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Viscous Dissipation Effect on MHD Free Convective Flow in the Presence of Thermal Radiation and Chemical Reaction

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ABSTRACT

The current paper illustrates the consequence of viscous dissipation on the unsteady MHD flow of an incompressible viscous fluid over a vertical permeable surface embedded in a porous medium. The roles of chemical reaction and thermal radiation has made the study more interesting. The Perturbation method has been applied to solve the coupled and nonlinear governing equations. It is found that the present solutions are in very good agreement with the previous solutions. The important findings are: increasing values of Eckert number (Ec) enhances the velocity of the fluid flow. The viscous dissipation convincingly increases the temperature. This analysis is of great interest in many applications such as polymer processing flows, condensation process of metallic plate in cooling bath, aerodynamic extrusion of plastic sheets etc.

1. INTRODUCTION

Magnetohydrodynamics (MHD) presents the magnetic properties of electrically conducting fluid such as plasma, liquid metals, salt water etc. These fluids also exhibit the viscosity property which is responsible for the flow of fluid. It takes energy from the motion of fluid and transfer it into internal energy. As a result, the fluid gets heated. This dissipation process has applications in polymer processing flows, aerodynamic heating in the thin boundary layer around high speed aircraft etc. Many researchers have studied the significant effects of viscous dissipation on MHD flow. Devi et al. [1] investigated the effect of viscous and joule dissipation on MHD flow past a stretching porous surface. Murugesan and Kumar [2], Kumari and Goyal [3], Kumar and Reddy [4] and Rani et al. [5] have considered the viscous dissipation effects under distinct flow domain.

Now-a-days, it is also of great interest to study thermal radiation and chemical reaction in MHD flow of the fluid. Fluid temperature greater than absolute zero emits thermal radiation. When the fluid flows it results in change acceleration a dipole oscillation which produces electro-magnetic radiation. Mass transfer with chemical reaction also acts a vital role in the fluid flow. Many authors ([6-12]) tried to study the influences of thermal radiation and chemical reaction on MHD flow along with viscous dissipation. They solved the governing equations either analytically or numerically.

Further, the roles of external magnetic field and porous medium in MHD heat transfer flow are very much of industrial importance. These types of engineering problems are very relevant in geothermal, energy extractions, oil exploration and the boundary layer control in the field of aerodynamics. Therefore, many investigations have been accomplished by the renowned authors. Makinde et al. [13-15] elucidated numerically the MHD fluid flows past a vertical plate as well as past a slippery stretching. In all of their study, they considered the flow system in a porous medium. Swain and Senapati [16] examined the mass transfer effect on MHD free convective flow embedded in a porous medium. Uddin [17] also carried out his research in a Porous Medium.

The objective of the present study is to analyze viscous dissipation effect on an unsteady MHD free convective flow embedded in a porous medium. Viscous dissipation effect was not attained by Prakash et al. [6], but here we consider it. The expressions for velocity, skin friction, Sherwood number, Nusselt number are obtained using the perturbation technique. These are compared with the previous results for different physical parameters. The present results agree well with the previous results. Moreover, the works of Prakash et al. [6] and Kim [18] have been discussed as special cases. The significance of the physical parameters also studied graphically. Here, all the calculations and graphs are carried out using MATLAB software.

2. MATHEMATICAL FORMULATION

The geometrical flow of the problem is shown in Figure 1. A two-dimensional boundary layer flow of a viscous incompressible fluid past a semi-infinite vertical permeable plate placed in a uniform porous medium is observed. The fluid is also assumed to be electrically conducting and heat absorbing. An external magnetic field B_0 is applied in the presence of thermal and concentration buoyancy effects. The Hall and ion slip effects are considered negligible. External electric field is assumed to be zero and the electric field due to the polarization of charges is negligible. The plate is maintained at constant temperature T_w and concentration C_w. which is higher than the ambient temperature T_∞ and concentration C_∞, respectively. Since we have taken semi-infinite plane surface, the flow variables are the functions of y^* and t^* only. Under these assumptions, the governing equations are given by;

$$\frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma {B_0}^2}{\rho} u^* + g\beta_T(T^* - T_\infty) + g\beta_C(C^* - C_\infty) - \frac{\nu}{K^*}u^*, \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{K}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*} - \frac{Q_0}{\rho C_p}(T^* - T_\infty) + \frac{Q_1^*}{\rho C_p}(C^* - C_\infty) + \frac{D_m}{C_s}\frac{K_T}{\rho C_p}\frac{\partial^2 C^*}{\partial y^{*2}} + \frac{\mu}{\rho C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2, \quad (3)$$

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - R(C^* - C_\infty). \quad (4)$$

The radiative heat flux is taken, which has been given by Pal and Talukdar [19] and Cogley et al. [20] as:

$$\frac{\partial q_r^*}{\partial y^*} = 4\,(T^* - T_\infty)I', \quad (5)$$

where, $I' = \int_0^\infty K_{w\lambda}\frac{\partial e_{b\lambda}}{\partial T^*}d\lambda$, $K_{w\lambda}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is Planck's function.

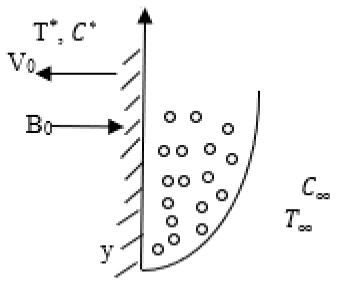

Figure 1. Flow geometry

Under the assumption stated above, the initial and boundary conditions for the velocity distribution involving slip flow, temperature, and concentration distributions are defined as:

$$u^* = u^*_{slip} = \frac{\sqrt{k}}{\alpha}\frac{\partial u^*}{\partial y^*}, T^* = T_w + \epsilon(T_w - T_\infty)e^{n^*t^*}, C^* = C_w + \epsilon(C_w - C_\infty)e^{n^*t^*} \quad \text{at } y^* = 0 \quad (6)$$

$$u^* = U^*_\infty = U_0\left(1 + \epsilon e^{n^*t^*}\right), T^* \to T_\infty, C^* \to C_\infty \text{ as } y^* \to \infty \quad (7)$$

From Eq. (1) we know that the suction velocity near the plate surface is either a constant or a function of time only. Consequently, it is considered that

$$v^* = -V_0(1 + \epsilon A e^{n^*t^*}), \quad (8)$$

where, V_0 is the mean suction velocity and $\epsilon A \ll 1$. The negative sign indicates that the suction velocity is pointed towards the plate.

