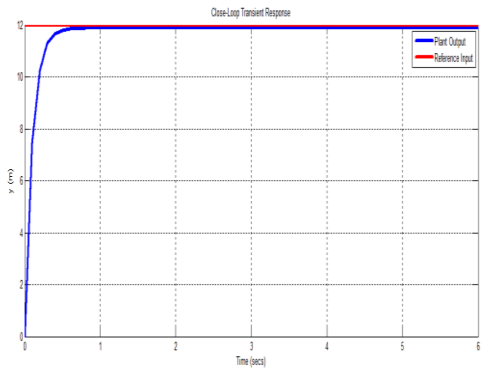

Figure 15. Closed-loop transient response of the system with H_∞ controller

5. CONCLUSIONS

This paper presents a control oriented LFT modeling framework for the n^{th} order linear dynamical system. At the outset of the modeling, parametric uncertainties have been introducing to transform the generalized transfer function of the linear dynamical system into a comprehensive model consisting of the nominal system model and transfer function matrix to account for the various model uncertainties to represent the system model in LFT framework. The proposed modeling algorithm is essential for the application of the modern robust control technique like μ-analysis and synthesis in addition to H_∞ -Control and H_∞-Loop Shaping in order to obtain optimal control performance. The uncertain physical parameters are not known precisely and it is assumed that the parameter values are known within an interval to express in terms of possible relative error. The LFT modeling realization of a dynamical system is a minimal representation refers to the smallest possible dimension of the uncertainty matrix. The model of dynamical systems varies due to changes in the system configuration and the operating conditions. This system variable characterized as model uncertainties to facilitate and improve the effect that incorporates in the system modeling to improve the model quality, increase the system reliability and better utilization of appropriate control law.

The effectiveness of the proposed modeling algorithm has been verified on a BLDC motor to validate the feasibility of the modeling technique. This mathematical framework permits the desired accuracy of the proposed modeling structure based on the availability of reliable model parameter values and establishes the fact that the generalized modeling algorithm is suitable for real-time applications. H_∞ control law has been implemented to the upper LFT structure of the BLDC motor and it has been observed that the closed loop system achieves the robust stability as well as robust performance even in the presence of all parametric uncertainties and disturbances. Moreover, closed loop system exhibits satisfactory transient response in time domain analysis. Finally, it has been observed that the generalized H_∞ control-oriented modeling technique is a very convenient and quite effective approach for robust control law.

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