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Analysis of Electric Power Transmission Line Wave Voltage Components in Polar Form

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ABSTRACT

In this paper, a well-known mathematical model of electric power transmission line under steady state conditions is considered. From this model, the mathematical expressions that describe the two components of the resultant voltage i.e. voltage travelling and refracted waves along a power transmission line have been developed taking as starting point the end of the line.
We use the fore-mentioned mathematical expressions and the data of a typical electric transmission line to calculate how the voltage travelling and refracted waves vary. The results are also graphed in order to have an optical view of how the voltage travelling and refracted waves behave. Finally, the results are analysed and the relative conclusions are drawn.

1. INTRODUCTION

Most people think of the voltage as an element that when it is put on, it is applied immediately. They cannot imagine that the voltage is a wave (an electromagnetic wave) that travels and refracts with almost the speed of light. This understanding is due to the length of line and the inability that people have to perceive the very small time intervals (psecs, μsecs, msecs depending on the line length) that the wave needs to cover these distances.

In this paper, the length under consideration is that of a power transmission line of an electric power system [1-9], a length of some hundred kilometers. The equivalent electric circuit under steady state conditions is drawn and the respective differential equations are extracted from it using as independent variable the distance x from either the rears of the line. The above mathematical model already exists in the literature and can easily be found [1-5].

Solving the differential equations, the mathematical expressions describing the voltage travelling and refracted waves are obtained (section 2). The proof that the above voltages are the travelling and refracted wave respectively is the mathematical expressions themselves. They are the mathematical expressions of a travelling and refracted wave respectively.

As far as I know and search in the literature, I could not find calculation and graphical representation of the voltage travelling and refracted waves along an electric power transmission line. Thus, in this paper, the above mathematical expressions are tested on a typical electric power transmission line and the results are presented in section 3. Furthermore, in section 3, the above results are graphed in order to have an optical image of how the voltage travelling and refracted waves along the line behave. Finally, in section 4, a discussion is developed, the results are studied, analysed and in section 5, the relative conclusions are drawn.

2. DEVELOPMENT AND ANALYSIS OF THE MATHEMATICAL EXPRESSIONS OF VOLTAGE TRAVELLING AND REFRACTED WAVES

In Figure 1, the electric equivalent representation of power transmission line under steady state conditions and using divided elements has been drawn.

Where z dx=the infinitesimal long-wise complex impedance of dx

y dx=the infinitesimal transversal complex conductance of dx

From the infinitesimal element dx, the following equations are drawn:

1^{st} law of Kirchhoff: [I(x)+dI(x)]=I(x) + dI(x)

2^{nd} law of Kirchhoff: [V(x)+dV(x)]=V(x) + dV(x)

Voltage drop on element zdx:

$$dV(x)=[I(x)+dI(x)]zdx\cong I(x)zdx\rightarrow\frac{dV(x)}{dx}=I(x)\,z \quad (1)$$

Voltage drop on element ydx:

$$dI(x)=V(x)\,ydx \rightarrow \frac{dI(x)}{dx}=V(x)\,y \quad (2)$$

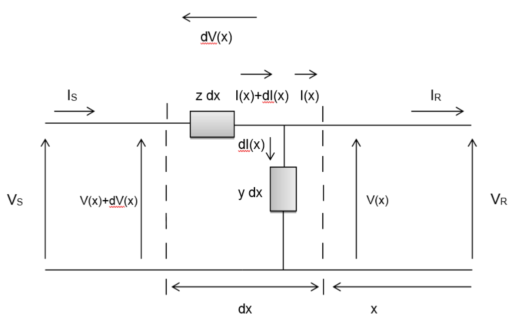

Figure 1. Electric equivalent representation of electric power transmission line

Differentiating Eq. (1) and replacing it into Eq. (2), we get:

$$\frac{d^2V(x)}{dx^2}=yz\,V(x) \tag{3}$$

Differentiating Eq. (2) and replacing it into Eq. (1), we also get:

$$\frac{d^2I(x)}{dx^2}=yz\,I(x) \tag{4}$$

From Eqns. (3) and (4), V(x) and I(x) are described by the same differential equations. The above implies that V(x) and I(x) are described by similar mathematical functions.

We take as initial conditions:

$$V(x=0)=V_R \tag{5}$$

and

$$I(x=0)=I_R \tag{6}$$

i.e. we take as x=0 the end of electric power transmission line

Then, from Eqns. (3), (4), (5) and (6), we extract the following mathematical expressions of voltage travelling and refracted wave respectively:

$$V_{trav}(x)=\frac{V_R+I_R\,z_C}{2}\,e^{\gamma x} \tag{7}$$

$$V_{refr}(x)=\frac{V_R-I_R\,z_C}{2}\,e^{-\gamma x} \tag{8}$$

The above Eqns. (7) and (8) are the mathematical expressions of a wave.

