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Analysis of Electric Power Transmission Line Resultant Wave Current in Polar Form

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ABSTRACT

In this paper, a well-known mathematical model of electric power transmission line under steady state conditions is considered. From this model, the mathematical expression that describes the resultant current along a power transmission line has been developed taking as starting point the end of the line.
We use the fore-mentioned mathematical expression and the data of a typical electric transmission line to calculate how the current wave varies. The results are also graphed in order to have an optical view of how the current wave behaves. Finally, the results are analysed and the relative conclusions are drawn.

1. INTRODUCTION

In this paper, a power transmission line of an electric power system [1-8] is under consideration. Its equivalent electric circuit under steady state conditions is drawn and the respective differential equations are extracted from it using as independent variable the distance x from either the rears of the line. The above mathematical model already exists in the literature and can easily be found [1-4, 8].

Solving the differential equations, the mathematical expression in polar form describing the resultant current wave is obtained (section 2). The proof that the above resultant current is a wave is the mathematical expression itself. It is the mathematical expression of a wave.

As far as I know and search in the literature, I could not find calculation and graphical representation of the current wave along an electric power transmission line. Thus, in this paper, the above mathematical expression is tested on a typical electric power transmission line and the results are presented in section 3. Furthermore, in section 3, the above results are graphed in order to have an optical image of how the resultant current wave along the line behaves. In section 4, a low voltage laboratory electric power transmission line model is used to obtain experimental results to verify the equations of section 2. Finally, in section 5, a discussion is developed, the results are studied, analyzed and in section 6, the relative conclusions are drawn.

2. DEVELOPMENT OF THE MATHEMATICAL EXPRESSION OF RESULTANT CURRENT WAVE

In Figure 1, the electric equivalent representation of power transmission line under steady state conditions and using divided elements has been drawn.
Where
z dx = the infinitesimal long-wise complex impedance of dx
y dx = the infinitesimal transversal complex conductance of dx

From the infinitesimal element dx, the following equations are drawn:
1st law of Kirchhoff: [I(x)+dI(x)] = I(x) + dI(x)
2nd law of Kirchhoff: [V(x)+dV(x)] = V(x) + dV(x)
Voltage drop on element zdx:

$$dV(x)=[I(x)+dI(x)]zdx=\cong I(x)\, zdx \rightarrow \frac{dV(x)}{dx}=I(x)\, z \tag{1}$$

Voltage drop on element ydx:

$$dI(x) = V(x)\, ydx \rightarrow \frac{dI(x)}{dx} = V(x)\, y \tag{2}$$

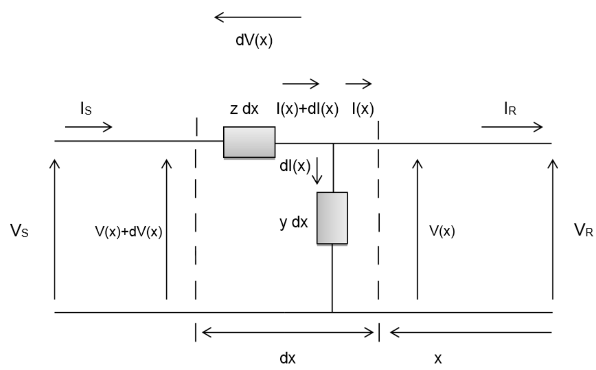

Figure 1. Electric equivalent representation of electric power transmission line

Differentiating Eq. (1) and replacing it into Eq. (2), we get:

$$\frac{d^2V(x)}{dx^2}=yz\, V(x) \tag{3}$$

Differentiating Eq. (2) and replacing it into Eq. (1), we also get:

$$\frac{d^2I(x)}{dx^2}=yz\ I(x) \qquad (4)$$

From Eqns. (3) and (4), V(x) and I(x) are described by the same differential equations. The above implies that V(x) and I(x) are described by similar mathematical functions.