In the free stream Eq. (2) gives

$$-\frac{1}{\rho}\frac{dp^*}{dx^*} = \frac{dU^*_\infty}{dt^*} + \frac{\sigma}{\rho}B_0^2 U^*_\infty + \frac{\nu}{K^*}U^*_\infty \quad (9)$$

The non- dimensional quantities used in the text are:

$$u = \frac{u^*}{U_0}, \quad v = \frac{v^*}{V_0}, y = \frac{v_0 y^*}{\nu}, U_\infty = \frac{U^*_\infty}{U_0}, t = \frac{V_0^2 t^*}{\nu}, \theta = \frac{T^* - T_\infty}{T_w - T_\infty}, \quad C = \frac{C^* - C_\infty}{C_w - C_\infty}, n = \frac{n^*\nu}{V_0^2}, Gr = \frac{\rho g\nu(T_w - T_\infty)\beta_T}{U_0 V_0^2},$$
$$M = \frac{\sigma\nu B_0^2}{\rho V_0^2}, Sc = \frac{\nu}{D}$$
$$Q_1 = \frac{Q_1^*\nu(C_w - C_\infty)}{\rho C_p V_0^2(T_w - T_\infty)}, \gamma = \frac{R\nu}{V_0^2}, Du = \frac{D_m k_T(C_w - C_\infty)}{C_s K(T_w - T_\infty)}, K = \frac{V_0^2 K^*}{\nu^2}, \quad (10)$$
$$Ec = \frac{V_0^2}{Cp(T_w - T_\infty)}, Gm = \frac{\rho g\nu(C_w - C_\infty)\beta_C}{U_0 V_0^2}, Pr = \frac{\mu C_p}{k},$$
$$\phi = \frac{Q_0\nu}{\rho C_p V_o^2}, F = \frac{4\nu I'}{\rho C_p V_o^2}$$

Considering the above dimensionless variables, the Eq. (2) - (4) can be written in a dimensionless form as:

$$\frac{\partial u}{\partial t} - (1 + \epsilon A e^{nt})\frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + M_1(U_\infty - U) + \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC, \quad (11)$$

$$\frac{\partial \theta}{\partial t} - (1 + \epsilon A e^{nt})\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - F\theta + Q_1 C + \emptyset\theta + \frac{Du}{Pr}\left(\frac{\partial^2 C}{\partial y^2}\right) + Ec\left(\frac{\partial u}{\partial y}\right)^2, \quad (12)$$

$$\frac{\partial C}{\partial t} - (1 + \epsilon A e^{nt})\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - \gamma C. \quad (13)$$

where, $M_1 = M + \frac{1}{K}$, M and K represents the magnetic force intensity and porosity parameter.

The corresponding non-dimensional initial and boundary conditions are given by:

$$u = u_{slip} = \emptyset_1\frac{\partial u}{\partial y}, \theta = 1 + \epsilon e^{nt}, C = 1 + \epsilon e^{nt} \quad at\ y = 0 \quad (14)$$

$$U \to U_\infty = (1 + \epsilon e^{nt}), \theta \to 0, C \to 0 \qquad as\ y \to \infty \quad (15)$$

where, $\emptyset_1 = \frac{\sqrt{K}}{\alpha}\frac{V_0}{\nu}$ is the permeability parameter.

3. METHOD OF SOLUTION

To solve the coupled non-linear partial differential Eqns. (11)-(13) subject to the boundary conditions (14) and (15), the Perturbation method [21] is adopted.

We assume that

$$u = f_0(y) + \epsilon e^{nt} f_1(y) + O(\epsilon^2), \quad (16)$$

$$\theta = g_0(y) + \epsilon e^{nt} g_1(y) + O(\epsilon^2), \quad (17)$$

$$C = h_0(y) + \epsilon e^{nt} h_1(y) + O(\epsilon^2). \quad (18)$$

Putting Eqns. (16)-(18) into Eqns. (11)-(13) and comparing the coefficients (after neglecting the higher order terms of ϵ), we get

$$f_0'' + f_0' - M_1 f_0 = -M_1 - Gr g_0 - Gm h_0, \quad (19)$$

$$f_1'' + f_1' - (M_1+n)f_1 = -(M_1+n) - Af_0' - Grg_1 - Gmh_1, \quad (20)$$

$$g_o'' + Prg_0' - \Pr(F+\emptyset)\, g_0 = -PrQ_1h_0 - Duh_0'' - E_c(f_0')^2 Pr\,, \quad (21)$$

$$g_0'' + Prg_1 - \Pr(F+\emptyset+n)\, g_1 = -PrQ_1h_1 - Duh_1'' - PrAg_0' - 2E_cPrf_0'f_1'\,, \quad (22)$$

$$h_0'' + Sch_0' - Sc\gamma h_0 = 0\,, \quad (23)$$

$$h_1'' + Sch_1' - Sc(\gamma+n)h_1 = -ASch_0'\,. \quad (24)$$

Further, the boundary conditions (14) and (15) reduce to:

$$f_0 = \emptyset_1 f_0', f_1 = \emptyset_1 f_1', g_0 = 1, g_1 = 1, h_o = 1, h_1 = 1 \quad \text{at} \quad y = 0\,, \quad (25)$$

$$f_0 = 1, f_1 = 1, g_0 \to 0, g_1 \to 0, h_0 \to 0, h_1 \to 0 \; as\; y \to \infty\,. \quad (26)$$

Solving Eqns. (23) and (24) under the boundary conditions (25) and (26) i.e. $h_0 = 1, h_1 = 1 \; at\; y = 0$ and $h_0 \to 0, h_1 \to 0 \; as\; y \to \infty$; we get $h_0 = e^{m_2 y}, h_1 = (1-L)e^{m_4 y} + Le^{m_2 y}$.