Then, the voltage refraction co-efficient $\rho_V(x)$ can be defined as a function of distance x. The voltage refraction co-efficient is set as :

$$\rho_{V(x)}=\frac{V_{refr}(x)}{V_{trav}(x)} \tag{9}$$

3. CALCULATION AND GRAPHICAL PRESENTATION OF VOLTAGE TRAVELLING AND REFRACTED WAVES

We consider a typical electric power transmission line with the following parameters:

$R=0.107\ \Omega/km$ $L=1.362\ mH/km$

$G=0\ S/km$ $C=0.0085\ \mu F/km$

$f=50\ Hz$ $l=360\ km$

$V_R=115470< 0°\ V$ $I_R=360.844< 0°\ A$

Then using the list of symbols and the analysis of section 2, we can calculate the other complex parameters of the above line in polar and/or cartesian form:

$$\gamma=1.085x10^{-3} <82.98°\ km^{-1}=(0.1326x10^{-3} + j\ 1.07687x10^{-3})\ km^{-1}$$

$\alpha=0.1326x10^{-3}$ neper/km $\beta=1.07687x10^{-3}$ rad/km

$$z_C=406.41 <-7.02°\ \Omega$$

$$\frac{V_R+I_R\,z_C}{2}=130817.935 <-3.93°\ V$$

$$\frac{V_R-I_R\,z_C}{2}=17507.97 <149.213°\ V$$

$$\lambda=5834.674\ km$$

$$\upsilon=291733.696\ km/sec\ \ \tau=1.234\ msecs$$

$$\Delta=22.212°\ \ \Delta/l=0.0617°/km$$

Then, Eqns. (7), (8) and (9) using the above parameters become:

$$V_{trav}(x)=130817.935 <-3.93°\ e^{(0.1326x10\text{-}3 + j\ 1.07687x10\text{-}3)x}\ V \tag{10}$$

$$V_{refr}(x)=17507.97 <149.213°\ e^{-(0.1326x10\text{-}3 + j\ 1.07687x10\text{-}3)x}\ V \tag{11}$$

$$\rho_V(x) = \frac{17507.97 <149.213°\ e^{-(0.1326x10-3 + j\ 1.07687x10-3)x}}{130817.935 <-3.93°\ \ e^{(0.1326x10-3 + j\ 1.07687x10-3)x}} \tag{12}$$

Using Eqns. (10), (11) and (12) and taking step Δx=10km, we calculate the values of voltage travelling and refracted wave as well as voltage refraction co-efficient and the results are presented in Table 1. Since the voltages are vectors, the results are complex numbers and are given in polar form i.e. in voltage magnitude (Volts) and voltage phase (°) representation. The voltage refraction co-efficient $\rho_V(x)$ is a pure complex number since is derived from the division of the voltage waves and is also given in table 1 in polar form ie. in magnitude(pure real number) and phase (°) form.

The graphical presentations of results obtained in table 1 are given in Figures 2 to 4.

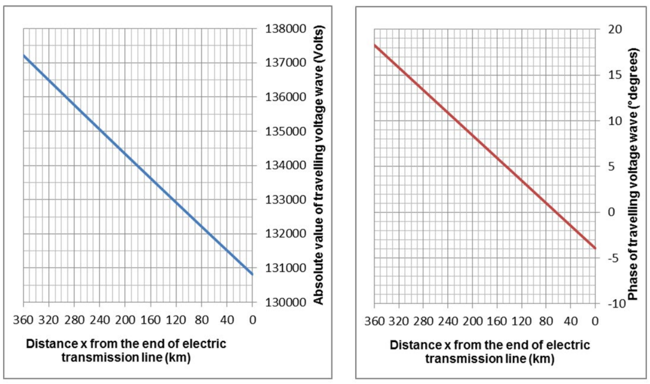

Figure 2. Absolute value (intensity) and phase (angle) of voltage travelling wave from the beginning towards the end of line i.e. along the direction the travelling wave moves (direction right to left of electric power transmission line of figure 1)