We take as initial conditions:

$$V(x=0)=V_R \qquad (5)$$

and

$$I(x=0)=I_R \qquad (6)$$

i.e. we take as x=0 the end of electric power transmission line

Then, from Eqns. (3), (4), (5) and (6), we extract the following mathematical expression of the resultant current wave:

$$I(x)=\frac{\frac{V_R}{z_C}+I_R}{2}\ e^{\gamma x}-\frac{\frac{V_R}{z_C}-I_R}{2}\ e^{-\gamma x} \qquad (7)$$

The above Eq. (7) is the mathematical expressions of a wave. Eq. (7) can also be written in hyperbolic form:

$$I(x) = I_R\cosh(\gamma x) +V_{R/}z_C\ \sinh(\gamma x) \qquad (8)$$

The term cosh(γx) can be written as :

$$\cosh(\gamma x)=\cosh[(\alpha+j\beta)x]=\cosh(\alpha x+j\beta x)=\cosh(\alpha x).\cos(\beta x)+j\sinh(\alpha x).\sin(\beta x)$$

The term sinh(γx) can also be written as :

$$\sinh(\gamma x)=\sinh[(\alpha+j\beta)x]=\sinh(\alpha x+j\beta x) =\sinh(\alpha x).\cos(\beta x)+j\cosh(\alpha x).\sin(\beta x)$$

3. CALCULATION AND GRAPHICAL PRESENTATION OF RESULTANT CURRENT WAVE

We consider a typical electric power transmission line with the following parameters:

R= 0.107 Ω/km	L = 1.362 mH/km
G = 0 S/km	C = 0.0085 μF/km
f = 50 Hz	l = 360 km
V_R = 115470< 0° V	I_R = 360.844< 0° A

Then using the list of symbols and the analysis of section 2, we can calculate the other complex parameters of the above line in polar and/or cartesian form:

$$\gamma = 1.085\text{x}10^{-3} <82.98^\circ\ \text{km}^{-1} = (0.1326\text{x}10^{-3} + j\ 1.07687\text{x}10^{-3})\ \text{km}^{-1}$$

$$\alpha=0.1326\text{x}10^{-3}\ \text{neper/km}$$
$$\beta=1.07687\text{x}10^{-3}\ \text{rad/km}$$

$$z_C = 406.41 <-7.02^\circ\ \Omega$$

$$\frac{\frac{V_R}{z_C}+I_R}{2} = 321.886<3.092^\circ \quad A$$

$$-\frac{\frac{V_R}{z_C}-I_R}{2}= 43.079<-23.767^\circ \quad A$$

λ=5834.674 km

υ = 291733.696 km/sec

τ= 1.234 msecs

Δ= 22.212°

Δ/l=0.0617°/km

V_S=132807.0<25.12620° V

Then, Eq. (7) using the above parameters becomes:

$$I(x) =321.886 < 3.092^\circ\ e^{(0.1326\text{x}10\text{-}3\ +\ j\ 1.07687\text{x}10\text{-}3)x} +43.079<-23.767^\circ\ e^{-(0.1326\text{x}10\text{-}3\ +\ j\ 1.07687\text{x}10\text{-}3)x}\quad A \qquad (9)$$

Table 1. Calculation results of resultant current wave

α/α	x (km)	I(x) (Amps)	$\varphi_{I(x)}$ (°)
1	0	360.8439	0
2	10	360.8365	0.490425
3	20	360.8144	0.982528
4	30	360.7778	1.476348
5	40	360.7268	1.971925
6	50	360.6617	2.469300
7	60	360.5827	2.968510
8	70	360.4899	3.469596
9	80	360.3837	3.972596
10	90	360.2641	4.477549
11	100	360.1315	4.984494
12	110	359.9860	5.493468
13	120	359.8280	6.004509
14	130	359.6575	6.517655
15	140	359.4749	7.032943
16	150	359.2805	7.550409
17	160	359.0744	8.070091
18	170	358.8570	8.592025
19	180	358.6285	9.116245
20	190	358.3891	9.642788
21	200	358.1392	10.17169
22	210	357.8791	10.70298
23	220	357.6089	11.23670
24	230	357.3291	11.77288
25	240	357.0399	12.31155
26	250	356.7416	12.85275
27	260	356.4345	13.39651
28	270	356.1190	13.94285
29	280	355.7952	14.49182
30	290	355.4637	15.04343
31	300	355.1246	15.59773
32	310	354.7784	16.15473
33	320	354.4253	16.71448
34	330	354.0658	17.27699
35	340	353.7000	17.84229
36	350	353.3285	18.41041
37	360	352.9515	18.98137

Using Eq. (8) and taking step Δx=10km, we calculate the values of resultant current wave and the results are presented in Table 1. Since the current is vector, the results are complex numbers and are given in polar form ie. in current magnitude (Amps) and current phase (°) representation.

The graphical presentations of results obtained in Table 1 are given in Figure 2.

