Modelling, Measurement and Control A
Vol. 92, No. 2-4, December, 2019, pp. 94-97
Journal homepage: http://iieta.org/journals/mmc_a

Analysis of Electric Power Transmission Line Presenting Only Long-Wise Inductance

Georgios Leonidopoulos

Electrical and Electronic Engineering Dept., University of West Attica, Thevon 250, Aegaleo 12244, Greece

Corresponding Author Email: gleon@uniwa.gr

https://doi.org/10.18280/mmc_a.922-409

Received: 5 March 2019
Accepted: 8 June 2019

Keywords:
electric power transmission line, modelling, power, electrical analysis, mathematical analysis

ABSTRACT

In this paper, a simple electric power transmission line presenting only long-wise inductance is studied electrically and is analysed mathematically in order to find the mathematical equations, regarding mainly voltage, current and power flowing in the line in steady state condition, that describe it. The information that one and particularily an electrical engineer can dig out regarding its electric behaviour are obtained and analysed in detail.

1. INTRODUCTION

The simplest representation of an electric power transmission line [1-8] is the electric equivalent that presents only long-wise inductance.

In next section 2, the above power transmission line is examined carefully and thoughtfully from the electrical point of view that means what an electrical engineer expects from this type of line having the above mentioned characteristics and to write them down.

Then, in section 3 that follows, the above line is analyzed mathematically and the equations drawn are coming to verify or not the expectations of the previous section 2.

In section 4, the experimental results obtained from a low voltage laboratory model of an electric power transmission line presenting only long-wise inductance are presented. The results verify the theoretical results obtained in section 3.

Finally, in section 5, the relative discussion is developed and the respective conclusions are drawn.

2. ELECTRIC ANALYSIS OF THE ELECTRIC POWER TRANSMISSION LINE

In Figure 1, the electric equivalent of the above mentioned electric power transmission line is presented. The data given are the voltages at the beginning and at the end of line as well as the inductance of line.

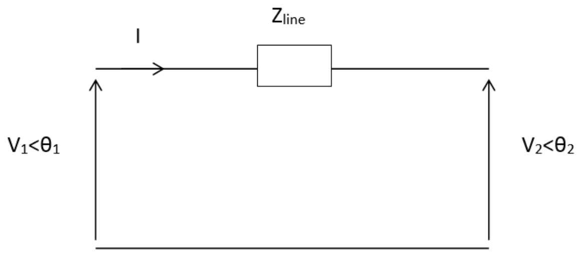

Figure 1. Electric equivalent representation of electric power transmission line

If the above power transmission line (Figure 1) is examined carefully and thoughtfully from the electrical point of view, an electrical engineer expects the following:

(1) the active power P_1 at the beginning of the line and the active power P_2 at the end of the line must be both positive since the active power is actually Joule losses i.e. ohmic losses or heat losses that are consumed.

(2) the active power P_1 at the beginning of the line must be equal to the active power P_2 at the end of the line. That is because the line has no ohmic resistance and therefore no active power is consumed in the line. In other words, the active power of the line P_{line} is zero.

(3) the reactive power of the line Q_{line} must be always positive. The above statement is based on the fact that the line presents only inductance and the inductance absorbs reactive power.

(4) the reactive power Q_1 at the beginning of the line must be equal to the addition of the reactive power Q_2 at the end of the line plus the reactive power of the line Q_{line}. In other words, the reactive power Q_1 minus the reactive power Q_2 must be equal to Q_{line}.

(5) the apparent resistance in complex form Z_1 at the beginning of the line must be equal to the apparent resistance in complex form Z_2 at the end of the line plus the resistance in complex form of the line Z_{line}. This is due to the connection in series of all the above complex resistances. In other words, the complex resistance Z_{line} is equal to Z_1 minus Z_2.

(6) either from the mathematical expression of the apparent power S_2 at the end of the line or the apparent complex resistance Z_2 at the end of the line a criterion must and can be developed i.e. a mathematical relationship among the quantities V_1, θ_1, V_2, θ_2, ω, L_{line} to indicate when the load is ohmic-inductive, ohmic-capacitive or pure ohmic. Logically, the criterion drawn by either of the above S_2 and Z_2 must be the same. We expect to draw the above criterion from either expression S_2 or Z_2 because they refer not only to the end of the line but also to the load which is connected to the end of the line.

