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Workspace Analysis of an Over-constrained 2-RPU&SPR Parallel Manipulator

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ABSTRACT

In this study, an over-constrained 2-RPU&SPR parallel manipulator was researched, numerical analysis approach was used to analyze the manipulator's reachable workspace. Firstly, structural constraints, such as the pose extreme and the inputting parameters extreme, were determined by structure parameters given. Next,boundary research method was adopted to estimate whether the calculated position of the mobile platform and the length of each leg satisfy the structural constraints by Matlab. Finally, the set of all the satisfied position of the mobile platforms the reachable workspace of the 2-RPU&SPR parallel manipulator. This provides a significant theoretical basis for the subsequent optimization design and application of the mechanism.

Keywords: Over-constrained mechanism, Workspace, Simulation analysis.

1. INTRODUCTION

Under different driving element of the inputting parameters, the workspace of parallel mechanism is mapping established by the kinematic relations input parameters and the output results. Its size and shape directly reflect the activity space of the end effector of parallel mechanism, and it is one of the most important indexes to measure the working performance of parallel mechanism. Analytic geometry method and numerical analysis method are all used to solve the working space of the parallel manipulators. Although analytic geometry method has more accurate solutions, but because of its geometric derivation and computational complexity, and the results are related to its direct position, so it can only be applied to some special structure of parallel mechanism. However, the structure of parallel mechanism is becoming progressively complex and diverse. The numerical analysis method has been received more and more widely used. Because of its wide application, simple calculation and limited conditions for location analysis.

In this paper, the workspace of 2-RPU&SPR parallel mechanism has been studied, and the numerical analysis method is used to analyze the manipulator's reachable workspace. Firstly, by use of the structure parameter given, we determine the mechanism of structural constraints, namely extreme configuration and the limit of inputting parameters; secondly, the searching method of limit position has been adopted to solve positional solutions of moving platform by Matlab. So we can estimate whether the solutions can satisfy the structural constraints. Finally, the positional set of the moving platform position which meet the conditions of the constraints is the reachable workspace of 2-RPU&SPR. This provides a significant ttheoretical basis for the optimization design and application of the following mechanism.

2. THE DESCRIPTION OF THE OVER-CONSTRAINED 2-RPU&SPR PARALLEL MECHANISM

As is depicted in Figure 1, the type of over-constrained 3-DOF PM 2-RPU&SPRcomprises of a base platform and a moving platform connected by two identical RPU limbs and one SPR limb. Here, R, U and S represent the revolute, universal and spherical joints, and the underlined P denotes the actuated prismatic joint. Place the reference frame *B-xyz* attached to the base and the moving frame *A-uvw* attached to the moving platform with *B* and *A* being the origins located at the center point of lines B_1B_2 and A_1A_3 with the x and u axes being parallel toB_1B_2andA_1A_3, and the *z* and waxes being normal to the base plate and the moving plate. Here, B_i (i=1, 2) and $A_3$are the intersection of the axes of the revolute joints and actuated prismatic joints, B_3 is the center of the spherical joint, A_1 and A_2 are the centers of the universal joints, respectively. It should be pointed out that, for the type of 2-2RPU&SPR PM, two universal joints of RPU limbs are common used and attached on the moving platform.

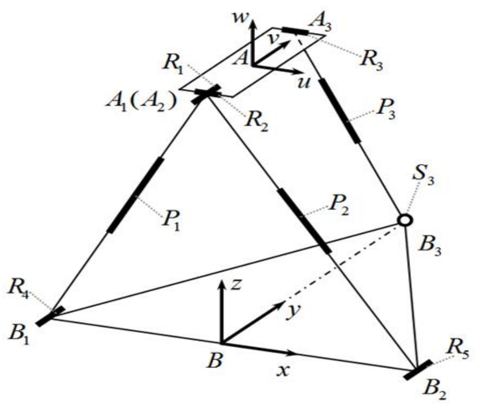


Figure 1. The type of 2-RPU&SPR PM

Point B_i is on the fixed base of parallel mechanism. B_i^B indicates the position vector of point B_i in the base coordinate system *B-xyz*. On the moving platform, the position vectors of point A_i relative to the moving coordinate system *A-uvw* and Base coordinate system *B-xyz* are A_i^A, A^B respectively.