Now, since Eqns. (19)-(22) are non-linear, we again assume

$$g_0(y) = g_{00}(y) + E_c g_{01}(y) + O(E_c{}^2), \quad (27)$$

$$g_1(y) = g_{10}(y) + E_c g_{11}(y) + O\big(E_c{}^2\big), \quad (28)$$

$$f_0(y) = f_{00}(y) + E_c f_{01}(y) + O(E_c{}^2), \quad (29)$$

$$f_1(y) = f_{10}(y) + E_c f_{11}(y) + O(E_c{}^2), \quad (30)$$

where, $E_c \ll 1$.

Substituting (27)-(30) in (19)-(22) respectively, we find the following equations:

Zeroth order

$$g_{00}'' + Prg_{00}' - Pr(F+\emptyset)g_{00} = -PrQ_1h_0 - Duh_0''\,, \quad (31)$$

$$g_{10}'' + Prg_{10}' - Pr(F+\emptyset+n)\, g_{10} = -PrQ_1h_1 - Duh_1'' - PrAg_{00}'\,, \quad (32)$$

$$f_{00}'' + f_{00}' - M_1 f_{00} = -M_1 - Grg_{00} - Gmh_0\,, \quad (33)$$

$$f_{10}'' + f_{10}' - (M_1+n)f_{10} = -(M_1+n) - Af_{00}' - Grg_{10} - Gmh_1. \quad (34)$$

First order

$$g_{01}'' + Prg_{01}' - Pr(F+\emptyset)g_{01} = -{f_{00}}'^2\,, \quad (35)$$

$$g_{11}'' + Prg_{11}' - Pr\,(F+\emptyset+n)g_{11} = -PrAg_{01}' - 2Prf_{00}'f_{10}', \quad (36)$$

$$f_{01}'' + f_{01}' - M_1 f_{01} = -Grg_{01}\,, \quad (37)$$

$$f_{11}'' + f_{11}' - (M_1+n)f_{11} = -Af_{01}' - Grg_{11}\,. \quad (38)$$

Similarly, the corresponding boundary conditions are as follows:

$$g_{00} = 1, g_{01} = 0, g_{10} = 1, g_{11} = 0, f_{00} = \emptyset_1{f_{00}}', f_{01} = \emptyset_1{f_{01}}', f_{10} = \emptyset_1{f_{10}}', f_{11} = \emptyset_1{f_{11}}' \quad \text{at} \quad y = 0, \quad (39)$$

$$\text{and}\, g_{00} \to 0, g_{01} \to 0, g_{10} \to 0, g_{11} \to 0, f_{00} \to 1, f_{01} \to 0, \;\; f_{10} \to 1, f_{11} \to 0 \qquad \text{as } y \to \infty\,. \quad (40)$$

Now, solving the Eqns. (31)-(38) under the boundary conditions (39) and (40) we get;

$$g_{00} = C_6 e^{m_6 y} + K_1 e^{m_2 y}\;, \quad (41)$$

$$g_{01} = C_{10}e^{m_{10}y} + K_5 e^{2m_6 y} + K_6 e^{2m_2 y} + K_7 e^{2m_8 y} + K_{17}e^{(m_2+m_6)y} + K_{18}e^{(m_2+m_8)y} + K_{19}e^{(m_6+m_8)y}, \quad (42)$$

$$g_{10} = C_{12}e^{m_{12}y} + K_8 e^{m_2 y} + K_9 e^{m_4 y} + K_{10}e^{m_6 y}\,, \quad (43)$$

$$g_{11} = C_{16}e^{m_{16}y} + K_{20}e^{m_{10}y} + K_{21}e^{2m_6 y} + K_{22}e^{2m_2 y} + K_{23}e^{2m_8 y} + K_{24}e^{(m_2+m_6)y} + K_{25}e^{(m_2+m_8)y} + K_{26}e^{(m_6+m_8)y} + K_{27}e^{(m_6+m_{14})y} + K_{28}e^{(m_2+m_6)y} + K_{29}e^{(m_4+m_6)y} + K_{30}e^{(2m_6 y)} + K_{31}e^{(m_6+m_8)y} + K_{32}e^{(m_6+m_{12})y} + K_{33}e^{(m_2+m_{14})y} + K_{34}e^{2m_2 y} + K_{35}e^{(m_2+m_4)y} + K_{36}e^{(m_2+m_6)y} + K_{37}e^{(m_2+m_8)y} + K_{38}e^{(m_2+m_{12})y} + K_{39}e^{(m_8+m_{14})y} + K_{40}e^{(m_2+m_8)y} + K_{41}e^{(m_4+m_8)y} + K_{42}e^{(m_6+m_8)y} + K_{43}e^{(2m_8 y)} + K_{44}e^{(m_8+m_{12})y}\,, \quad (44)$$

$$f_{00} = 1 + K_2 e^{m_6 y} + K_3 e^{m_2 y} + K_4 e^{m_8 y}\,, \quad (45)$$

$$f_{01} = C_{18}e^{m_{18}y} + K_{45}e^{m_{10}y} + K_{46}e^{2m_6 y} + K_{47}e^{2m_2 y} + K_{48}e^{2m_8 y} + K_{49}e^{(m_2+m_6)y} + K_{50}e^{(m_2+m_8)y} + K_{51}e^{(m_6+m_8)y}, \quad (46)$$

$$f_{10} = 1 + K_{16}e^{m_{14}y} + K_{11}e^{m_2 y} + K_{12}e^{m_4 y} + K_{13}e^{m_6 y} + K_{14}e^{m_8 y} + K_{15}e^{m_{12}y}, \quad (47)$$