In the following section, the mathematical expressions of all the above quantities are going to be developed in order to verify or not all the electrical conclusions drawn logically in this section. The relative mathematical expressions will be based on the electric equivalent of the electric power transmission line (Figure 1).

3. MATHEMATICAL ANALYSIS OF THE ELECTRIC POWER TRANSMISSION LINE

From Figure 1, using Kirchoff's 2nd law and Ohm's law, we have:

$$I = \frac{V_1<\theta_1 - V_2<\theta_2}{Z_{line}} = \frac{(V_1 sin\theta_1 - V_2 sin\theta_2) - j(V_1 cos\theta_1 - V_2 cos\theta_2)}{\omega L} \quad (1)$$

$$S_1 = V_1<\theta_1\, I^* = \frac{V_1 V_2 \sin(\theta_1-\theta_2) + j[V_1^2 - V_1 V_2 \cos(\theta_1-\theta_2)]}{\omega L} \quad (2)$$

$$P_1 = \frac{V_1 V_2 \sin(\theta_1-\theta_2)}{\omega L} \quad (3)$$

It must be:

$$P_1>0 \rightarrow \sin(\theta_1-\theta_2)>0 \rightarrow \theta_1>\theta_2 \quad (4)$$

$$Q_1 = \frac{V_1^2 - V_1 V_2 \cos(\theta_1-\theta_2)}{\omega L} \quad (5)$$

$$S_2 = V_2<\theta_2\, I^* = \frac{V_1 V_2 \sin(\theta_1-\theta_2) + j[V_1 V_2 \cos(\theta_1-\theta_2) - V_2^2]}{\omega L} \quad (6)$$

$$P_2 = \frac{V_1 V_2 \sin(\theta_1-\theta_2)}{\omega L} \quad (7)$$

It must be:

$$P_2>0 \rightarrow \sin(\theta_1-\theta_2)>0 \rightarrow \theta_1>\theta_2 \quad (8)$$

$$Q_2 = \frac{V_1 V_2 \cos(\theta_1-\theta_2) - V_2^2}{\omega L} \quad (9)$$

$$V_{line} = V_1<\theta_1 - V_2<\theta_2 = (V_1 cos\theta_1 - V_2 cos\theta_2) + j(V_1 sin\theta_1 - V_2 sin\theta_2) \quad (10)$$

$$V^2_{line,magnitude} = V_1^2 + V_2^2 - 2V_1V_2cos(\theta_1-\theta_2) \quad (11)$$

$$S_{line} = V_{line}\, I^* = \frac{V^2{}_{line,magnitude}}{Z^*{}_{line}} = j\frac{V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2)}{\omega L} \quad (12)$$

$$P_{line} = 0 \quad (13)$$

$$Q_{line} = \frac{V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2)}{\omega L} \quad (14)$$

It must be:

$$Q_{line}>0 \rightarrow V_1^2+V_2^2-2V_1V_2cos(\theta_1-\theta_2)>0 \rightarrow V_1^2+V_2^2 >^? 2V_1V_2cos(\theta_1-\theta_2)$$

This is true due to Eq. (15) below:

$$(V_1-V_2)^2 \geq 0 \rightarrow V_1^2+V_2^2-2V_1V_2 \geq 0 \rightarrow V_1^2+V_2^2 \geq 2V_1V_2 \geq 2V_1V_2cos(\theta_1-\theta_2) \rightarrow V_1^2+V_2^2 \geq 2V_1V_2cos(\theta_1-\theta_2) \quad (15)$$

$$S_{line} = S_1 - S_2 = j\frac{V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2)}{\omega L} \quad (16)$$

$$Z_1 = \frac{V_1<\theta_1}{I} = \omega L\frac{V_1V_2\sin(\theta_1-\theta_2) + j[V_1^2 - V_1V_2\cos(\theta_1-\theta_2)]}{V_1^2+V_2^2-2V_1V_2\cos(\theta_1-\theta_2)} \quad (17)$$

$$Z_2 = \frac{V_2<\theta_2}{I} = \omega L\frac{V_1V_2\sin(\theta_1-\theta_2) + j[V_1V_2\cos(\theta_1-\theta_2) - V_2^2]}{V_1^2+V_2^2-2V_1V_2\cos(\theta_1-\theta_2)} \quad (18)$$

$$Z_{line} = Z_1 - Z_2 = j\omega L \quad (19)$$

(1) If the load is ohmic-inductive, it must be: $Q_2 > 0$ (Eq. (9)) or the imaginary part of Z_2 must be > 0 (Eq. (18))

$$\text{then } V_1V_2cos(\theta_1-\theta_2)-V_2^2 > 0 \rightarrow \frac{V_2}{V_1} < cos(\theta_1-\theta_2) \quad (20)$$

$$\text{since } 0 \leq cos(\theta_1-\theta 2) \leq 1, \text{ then } \frac{V_2}{V_1} < 1 \rightarrow V_2<V_1 \quad (21)$$