Those points can be expressed as followed:

$$B_i^B = \begin{bmatrix} x_{Bi} \\ y_{Bi} \\ z_{Bi} \end{bmatrix},\ A_i^A = \begin{bmatrix} u_{Ai} \\ v_{Ai} \\ w_{Ai} \end{bmatrix},\ A_i^B = \begin{bmatrix} x_{Ai} \\ y_{Ai} \\ z_{Ai} \end{bmatrix},$$

$$A^B = \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix},\ R_A^B = \begin{bmatrix} x_l & y_l & z_l \\ x_m & y_m & z_m \\ x_n & y_n & z_n \end{bmatrix} \quad (1)$$

$$A_i^B = R_A^B A_i^A + A^B,$$

$$A_1^A = A_2^A = \begin{bmatrix} 0 \\ -k \\ 0 \end{bmatrix},\ A_3^A = \begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix},\ B_1^B = \begin{bmatrix} -m \\ 0 \\ 0 \end{bmatrix},$$

$$B_2^B = \begin{bmatrix} m \\ 0 \\ 0 \end{bmatrix}\ B_3^B = \begin{bmatrix} 0 \\ n \\ 0 \end{bmatrix} \quad (2)$$

$$A_1^B = A_2^B = \begin{bmatrix} -ky_l + x_A \\ -ky_m + y_A \\ -ky_n + z_A \end{bmatrix},\ A_3^B = \begin{bmatrix} ky_l + x_A \\ ky_m + y_A \\ ky_n + z_A \end{bmatrix} \quad (3)$$

where, vector $A^B{}_I$ is a positional vector which fixed on the base coordinate system *B-xyz* relative to moving coordinate system *A-uvw*; the matrix R_A^B is The orientation matrix of the *A-uvw* with respect to the *B-xyz*; the distance between A_i and A is K; similarly, m indicates the distance between point B_1 and B_2 and n the distance between pointB_3 and B.

That the platform center of the moving platform can reach the collection of location defined as the work space of this parallel manipulator. The working space of parallel mechanism is closely related to the structure parameters and the constraints of the mechanism. The structure parameters include the limit value of the length of the brands and the rotation angle of the rotating pair, the shape and size of the moving platform and the fixed platform. The constraint conditions include the constraint of degree of freedom, the size of the chain and the shape of the platform. Therefore, on the basis of the inverse position solution algorithm, according to satisfying the constraint condition, the point can be determined whether the point is in the working space, and the point is outside of the working space.

Due to rotation the moving platform has the X and Y direction of translation, and Z direction of rotation. Therefore, it is necessary to consider the limiting values of the two rotation angles generated by the two directions. Under the condition that the structural parameters have been determined, the values of α change with the change of the Y_A value, but the value of β is determined by the X_A value and the Z_A value. The limit conditions of the two limit angle can be determined by the limit conditions of the length of each branch. According to the geometric method when the long rod of the first and second branched chain reach minimum value, rod long of third branch chain reach maximum value, the value reached the maximum, as shown in Figure 2; on the contrary, when the first and second branched chain long rod reached a maximum value, the third branched chain rod long reaches a minimum value, the value of minimum, as shown in Figure 3. Due to the first and second branched chain can interfere with moving platform, here, the hook hinge rotating shaft size has been neglected when first branched long rod reaches a minimum value, the length of the second branched chain reaches maximum value, the value reached the maximum; conversely, value has a minimum value, as shown in Figure 4.

The structure properties of the parallel mechanism and the limit condition of the length of the branch rod are known, and the maximum can be obtained from the surface of *yz*, and the expression of the extreming value can be obtained. Because the structure of the mechanism has symmetry in the *xz* plane and the A point is only moving in the *yz* plane, the extreme value of β has the characteristic of symmetry. The calculation formula of the extreme value can be introduced by the cosine theorem.