$$f_{11} = C_{20}e^{m_{20}y} + K_{52}e^{m_{18}y} + K_{53}e^{m_{10}y} + K_{54}e^{2m_6 y} + K_{55}e^{2m_2 y} + K_{56}e^{2m_8 y} + K_{57}e^{(m_2+m_6)y} + K_{58}e^{(m_2+m_8)y} + K_{59}e^{(m_6+m_8)y} + K_{60}e^{m_{16}y} + K_{61}e^{m_{10}y} + K_{62}e^{2m_6 y} + K_{63}e^{2m_2 y} + K_{64}e^{2m_8 y} + K_{65}e^{(m_2+m_8)y} + K_{66}e^{(m_2+m_8)y} + K_{67}e^{(m_6+m_8)y} + K_{68}e^{(m_6+m_{14})y} + K_{69}e^{(m_2+m_6)y} + K_{70}e^{(m_4+m_6)y} + K_{71}e^{2m_6 y} + K_{72}e^{(m_6+m_8)y} + K_{73}e^{(m_6+m_{12})y} + K_{74}e^{(m_2+m_{14})y} + K_{75}e^{2m_2 y} + K_{76}e^{(m_2+m_4)y} + K_{77}e^{(m_2+m_6)y} + K_{78}e^{(m_2+m_8)y} + K_{79}e^{(m_2+m_{12})y} + K_{80}e^{(m_8+m_{14})y} + K_{81}e^{(m_2+m_8)y} + K_{82}e^{(m_4+m_8)y} + K_{83}e^{(m_6+m_8)y} + K_{84}e^{2m_8 y} + K_{85}e^{(m_8+m_{12})y}\,. \quad (48)$$

Therefore, from Eqns. (16)-(18) and (27)-(30), we get the values of u, θ and C as follows:

$$u = f_{00}(y) + E_c f_{01}(y) + \epsilon e^{nt}[f_{10}(y) + E_c f_{11}(y)] = [1 + K_2 e^{m_6 y} + K_3 e^{m_2 y} + K_4 e^{m_8 y}] + E_c\big[C_{18}e^{m_{18}y} + K_{45}e^{m_{10}y} + K_{46}e^{2m_6 y} + K_{47}e^{2m_2 y} + K_{48}e^{2m_8 y} + K_{49}e^{(m_6+m_2)y} + K_{50}e^{(m_2+m_8)y} + K_{51}e^{(m_6+m_8)y}\big] + \epsilon e^{nt}\{[1 + \quad (49)$$

$$K_{16}e^{m_{14}y} + K_{11}e^{m_2y} + K_{12}e^{m_4y} + K_{13}e^{m_6y} + K_{14}e^{m_8y} + K_{15}e^{m_{12}y}] + E_c[C_{20}e^{m_{20}y} + K_{52}e^{m_{18}y} + K_{53}e^{m_{10}y} + K_{54}e^{2m_6y} + K_{55}e^{2m_2y} + K_{56}e^{2m_8y} + K_{57}e^{(m_2+m_6)y} + K_{58}e^{(m_2+m_8)y} + K_{59}e^{(m_6+m_8)y} + K_{60}e^{m_{16}y} + K_{61}e^{m_{10}y} + K_{62}e^{2m_6y} + K_{63}e^{2m_2y} + K_{64}e^{2m_8y} + K_{65}e^{(m_2+m_8)y} + K_{66}e^{(m_2+m_8)y} + K_{67}e^{(m_6+m_8)y} + K_{68}e^{(m_6+m_{14})y} + K_{69}e^{(m_2+m_6)y} + K_{70}e^{(m_4+m_6)y} + K_{71}e^{2m_6y} + K_{72}e^{(m_6+m_8)y} + K_{73}e^{(m_6+m_{12})y} + K_{74}e^{(m_2+m_{14})y} + K_{75}e^{2m_2y} + K_{76}e^{(m_2+m_4)y} + K_{77}e^{(m_2+m_6)y} + K_{78}e^{(m_2+m_8)y} + K_{79}e^{(m_2+m_{12})y} + K_{80}e^{(m_8+m_{14})y} + K_{81}e^{(m_2+m_8)y} + K_{82}e^{(m_8+m_4)y} + K_{83}e^{(m_6+m_8)y} + K_{84}e^{2m_8y} + K_{85}e^{(m_8+m_{12})y}]\},$$

$$\theta = g_{00}(y) + E_c g_{01}(y) + \epsilon e^{nt}[g_{10}(y) + E_c g_{11}(y)] = C_6e^{m_6y} + K_1e^{m_2y} + E_c[C_{10}e^{m_{10}y} + K_5e^{2m_6y} + K_6e^{2m_2y} + K_7e^{2m_8y} + K_{17}e^{(m_2+m_6)y} + K_{18}e^{(m_2+m_8)y} + K_{19}e^{(m_6+m_8)y}] + \epsilon e^{nt}\{[C_{12}e^{m_{12}y} + K_8e^{m_2y} + K_9e^{m_4y} + K_{10}e^{m_6y}] + E_c[C_{16}e^{m_{16}y} + K_{20}e^{m_{10}y} + K_{21}e^{2m_6y} + K_{22}e^{2m_2y} + K_{23}e^{2m_8y} + K_{24}e^{(m_2+m_6)y} + K_{25}e^{(m_2+m_8)y} + K_{26}e^{(m_6+m_8)y} + K_{27}e^{(m_6+m_{14})y} + K_{28}e^{(m_2+m_6)y} + K_{29}e^{(m_4+m_6)y} + K_{30}e^{(2m_6y)} + K_{31}e^{(m_6+m_8)y} + K_{32}e^{(m_6+m_{12})y} + K_{33}e^{(m_2+m_{14})y} + K_{34}e^{2m_2y} + K_{35}e^{(m_2+m_4)y} + K_{36}e^{(m_2+m_6)y} + K_{37}e^{(m_2+m_8)y} + K_{38}e^{(m_2+m_{12})y} + K_{39}e^{(m_8+m_{14})y} + K_{40}e^{(m_2+m_8)y} + K_{41}e^{(m_4+m_8)y} + K_{42}e^{(m_6+m_8)y} + K_{43}e^{(2m_8y)} + K_{44}e^{(m_8+m_{12})y}]\}, \quad (50)$$