(2) If the load is ohmic-capacitive, it must be: $Q_2 < 0$ (Eq. (9)) or the imaginary part of Z_2 must be < 0 (Eq. (18))

$$\text{then } V_1V_2cos(\theta_1-\theta_2)-V_2^2 < 0 \rightarrow cos(\theta_1-\theta_2) < \frac{V_2}{V_1} \quad (22)$$

(3) If the load is omhic, it must be: $Q_2=0$ (Eq. (9)) or the imaginary part of Z_2 must be=0 (Eq. (18))

$$\text{then } V_1V_2cos(\theta_1-\theta_2)-V_2^2 = 0 \rightarrow cos(\theta_1-\theta_2) = \frac{V_2}{V_1} \quad (23)$$

4. EXPERIMENTAL EXAMINATION OF THEORETICAL ANALYSIS

A low voltage laboratory model of an electric power transmission line presenting only long-wise inductance is utilized in order to obtain experimental results. These results will then be compared to the theoretical ones in order to verify or not them.

The electric power transmission line model in steady state condition gave the following experimental measurements:

$Z_{line} = 400< 90°\ \Omega$
$V_1 = 142.4V$
$V_2 = 132V$
$I = 0.12A$
$\theta_1 = 22°$
$\theta_2 = 0°$
$P_1 = 18W$
$Q_1 = 6Var$
$P_2 = 18W$
$Q_2 = 0Var$

Then, calculating the following equations, we find:

Eq. (1): $I=0.13<-0.03°A$
Eq. (3): $P_1 = 17.6W$
Eq. (4): $\theta_1 > \theta_2 \rightarrow 22° > 0°$

$$\text{Eq. (5): } Q_1 = 7.12\text{Var}$$
$$\text{Eq. (7): } P_2 = 17.6\text{W}$$
$$\text{Eq. (8): } \theta_1 > \theta_2 \ \rightarrow 22° > 0°$$
$$\text{Eq. (9): } Q_2 = 0.01\text{Var}$$
$$\text{Eq. (13): } P_{line} = P_1 - P_2 = 0$$
$$\text{Eq. (14): } Q_{line} = 7.11\text{Var}$$
$$\text{Eq. (15): } 37701.76 > 34856.18$$
$$\text{Eq. (23): } 0.927 = 0.927$$

The values of Eqns. (3) and (7) are the same and very close to that of experimental result. The positive value of equations (3) and (7) are verified by the inequalities (4) and (8). The values of Eq. (5) and (14) and their positive value verify inequality (15) and with the value of Eq. (13) also indicate the inductive character of the line. Any small differences are due to the rounding of numbers and the precision of the instruments. The equality of Eq. (23) implies the ohmic character of the load.

5. DISCUSSION AND CONCLUSIONS

Comparing the electric analysis developed logically in section 2 to the mathematical analysis of the electric equivalent circuit of the electric power transmission line under discussion in section 3, we can draw the following:

(1) the active power P_1 at the beginning of the line is positive as we expect (section 2, case 1) only if $\theta_1 > \theta_2$ (Eqns. (3), (4)). Otherwise, there is no flow of active power from the source to the line. The same applies to the active power P_2 at the end of the line (Eq. (7)).

(2) the active power P_1 at the beginning of the line is equal to the active power P_2 at the end of the line (Eqns. (3), (7)) as stated in section 2, case 2. That means $P_{line}=0$ (Eq. (13)) as expected.

(3) the reactive power Q_{line} of the line is positive as expected (section 2, case 3) and shown in Eq. (14) and (15). In the case that $V_1=V_2$ and $\theta_1=\theta_2$, there is no flow of reactive power in the line and no flow of active power from the source to the line (case 1 above) since complex voltage $V_1<\theta_1$ at the beginning of the line is equal in magnitude and phase (angle) to the complex voltage $V_2<\theta_2$ at the end of the line and the current flow in the line is zero (Eq. (1)).