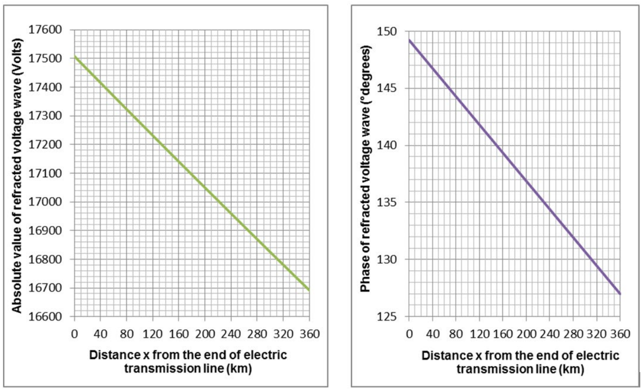

Figure 3. Absolute value (intensity) and phase (angle) of voltage refracted wave from the end towards the beginning of line i.e. along the direction the refracted wave moves (direction opposite to that of graph 1, i.e. left to right of electric power transmission line of figure 1)

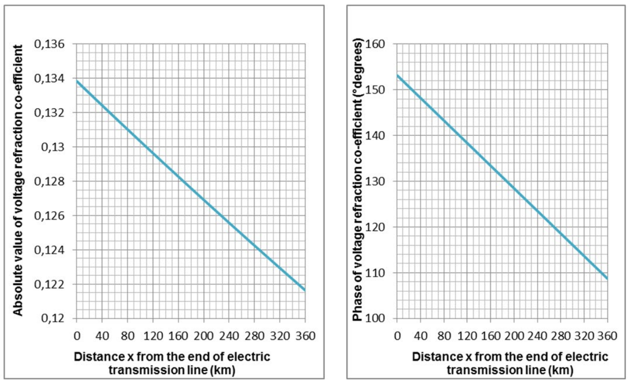

Figure 4. Absolute value and phase (angle) of voltage refraction co-efficient from the end where the refraction occurs towards the beginning of line (direction opposite to that of graph 1, i.e. left to right of electric power transmission line of figure 1)

Table 1. Calculation results of voltage travelling and refracted wave

α/α	x (km)	$V_{trav}(x)$ (Volts)	$\varphi_{Vtrav}(x)$ (°)	$V_{refr}(x)$ (Volts)	$\varphi_{Vrefr}(x)$ (°)	$\rho_V(x)$	$\varphi_{\rho V}(x)$ (°)
1	0	130818.0	-3.928010	17507.96	149.2129	0.1338345	153.1409
2	10	130991.6	-3.310870	17484.75	148.5958	0.1334799	151.9066
3	20	131165.5	-2.693720	17461.58	147.9786	0.1331263	150.6723
4	30	131339.6	-2.076570	17438.43	147.3615	0.1327736	149.4381
5	40	131513.9	-1.459430	17415.32	146.7443	0.1324219	148.2038
6	50	131688.4	-0.842280	17392.23	146.1272	0.1320711	146.9695
7	60	131863.2	-0.225130	17369.18	145.5100	0.1317212	145.7352
8	70	132038.2	0.392014	17346.16	144.8929	0.1313722	144.5009
9	80	132213.5	1.009160	17323.17	144.2757	0.1310242	143.2666
10	90	132389.0	1.626307	17300.21	143.6586	0.1306771	142.0323
11	100	132564.7	2.243454	17277.28	143.0415	0.1303309	140.7980
12	110	132740.6	2.860600	17254.37	142.4243	0.1299856	139.5637
13	120	132916.8	3.477747	17231.50	141.8072	0.1296413	138.3294
14	130	133093.2	4.094893	17208.67	141.1900	0.1292979	137.0951
15	140	133269.8	4.712040	17185.86	140.5729	0.1289553	135.8608
16	150	133446.7	5.329186	17163.08	139.9557	0.1286137	134.6265
17	160	133623.8	5.946333	17140.33	139.3386	0.1282730	133.3922
18	170	133801.2	6.563480	17117.61	138.7214	0.1279332	132.1579
19	180	133978.8	7.180626	17094.92	138.1043	0.1275942	130.9237
20	190	134156.6	7.797773	17072.26	137.4871	0.1272562	129.6894
21	200	134334.6	8.414919	17049.63	136.8700	0.1269191	128.4551
22	210	134512.9	9.032066	17027.03	136.2528	0.1265829	127.2208
23	220	134691.5	9.649212	17004.46	135.6357	0.1262475	125.9865
24	230	134870.2	10.266360	16981.93	135.0185	0.1259131	124.7522
25	240	135049.2	10.883510	16959.42	134.4014	0.1255795	123.5179
26	250	135228.5	11.500650	16936.94	133.7843	0.1252468	122.2836
27	260	135408.0	12.117800	16914.49	133.1671	0.1249150	121.0493
28	270	135587.7	12.734950	16892.07	132.5500	0.1245841	119.8150
29	280	135767.6	13.352090	16869.68	131.9328	0.1242541	118.5807
30	290	135947.8	13.969240	16847.32	131.3157	0.1239249	117.3464
31	300	136128.3	14.586390	16824.99	130.6985	0.1235966	116.1121
32	310	136308.9	15.203530	16802.69	130.0814	0.1232692	114.8778
33	320	136489.8	15.820680	16780.42	129.4642	0.1229426	113.6436
34	330	136671.0	16.437820	16758.17	128.8471	0.1226169	112.4093
35	340	136852.4	17.054970	16735.96	128.2299	0.1222921	111.1750
36	350	137034.0	17.672120	16713.78	127.6128	0.1219681	109.9407
37	360	137215.9	18.289260	16691.62	126.9956	0.1216450	108.7064