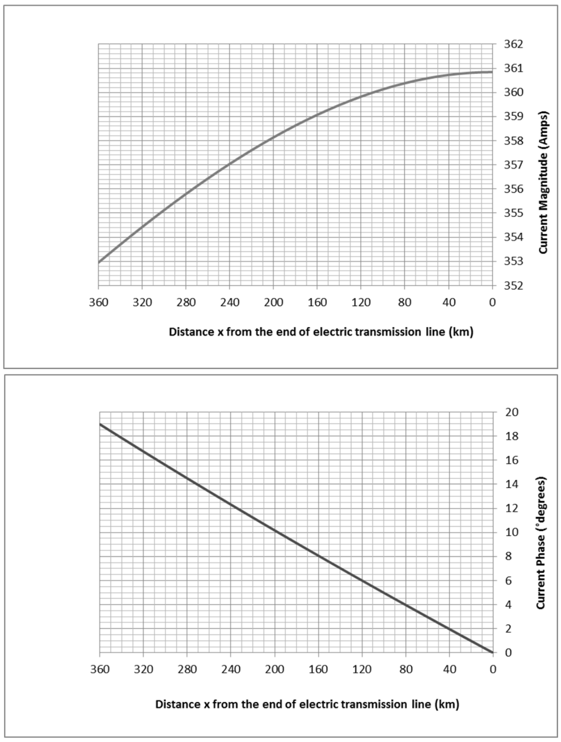

Figure 2. Current magnitude and phase (angle) from the beginning towards the end of line

4. EXPERIMENTAL RESULTS AND DISCUSSION

A low voltage laboratory model of an electric power transmission line is used to obtain the experimental results. The electric power transmission line model in steady state condition gave the following experimental measurements:

$$Z_{line}=400<90° \ \Omega$$
$$V_S=142.4<22° \ V$$
$$V_R=132<0° \ V$$
$$I_{line}= 0.133<0° \ A$$

Then, the equations that give the resultant current and voltage of the line, from the respective equations, become:

$$I(x)=I_R$$
$$V(x)=V_R+I_R \ z \ x$$

for x=l, we have:

$$I(x=l)=I_S=I_R=0.133<0° \ A$$
$$V(x=l)=V_S=142.44<21.931° \ V$$

The above results are the same with the results that the experiment gives. Any small differences are due to the rounding of numbers and the precision of the instruments. The values of V_R and I_{line} imply the ohmic character of the load.

5. DISCUSSION

The curves of graph 1 may appear common but they are not. One may look straight line or almost straight line but it is not. The above quantities have an exponential behaviour as someone can verify from the respective equation (7) in section 2. Their graphical representations depend on the values of their exponential constant factors (α and β). If their values are small and as variable x increases, the values αx and βx do not change enough in order their exponential behaviour to appear on the graphs. This is the reason they seem to be straight or almost straight lines.

The above explanation is given regarding their form. Regarding now their variation, the following reasoning is developed.

On one hand, the terms (V_R/z_C+I_R) and (V_R/z_C-I_R) are constant complex numbers since V_R, I_R and z_C are constant complex numbers. That implies that they have a constant magnitude and a constant phase.

On the other hand, the terms $e^{\gamma x}$ and $e^{-\gamma x}$ vary with distance x from the end of electric power transmission line.

5.1 First term of resultant current

The term $e^{\gamma x}$ can be written as $e^{(\alpha+j\beta)x} = e^{\alpha x} e^{j\beta x} = e^{\alpha x}[\cos(\beta x) + j \sin(\beta x)]$

The values of α and β are real positive numbers for a typical real power transmission line. This will be understood from the following analysis.

The $e^{\alpha x}$ is the magnitude of the above term while the $e^{j\beta x}$ is the phase (angle) of the same term.

The term $e^{\alpha x}$ increases as x increases i.e. the magnitude of current increases as we approach the beginning of line. In other words, the magnitude (intensity) of current diminishes as the wave travels from the beginning of line (where the voltage is applied and the current wave starts) to the end of line as one expects in real world (the intensity of signal diminishes as it moves away from source).

The term βx similarly increases as x increases. With similar as above reasoning, the term βx i.e. the phase of current wave diminishes as the wave travels from the beginning of line and moves to the end of line.