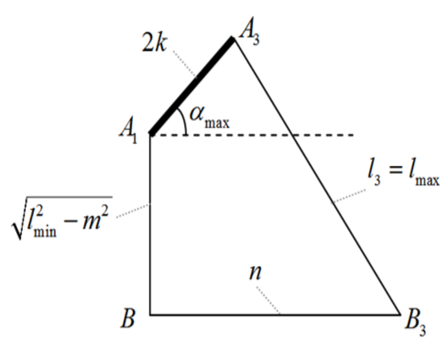


Figure 2. The geometric relationship of the maximum value (the y-z plane)

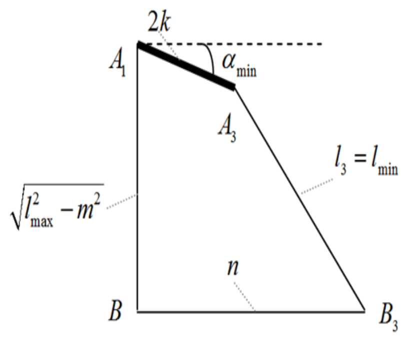


Figure 3. the geometric relationship of the minimum value (the y-z plane)

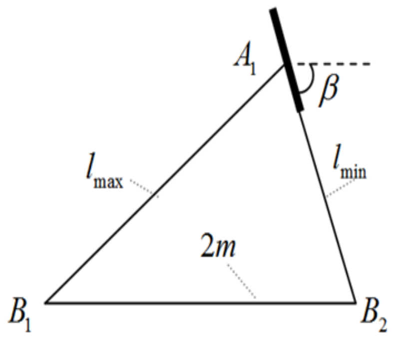


Figure 4. the geometric relation of the extreme value (the x-z plane)

According to the limit conditions of the branch long rod and the structural properties of the mechanism, the formula for calculating the ultimate position of α can be listed as follows:

$$(n-2k\cos\alpha_{\max})^2+\left(\sqrt{l_{\min}^2-m^2}+2k\sin\alpha_{\max}\right)^2=l_{\max}^2 \tag{4}$$

$$(n-2k\cos\alpha_{\min})^2+\left(\sqrt{l_{\max}^2-m^2}-2k\sin\alpha_{\min}\right)^2=l_{\min}^2 \tag{5}$$

Since the structure of the first and the second branch are the same, and the two branched chain can only move in the *X-Z* plane, the extreme value of β is symmetrical, and $\beta_{max}= -\beta_{min}$.

Based on the geometric relation of Figure 4, the two limit positions of the β angle are obtained by using the cosine theorem:

$$l_{\max}^2=4m^2+l_{\min}^2-4ml_{\min}\cos\beta_{\max} \tag{6}$$

3. THE SIMULATION OF THE MECHANICAL WORKSPACE

Using four groups of structural parameters relative to m, *N* and *K*, the workspace simulation was carried out respectively. The driving links of all the branches have the same structure and constraint conditions. such as $l_{\max}$ and $l_{\min}$, as shown in Table 1

Table 1. structural parameters and constraint parameters (units: mm)

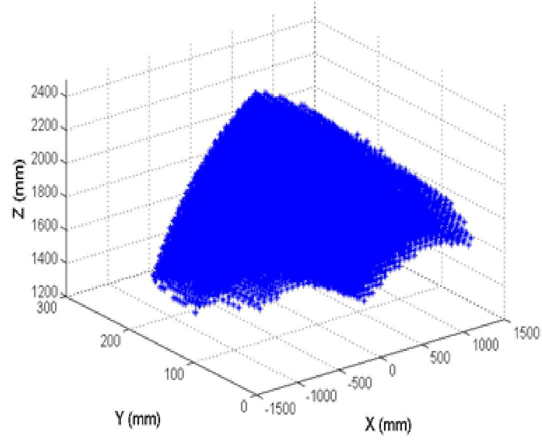
m	n	k	l_{max}	l_{min}
800	800	200	2500	1500
800	1000	200	2500	1500
800	1000	300	2500	1500
1000	1000	300	2500	1500

The structure parameters of each group and their constraint parameters were inputted into the working space program to solve the problem by Matlab. The solution procedure can be shown as followed:

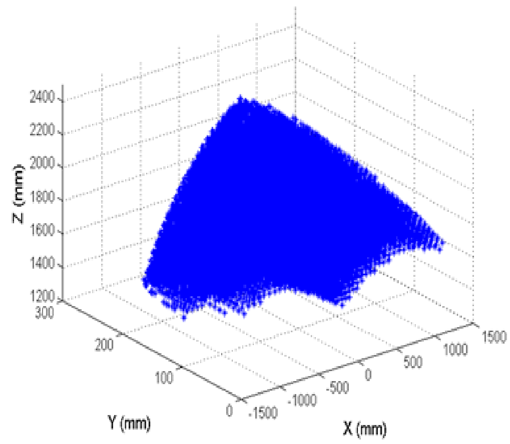
(1) the limit position parameters of the mechanism are obtained by the structural parameters and the constraints of driving element;

(2) the scanning step size is set, and the position of the moving platform of the mechanism is obtained by searching in the range of the effective position;

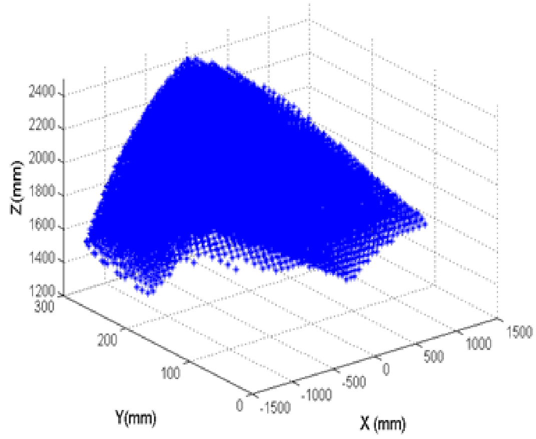
(3) draw the position coordinates of the satisfied condition and all the marks of the work space of the mechanism, so as to draw the following 4 groups of working space diagram, as shown in figure 5.



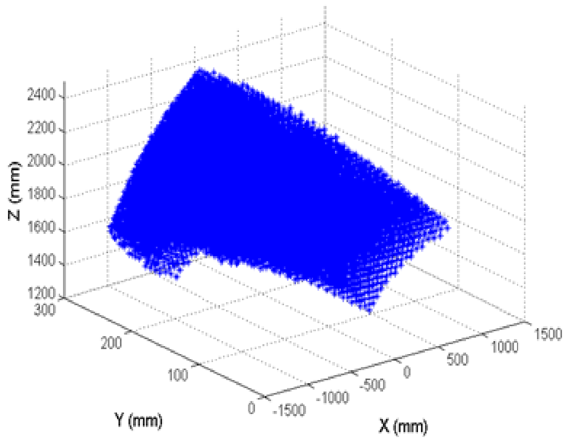
(a) m=800, n=800, k=200



(b) m=800, n=1000, k=200



(c) m=800, n=1000, k=300



(d) m=1000, n=1000, k=300

Figure 5. Schematic diagram of the reachable workspace of the mechanism under different structural parameters

4. CONCLUSIONS

By comparing the reachable workspace of different structure parameters, some conclusions can be listed as followed:

(1) when the structure parameter *K*, the value of *M* is the same, the value of n is not the same, the mechanism can obtain the reachable working space in the *Y* direction, the projection range is mm 0~k. through the analysis of the structure constraint equation (3-1) , The reachable workspace volume is larger: when the value of *n* decreases, the absolute values of *Y* direction limit angle α_{max} and α_{min} are become lager, the range of *A* are also getting lager.

(2) when the value of the structural parameters *m, n* are the same, the *K* value is not the same, the larger value of *K* the lager reachable workspace volume, because by the moving platform positional formula (2-16) analysis: when *k* decreases, the positional solution of *Y* direction decreases, scope of the institutions on a point of the reachable position also getting smaller.

(3) when the structure parameter *K*, the value of *n* is the same, the value of *M* is not the same, the mechanism can obtain the reachable working space in the *Y* direction, the projection range is *0~k* mm. The reachable workspace volume is larger, by the analysis of the structure constraint equation (3-1) ~ (3-3) , when the value of *M* decreases and the absolute value of *Y* direction limit angle and *β* increases, the range of *A* on the manipulator can also increases.

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