4. DISCUSSION

The curves of graphs 1, 2 and 3 may appear common but they are not. Some of them may look straight lines or almost straight lines but they are not. The above quantities have an exponential behavior as someone can verify from the respective equations in section 2. Their graphical representations depend on the values of their exponential constant factors (α and β). If their values are small and as variable x increases, the values αx and βx do not change enough in order their exponential behavior to appear on the graphs. This is the reason they seem to be straight or almost straight lines.

The above explanation is given regarding their form. Regarding now their variation, the following reasoning is developed.

On one hand, the terms $(V_R+I_R z_C)$ and $(V_R-I_R z_C)$ of Eqns. (7) and (8) in section 2 are constant complex numbers since V_R, I_R and z_C are constant complex numbers. That implies that they have a constant absolute value and a constant phase as shown in section 3.

On the other hand, the terms $e^{\gamma x}$ and $e^{-\gamma x}$ vary with distance x from the end of power transmission line.

The term $e^{\gamma x}$ can be written as $e^{(\alpha+j\beta)x}=e^{\alpha x}\, e^{j\beta x}=e^{\alpha x}[\cos(\beta x) + j\sin(\beta x)]$

The values of α and β are real positive numbers for a typical real power transmission line. This will be understood from the following analysis.

The term $e^{\alpha x}$ is the absolute value of the above term while the $e^{j\beta x}$ is the phase (angle) of the above term.

The term $e^{\alpha x}$ increases as x increases i.e. the absolute value of voltage travelling wave increases as we approach the beginning of line. In other words, the absolute value (intensity) of voltage travelling wave (Eq. (7)) diminishes as the wave travels from the beginning of line (where the voltage is applied and the voltage travelling wave starts) to the end of line as one expects in real world (the intensity of signal diminishes as it moves away from source).

The term βx similarly increases as x increases. With similar as above reasoning, the term βx i.e. the phase of voltage travelling wave (Eq. (7)) diminishes as the wave travels from the beginning of line and moves to the end of line.

Similarly, the term $e^{-\gamma x}$ can be written as $e^{-(\alpha+j\beta)x}=e^{-\alpha x}\, e^{-j\beta x}=e^{-\alpha x}[\cos(-\beta x) + j\sin(-\beta x)]$

At the end of electric power transmission line a part of voltage travelling wave is refracted and moves in the opposite direction of that of the voltage travelling wave ie. from the end towards the beginning of the line. This is implied by the negative value of -γx. With similar as above reasoning, the term $e^{-\alpha x}$ decreases as x increases. In other words, the absolute value (intensity) of voltage refracted wave (Eq. (8)) decreases as the wave moves from the end towards the beginning of line as one expects. It is really the part of voltage travelling wave that arrives at the end of line and refracts travelling in the opposite direction of line.

Additionally, the term -βx decreases as x increases i.e. the phase (angle) of voltage refracted wave (Eq. (8)) decreases as the wave moves from the end towards the beginning of line.

Using similar thinking, the term $e^{-2\gamma x}$ of Eq. (9) regarding the voltage refraction co-efficient can be written as follows:

$$e^{-2(\alpha+j\beta)x}=e^{-2\alpha x}\, e^{-j2\beta x}=e^{-2\alpha x}[\cos(-2\beta x) + j\sin(-2\beta x)]$$

Thus, using similar as above reasoning, both the magnitude and the phase angle of the voltage refraction co-efficient decrease as we move from the end (where the refraction occurs) towards the beginning of line as one expects.