$$C = e^{m_2y} + \epsilon e^{nt}[(1-L)e^{m_4y} + Le^{m_2y}]. \quad (51)$$

The coefficient of skin friction is given by

$$\left.\frac{\partial u}{\partial y}\right]_{y=0} = K_2m_6 + K_3m_2 + K_4m_8 + E_c[C_{18}m_{18} + K_{45}m_{10} + K_{46}2m_6 + K_{47}2m_2 + K_{48}2m_8 + K_{49}(m_2+m_6) + K_{50}(m_2+m_8) + K_{51}(m_6+m_8)] + \epsilon e^{nt}\{[K_{16}m_{14} + K_{11}m_2 + K_{12}m_4 + K_{13}m_6 + K_{14}m_8 + K_{15}m_{12}] + E_c[C_{20}m_{20} + K_{18}m_{18} + K_{53}m_{10} + K_{54}2m_6 + K_{55}2m_2 + K_{56}2m_8 + K_{57}(m_2+m_6) + K_{58}(m_2+m_8) + K_{59}(m_6+m_8) + K_{60}m_{16} + K_{61}m_{10} + K_{62}2m_6 + K_{63}2m_2 + K_{64}2m_8 + K_{65}(m_2+m_8) + K_{66}(m_2+m_8) + K_{67}(m_6+m_8) + K_{68}(m_6+m_{14}) + K_{69}(m_2+m_6) + K_{70}(m_4+m_6) + K_{71}2m_6 + K_{72}(m_6+m_8) + K_{73}(m_6+m_{12}) + K_{74}(m_2+m_{14}) + K_{75}2m_2 + K_{76}(m_2+m_4) + K_{77}(m_2+m_6) + K_{78}(m_2+m_8) + K_{79}(m_2+m_{12}) + K_{80}(m_8+m_{14}) + K_{81}(m_2+m_8) + K_{82}(m_4+m_8) + K_{83}(m_6+m_8) + K_{84}2m_8 + K_{85}(m_8+m_{12})]\}. \quad (52)$$

The rate of heat transfer in terms of Nusselt number

$$\frac{Nu}{Re_x} = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = C_6m_6 + K_1m_2 + E_c[C_{10}m_{10} + K_52m_6 + K_62m_2 + K_72m_8 + K_{17}(m_2+m_6) + K_{18}(m_2+m_8) + K_{19}(m_6+m_8)] + \epsilon e^{nt}\{[C_{12}m_{12} + K_8m_2 + K_9m_4 + K_{10}m_6] + E_c[C_{16}m_{16} + K_{20}m_{10} + K_{21}2m_6 + K_{22}2m_2 + K_{23}2m_8 + K_{24}(m_2+m_6) + K_{25}(m_2+m_8) + K_{26}(m_6+m_8) + K_{27}(m_6+m_{14}) + K_{28}(m_2+m_6) + K_{29}(m_4+m_6) + K_{30}2m_6 + K_{31}(m_6+m_8) + K_{32}(m_6+m_{12}) + K_{33}(m_2+m_{14}) + K_{34}2m_2 + K_{35}(m_2+m_4) + K_{36}(m_2+m_6) + K_{37}(m_2+m_8) + K_{38}(m_2+m_{12}) + K_{39}(m_8+m_{14}) + K_{40}(m_2+m_8) + K_{41}(m_4+m_8) + K_{42}(m_6+m_8) + K_{43}2m_8 + K_{44}(m_8+m_{12})]\} \quad (53)$$

Finally the rate of mass transfer in terms of Sherwood number (Sh) is given by

$$\frac{Sh}{Re_x} = \left(\frac{\partial C}{\partial y}\right)_{y=0} = m_2 + \epsilon e^{nt}[m_4(1-L) + Lm_2] \quad (54)$$

4. RESULT AND DISCUSSION

The perturbation solutions are obtained for the MHD free convective flow under the appropriate boundary conditions and are presented through graphs and tables. The effect of viscous dissipation is of special interest which has not been taken care of in the earlier studies.

In the Figure 2, velocity profile is depicted for distinct values of Eckert number (Ec). Increasing values of Ec enhances the velocity. This is happened by the increase in kinetic energy resulted by viscous dissipation in the boundary layer.

Velocity of fluid flow for distinct values of Gm and Gr are drawn in the Figures (3) and (4). It is noted that the increasing values of Gm and Gr help to improve the velocity. For both thermal and solutal Grasshoff number, buoyancy force dominates the viscous force which gives higher velocity. The results agree well with the study of Prakash et. al. [5].

From Figures (5) and (6), the effects of porous permeability parameters K and $\emptyset_1$ on velocity are observed. Here, $\emptyset_1$ is directly proportional to the square root of K. In the present study higher permeability K allows the fluids to flow rapidly. As a result, increasing values of $\emptyset_1$ also enhance the fluid velocity. At K=0.001, we get approximately linear flow after some elevation.

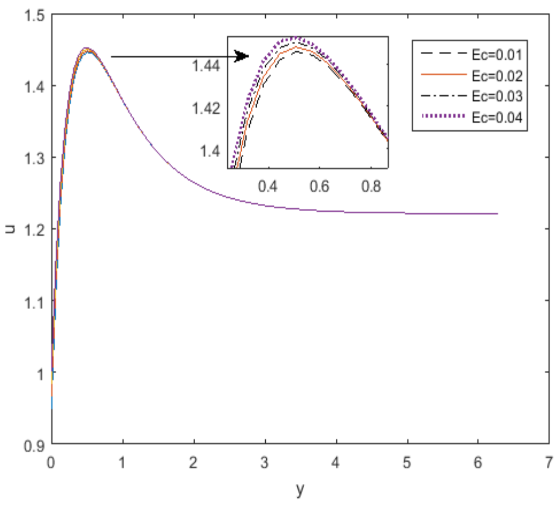

Figure 2. Velocity profiles for Ec, Sc=0.6, F=2, M=2, Gr=4, Gm=2, t=1, Pr=0.2, A=0.5, n1=0.1, Q1=2, Du=0.5,K=0.1