(4) the difference between the reactive power Q_1 and Q_2 (Eqns. (5), (9)) at the beginning and the end of the line respectively gives as result the reactive power of the line Q_{line} (Eq. (14)) as expected in section 2, case 4.

(5) similarly, the difference between the apparent complex resistance Z_1 and Z_2 (Eqns. (17), (18)) at the beginning and the end of the line respectively gives as result the complex resistance of the line Z_{line} (Eq. (19)) as expected in section 2, case 5.

(6) looking at the expressions of the apparent power of the load S_2 (Eq. (6)) and the load Z_2 in complex form (Eq. (18)) since $\omega L>0$ and the denominator in Eq. (18) is always positive (Eq. (15)), one can see that the same complex numerator in both Eq. (6) and (18) define the criterion regarding the type of load as expected in section 2, case 6. The real part of the numerator is always positive (Eq. (4)) showing the existence of the ohmic part of the load. If the imaginary part of the numerator is positive implying the presence of load with inductive character, both inequalities of Eq. (20) and (21) must be valid. If the above imaginary part is negative implying the presence of load with capacitive character only the inequality in Eq. (22) must be valid. If the above imaginary part is zero implying the presence of no imaginary load or the presence of both inductive and capacitive loads that nullify each other the equality of Eq. (23) is valid.

Finally, the experimental results of section 4 as analyzed and discussed in detail in that section come to verify the theoretical results and the relative analysis.

REFERENCES

[1] Gonen, T. (2014). Electric Power Transmission System Engineering: Analysis and Design. CRC Press.

[2] Weedy, B. M., Cory, B. J., Jenkins, N., Ekanayake, J. B., Strbac, G. (2012). Electric power systems. John Wiley & Sons.

[3] Nasar, S.A. (1996). Electric Energy Systems. Prentice Hall.

[4] Leonidopoulos, G. (2005). Modelling and simulation of electric power transmission line voltage. Modelling, Measurement and Control A, 88(1): 71-83.

[5] Leonidopoulos, G. (1989). Fast linear method and convergence improvement of load flow numerical solution methods. Electric Power Systems Research Journal, 16(1): 23-31. https://doi.org/10.1016/0378-7796(89)90034-5

[6] Leonidopoulos, G. (1991). Linear power system equations and security assessment. International Journal of Electrical Power and Energy Systems, 13(2): 100-102. https://doi.org/10.1016/0142-0615(91)90032-Q

[7] Leonidopoulos, G. (1994). Efficient starting point of load-flow equations. International Journal of Electrical Power and Energy Systems, 16(6): 419-422. https://doi.org/10.1016/0142-0615(94)90029-9

[8] Leonidopoulos, G. (2016). Modelling and simulation of electric power transmission line current as wave. Modelling, Measurement and Control A, 89(1): 1-12.

LIST OF SYMBOLS

$V_1<\theta_1$=voltage in complex form (polar) at the beginning of electric power transmission line

V_1= voltage magnitude (V)

θ_1= voltage phase(angle) (°)

$<\theta=e^{j\theta}$= $\cos\theta + j\sin\theta$ = Euler's equation

S_1=apparent power (VA) in complex form at the beginning of electric power transmission line

S_1=P_1+jQ_1

P_1=active power (W) at the beginning of electric power transmission line

Q_1 =reactive power (VAr) at the beginning of electric power transmission line

Z_1=apparent resistance in complex form at the beginning of electric power transmission line

$V_2<\theta_2$=voltage in complex form (polar) at the end of electric power transmission line

V_2=voltage magnitude (V)

θ_2=voltage phase(angle) (°)

S_2=apparent power (VA) in complex form at the end of electric power transmission line

S_2=P_2+jQ_2

P_2=active power (W) at the end of electric power transmission line

Q_2=reactive power (VAr) at the end of electric power transmission line
Z_2=apparent resistance in complex form at the end of electric power transmission line
V_{line}=voltage drop in complex form along the electric power transmission line
S_{line}=apparent power (VA) in complex form of electric power transmission line
S_{line}=P_{line}+jQ_{line}
P_{line}=active power (W) of electric power transmission line
Q_{line}=reactive power (VAr) of electric power transmission line
Z_{line}=jωL_{line} = resistance in complex form of electric power transmission line
L_{line}=inductance of electric power transmission line
Z^*_{line}=complex conjugate of Z_{line}
I=current in complex form of electric power transmission line