5. CONCLUSIONS

Studying the results presented in Table 1 and their graphs 1 to 3 of section 3, we can observe and conclude the following:

(1) the intensity (absolute value) of voltage travelling wave decreases as the wave travels from the beginning towards the end of line i.e. along the direction the voltage travelling wave moves

(2) the phase (angle) of voltage travelling wave decreases as the wave travels from the beginning towards the end of line i.e. along the direction the voltage travelling wave moves

(3) the intensity (absolute value) of voltage refracted wave decreases as the wave moves from the end towards the beginning of line i.e. along the direction the current refracted wave moves

(4) the phase (angle) of voltage refracted wave decreases as the wave moves from the end towards the beginning of line i.e. along the direction the current refracted wave moves

(5) the percentage (absolute value) of voltage refraction co-efficient decreases from the end (where the refraction occurs) towards the beginning of line

(6) the phase (angle) of voltage refraction co-efficient also decreases from the end towards the beginning of line

Regarding now the information that is drawn from the graphs 1 to 2 is discussed in the following paragraphs.

Looking at graphs 1 and 2, the magnitude of voltage travelling and refracted wave decreases as one moves from the left rear of the line where the power source is towards the right rear of the line where the load is and then back to the beginning. This observation implies that both line and load present an ohmic-inductive behaviour. In other words, we have a reactive power flow from the source to line and load. Regarding the load is pure ohmic as one can see in section 3 from the data of the typical power line given. Thus, the above statement is right.

The line from the data given in section 3 has an ohmic (R) as well as an inductive (L) long-wise elements plus a capacitive (C) transversal element. The above statement that the line presents an ohmic-inductive behaviour means that the capacitive element of the line does not produce enough reactive power to cover the needs of the inductive long-wise element of the line and thus the source comes to cover the rest reactive power needed.

Looking again at graphs 1 and 2, we can see that the phase of voltage travelling and refracted wave also decrease as one moves from the left rear of the line where the power source is towards the right rear of the line and then back to the beginning. The above observation implies and cannot be otherwise that we have an active power flow from the left rear of the line where the power source is towards the right rear of the line where the load is and back to the beginning in order to cover the needs in active power of both the ohmic element of the line and load.

Then, we can conclude that the above observations verify the analysis and discussion developed in section 4 of the paper. For better understanding of electric transmission line voltage as a wave, we propose to study it using cartesian co-ordinates. This will be the subject of a future paper.

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LIST OF SYMBOLS

R=long-wise omhic resistance of power transmission line (under sinusoidal voltage) per unit length of line (Ω/km)

L=long-wise inductance of power transmission line (under sinusoidal voltage) per unit length of line (H/km)

C=transversal capacitance of power transmission line (under sinusoidal voltage) per unit length of line (F/km)

G=transversal conductance of power transmission line (under sinusoidal voltage) per unit length of line (S/km)

l=length of power transmission line (km)

z=R+jωL=long-wise complex impedance of power transmission line per unit length of line (Ω/km)

y=G+jωC=transversal complex conductance of power transmission line per unit length of line (S/km)

Z=z.l=total long-wise complex impedance of power transmission line (Ω)

Y=y.l=total transversal complex conductance of power transmission line (S)

$\mathbf{V_S}$=complex line to earth voltage at the beginning of power transmission line, Sending voltage (V)

$\mathbf{V_R}$=complex line to earth voltage at the end of power transmission line, Receiving voltage (V)

$\mathbf{I_S}$=complex phase current at the beginning of power transmission line, Sending current (A)

$\mathbf{I_R}$=complex phase current at the end of power transmission line, Receiving current (A)

$\gamma=\sqrt{zy}=\alpha+j\beta$=transmission co-efficient of power transmission line (km^{-1})

α=reduction co-efficient of power transmission line (neper/km)

β=phase co-efficient of power transmission line (rad/km)

$\mathbf{z_C}=\sqrt{\frac{z}{y}}$=characteristic impedance of power transmission line (Ω)

$\mathbf{e^{j\varphi}}=\cos\varphi+j\sin\varphi$=Euler's equation

$\lambda=\frac{2\pi}{\beta}$=wave length of power transmission line (km)

υ=wave transmission velocity of power transmission line (km/sec)

τ=wave travelling time in order to cover the length of power transmission line (sec)

Δ=electric phase (angle) of power transmission line (rad)

$\frac{\mathbf{\Delta}}{\mathbf{l}}$=electric phase (angle) of power transmission line per unit length of line (rad/km)

$\mathbf{V_{trav}(x)}$=voltage travelling wave as a function of distance x (V)

$\mathbf{V_{refr}(x)}$=voltage refracted wave as a function of distance x (V)

$\mathbf{\rho_V(x)}=\frac{V_{refr}(x)}{V_{trav}(x)}$=voltage refraction co-efficient as a function of distance x

φ(x)=electric phase(angle) of respective complex quantity as function of distance x **(°)**