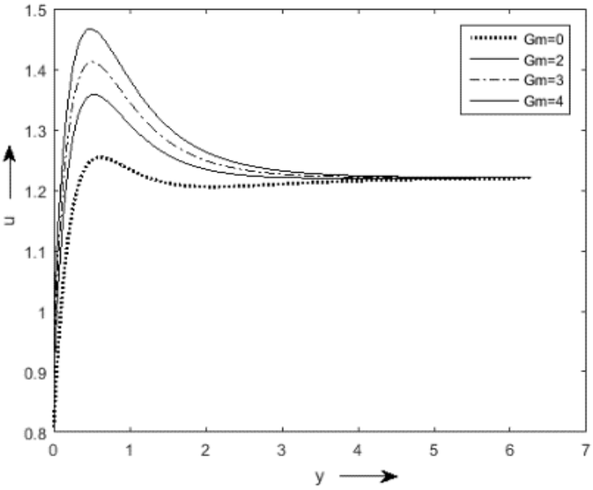

Figure 3. Velocity profiles for Gm, Sc=0.6, F=2, M=2, Gr=4, A=0.5, n1=0.1, t=1, Pr=0.2,K=0.1, Q1=2, Du=0.5

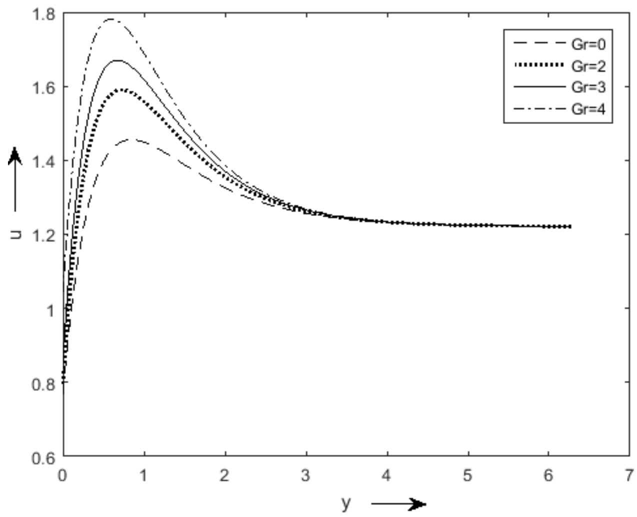

Figure 4. Velocity profiles for Gr, Sc=0.6, F=2, M=2, Gm=2, n1=0.1, t=1, Pr=0.2, K=0.1, A=0.5, Q1=2, Du=0.5

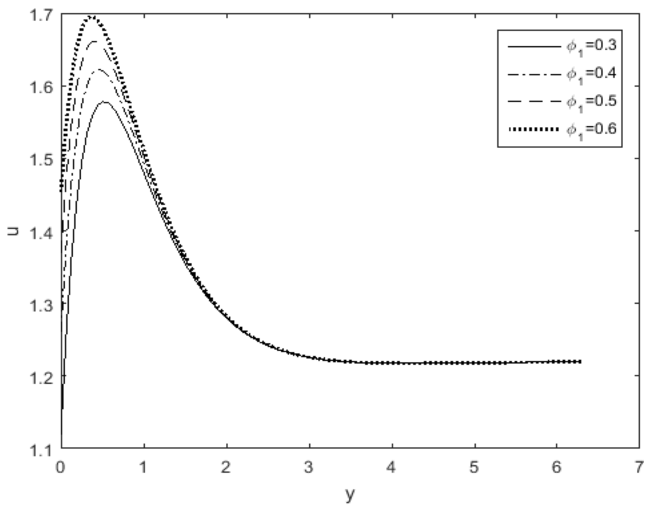

Figure 5. Velocity profiles for $\emptyset_1$, Sc=0.6, F=2, M=2, Gr=4, Gm=2, n1=0.1, t=1, Pr=0.2,K=0.1, A=0.5, Q1=2, Du=0.5

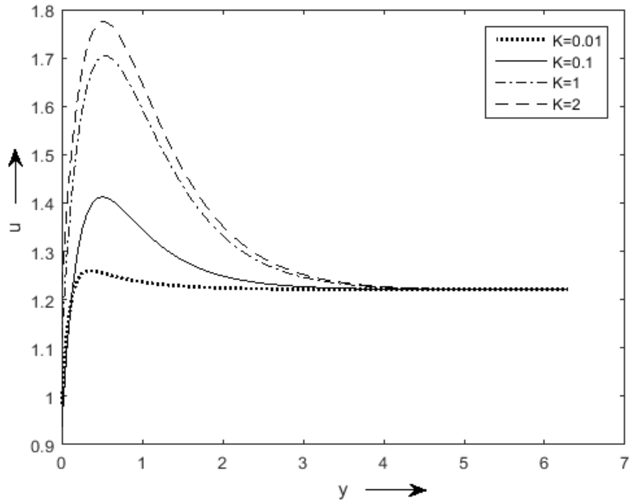

Figure 6. Velocity profiles for K, Sc=0.6, F=2, M=2, Gm=2, n1=0.1, t=1, Pr=0.2, Gr=4, A=0.5, Q1=2, Du=0.5

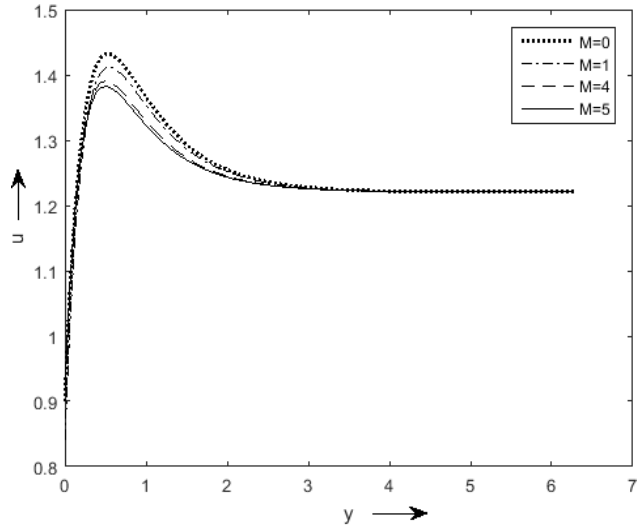

Figure 7. Velocity profiles for M, Sc=0.6, F=2, K=0.1, Gm=2, n1=0.1, t=1, Pr=0.2, Gr=4, A=0.5, Q1=2, Du=0.5, F=2