5.2 Second term of resultant current

Similarly, the term $e^{-\gamma x}$ can be written as $e^{-(\alpha+j\beta)x} = e^{-\alpha x} e^{-j\beta x} = e^{-\alpha x}[\cos(-\beta x) + j \sin(-\beta x)]$

With similar as above reasoning, the term $e^{-\alpha x}$ decreases as x increases. In other words, the magnitude (intensity) of current wave decreases as the wave moves from the end towards the beginning of line as one expects. It is really the part of current wave that arrives at the end of line and refracts travelling to the opposite direction of line. The opposite flow of current is indicated by the symbol minus (-) of the term.

Additionally, the term -βx decreases as x increases i.e. the phase (angle) of current wave decreases as the wave moves from the end towards the beginning of line.

5.3 Resultant current

Since from Eq. (7), the line current at point x is the algebraic summation of the above two terms and also taking into consideration the results of the above reasoning and depending on line parameters and the type of load at the end of line, we can state that in general the current magnitude and phase decrease from the beginning to the end of line. This implies having in mind the above that either or both the current magnitude and phase can also increase from the beginning to the end of line.

6. CONCLUSIONS

Studying the results presented in table 1 and the graph 1 of section 3, we can observe and conclude the following:

(1) the current magnitude (intensity) increases as we move from the beginning towards the end of line

(2) the current phase (angle) decreases as we move from the beginning towards the end of line Regarding now the information that is drawn from the graph 1 is discussed in the following paragraphs.

The above observations, the value of V_S in section 3 and that the current phase is behind the voltage phase imply that both line and load present an ohmic-inductive behaviour. In other words, we have an active and reactive power flow from the source to line and load.

Regarding the load is pure ohmic since load voltage and load current have the same angle as one can see in section 3.

The line from the data given in section 3 has an ohmic (R) as well as an inductive (L) long-wise elements plus a capacitive (C) transversal element. The above statement that the line presents an ohmic-inductive behaviour means that the capacitive element of the line does not produce enough reactive power to cover the needs of the inductive long-wise element of the line and thus the source comes to cover the rest reactive power needed. It also means that there is an active power flow from the source to cover the needs of the ohmic elements of line and load.

Then, we can conclude that the above observations verify the analysis and discussion developed in section 4 of the paper. Furthermore, the experimental results as developed and discussed in section 4 come to substantiate the equations drawn in section 2.

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LIST OF SYMBOLS

R=long-wise omhic resistance of power transmission line (under sinusoidal voltage) per unit length of line (Ω/km)

L=long-wise inductance of power transmission line (under sinusoidal voltage) per unit length of line (H/km)

C=transversal capacitance of power transmission line (under sinusoidal voltage) per unit length of line (F/km)

G=transversal conductance of power transmission line (under sinusoidal voltage) per unit length of line (S/km)

l=length of power transmission line (km)

z=R+jωL=long-wise complex impedance of power transmission line per unit length of line (Ω/km)

y=G+jωC=transversal complex conductance of power transmission line per unit length of line (S/km)

Z=z.l=total long-wise complex impedance of power transmission line (Ω)

Y=y.l= total transversal complex conductance of power transmission line (S)

$\mathbf{V_S}$=complex line to earth voltage at the beginning of power transmission line, Sending voltage (V)

$\mathbf{V_R}$=complex line to earth voltage at the end of power transmission line, Receiving voltage (V)

$\mathbf{I_S}$=complex phase current at the beginning of power transmission line, Sending current (A)

$\mathbf{I_R}$= complex phase current at the end of power transmission line, Receiving current (A)

$\gamma=\sqrt{zy}=\alpha+j\beta$= transmission co-efficient of power transmission line (km^{-1})

α=reduction co-efficient of power transmission line (neper/km)

β=phase co-efficient of power transmission line (rad/km)

$z_C=\sqrt{\frac{z}{y}}$=characteristic impedance of power transmission line (Ω)

$e^{j\varphi}=\cos\varphi + j\sin\varphi$ = Euler's equation

$\lambda=\frac{2\pi}{\beta}$= wave length of power transmission line (km)

υ=wave transmission velocity of power transmission line (km/sec)

τ=wave travelling time in order to cover the length of power transmission line (sec)

Δ=electric phase (angle) of power transmission line (rad)

$\frac{\Delta}{l}$=electric phase (angle) of power transmission line per unit length of line (rad/km)

I(x)=resultant line current wave as a function of distance x (A)

φ(x)=electric phase(angle) of respective complex quantity as function of distance x (°)