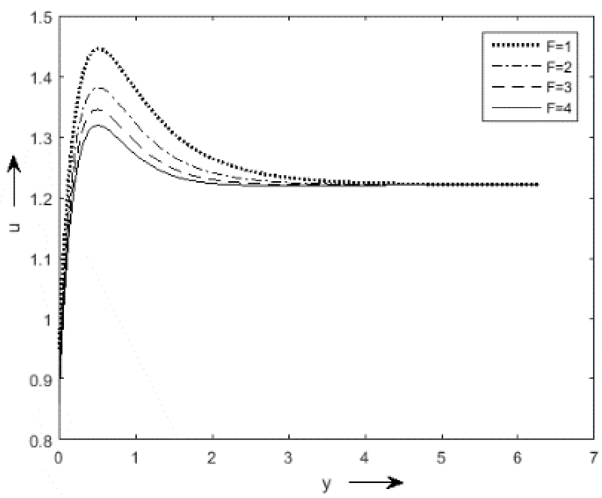

Figure 8. Velocity profiles for F, Sc=0.6, M=2, Gr=4, Gm=2, n1=0.1, t=1, Pr=0.2, K=0.1, Q1=2, Du=0.5, A=0.5

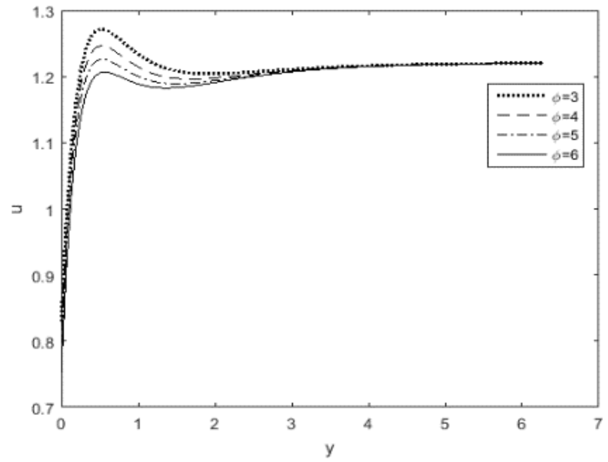

Figure 9. Velocity profiles for ϕ, Sc=0.6, M=2, Gr=4, Gm=2, n1=0.1, t=1, Pr=0.2, K=0.1, Q1=2, Du=0.5, A=0.5, F=2

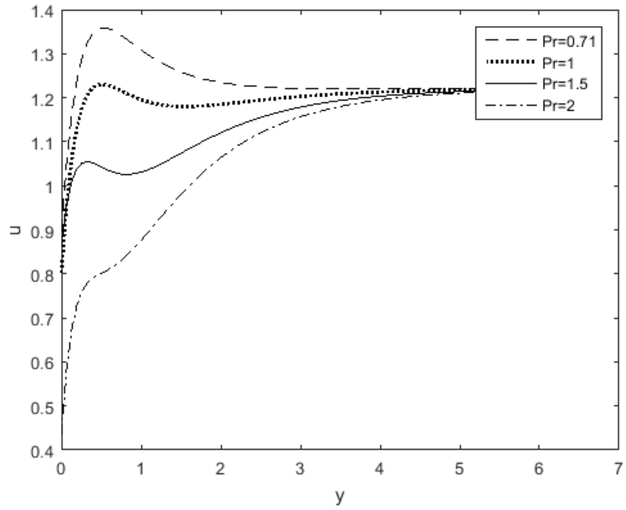

Figure 10. Velocity profiles for Pr, Sc=0.6, M=2, Gr=4, Gm=2, n1=0.1, t=1, K=0.1, Gr=4, Q1=2, Du=0.5, A=0.5, F=2

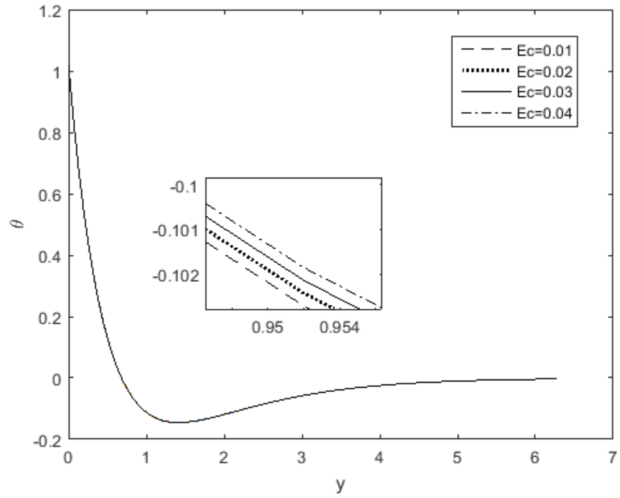

Figure 11. Temperature profiles for Ec, Sc=0.6, M= 2, Gr=4, Gm=2, n1=0.1, t=1, K=0.1, Pr=0.2, Q1=2, Du=0.5, A=0.5, F=2

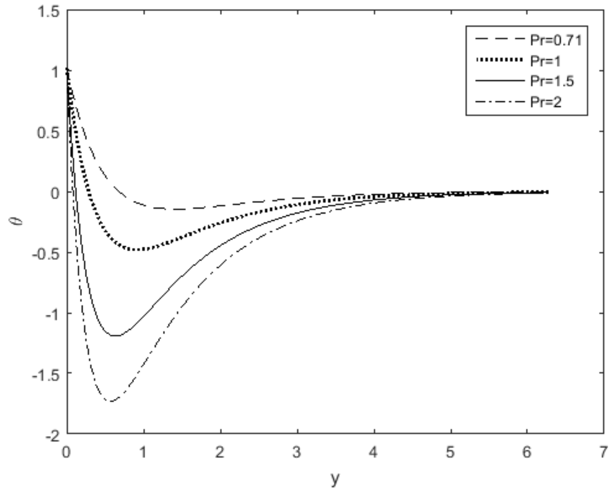

Figure 12. Temperature profiles for Pr, Sc=0.6, M=2, Gr=4, Gm=2, n1=0.1, t=1, K=0.1, F=2, Q1=2, Du=0.5, A=0.5

Table 1. Comparison between present results with previous results

γ	Result of Kim [18]		Result of Prakash et al. [6]		Present results	
	Nu	Sh	Nu	Sh	Nu	Sh
0.00	-1.3400	-0.8098	-1.3400	-0.8098	-1.3400	-0.8097
0.50	-1.4825	-1.1864	-1.4825	-1.1864	-1.4827	-1.1859
0.75	-1.5227	-1.3178	-1.5227	-1.3178	-1.5227	-1.3177
1.00	-1.5546	-1.4325	-1.5546	-1.4325	-1.5543	-1.4326

Figure 7 shows that magnetic parameter acts as a resistive force to the fluid flow. This results also agree well with previous study of Prakash et al. [5].

Figure 8 and 9, that velocity decreases when radiation parameter F and heat absorption parameter ∅ increase. So, we can control the flow of fluid by rising both the parameters.

In the Figure 10, effects of Prandtl number (Pr) on velocity is shown. Velocity is reduced with increasing values of Prandtl number. Here the momentum diffusivity influences behaviour of the fluid flow resulting lower velocity.

The increasing values of Eckert number (Ec) lead higher temperature which is shown in Figure 11. But in Figure 12, the Prandtl number (Pr) adversely affects the temperature. Here we can measure the relative importance of viscous dissipation to the thermal dissipation.

Table 2. Values of Nu and τ w.r.t. Ec

Ec	Nu	τ
0.00	-0.8243	2.3054
0.01	-0.8301	2.2896
0.02	-0.8359	2.2580
0.03	-0.8416	-1.4325
0.04	-0.8474	2.2422

In Table 1, our results with Ec=0, Sc=0.6, F=2, M=2, Gr=4, Gm=2, Q1=2, Pr=0.2, A=0.5, Du=0.5, n1=0.1, t=1, K=0.1 is compared with the previous results of Prakash et.al. [6], and Kim [18] and it agrees well. This supports our method and accuracy of calculation. From Table 2, the Nusselt number and Skin friction both diminish with higher Eckert number.

5. CONCLUSIONS

The study of viscous dissipation effect on the unsteady MHD flow of an incompressible viscous fluid over a vertical permeable surface fixed in a porous medium is executed in the proximity of thermal radiation and chemical reaction. After applying the perturbation technique, the governing partial differential equations lead some non-linear ordinary differential equations. To handle these non-linearities, Eckert number (Ec) is taken as a small parameter to perturb the equations again. This assumption gives us better result. Some results are given below.

- An increase in dissipative heat characterized by the parameter, Ec due to viscous dissipation leads to significant increase in temperature of the fluid.
- Increasing values of Eckert number enhances the velocity of fluid flow.
- For both thermal and solutal Grashoff number, buoyancy force dominates the viscous force.
- Higher porous permeability allows the fluid to flow rapidly.
- Magnetic parameter behaves as a resistive force to the fluid flow.
- Velocity is reduced with increasing values of Prandtl number.
- Velocity reduces when radiation parameter and heat absorption parameter rise.

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NOMENCLATURE

A	Suction velocity parameter
B_0	Magnetic field of uniform strength
C^*	Species concentration, kgm^{-3}
C_w	Species concentration at the plate
C_p	Specific heat at constant pressure, $Jkg^{-1}K$
C_S	Concentration susceptibility the plate
Cf	Skin friction coefficient
C_∞	Species concentration far away from the plate
	Chemical molecular diffusivity, m^2s^{-1}
D	Dufour number
Du	The coefficient of mass diffusivity
Dm	Eckert number
Ec	Thermal radiation parameter
F	Thermal Grashof number
Gr	Solutal Grashof number
Gm	Acceleration due to gravity, ms^{-2}
g	Permeability parameter
K	Thermal diffusion ratio
K_T	Absorption coefficient at the wall
$K_{\lambda w}$	The permeability of the porous medium
K^*	Hartmann number
M	Nusselt number
Nu	Prandtl number
Pr	Pressure
P^*	Heat source parameter
Q	Dimensional heat absorption coefficient
Q_0	Sink strength
Q^*	Radiation absorption parameter
Q_1	The coefficient of proportionality for the
Q_1^*	absorption
	Radiative heat flux
q_r	Chemical reaction parameter
R	Sherwood number
Sh	Schmidt number
Sc	Fluid temperature
T	Temperature of fluid near the plate, K
T^*	
T_w^*	Fluid temperature at the surface, K
T_∞^*	Fluid temperature in the free stream, K
T_w	Temperature at the wall
T_∞	The free stream dimensional temperature
	Radiative heat flux
u, v	Dimensionless velocity component, ms^{-1}

Greek symbols

α	Fluid thermal diffusivity, m^2s^{-1}
β_T	Thermal expansion coefficient, K^{-1}
β_C	Concentration expansion coefficient, K^{-1}
ϕ	Heat source parameter
ϕ_1	Porous permeability parameter
Θ	Dimensionless fluid temperature, K
μ	Dynamic viscosity, $kgm^{-1}s^{-1}$
ν	The kinematic viscosity, m^2s^{-1}
$\boldsymbol{\sigma}$	The fluid-electrical conductivity
ϕ_1	Permeability parameter
η	Dimensionless normal distance
$\boldsymbol{\tau}$	Skin friction coefficient
$\boldsymbol{\rho}$	Density of the fluid, kgm^{-3}

Subscripts

w	Conditions on the wall
∞	Free stream condition