APPLICATION OF ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM FOR THE ESTIMATION OF ROUGHNESS COEFFICIENT OF A MEANDERING OPEN-CHANNEL FLOW

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ABSTRACT

An experimental investigation concerning the variation in roughness for meandering channels with flow depths, aspect ratio and sinuosity is presented. Test results revealed that the value of roughness coefficient in terms of Chezy's C increases with increase in aspect ratio and sinuosity. Adaptive neuro-fuzzy-based inference system (ANFIS), an integrated system, a combination of fuzzy logic and neural network is employed to find out the roughness coefficient of a meandering channel. Estimation of roughness coefficient is important for forecasting of discharge because its flexibility to resolve issues supported nonlinearity, randomness and uncertainty of knowledge. In the present work, an ANFIS-based model is developed for the prediction of the roughness coefficient of a meandering channel in terms of Chezy's C. Different standard methods to predict this variable are conjointly tested and verified with the laboratory findings as well as global data moreover. By comparing the results with the established standard methods available in the literature, it was observed that traditional methods could not provide satisfactory output at different surface and hydraulic conditions. Statistical error analysis is also carried out in which it was found that ANFIS model performed more accurately giving results with less error than different existing strategies. The analysis shows a high level of accuracy with regard to the ANFIS-based model developed for predicting the Chezy's C especially coefficients of determination are found to be more encouraging.
Keywords: Aspect ratio, flow resistance, fuzzy inference system, meandering channel, sinuosity.

1 INTRODUCTION

Estimation of roughness coefficients in a natural channel is the fundamental problem in open-channel hydraulics. It involves in itself directly to the entire flow problem ranging from the prediction of flood, water level management, to the protection measures. Although much research has been performed to supplement basic roughness coefficients for a simple open-channel flow [1], less has been done for a meandering channel. According to researchers, the behavior of the flow is surprisingly complex. Consequently, the use of design methods based on straight simple channels is inappropriate when applied to single meandering channels and will result in large errors when estimating the discharge.

There is a tendency with regard to the selection of roughness coefficients as either an arbitrary or an intuitive process. Specific procedures can be used to determine the values for roughness coefficients in channels. Values of the roughness coefficient may be assigned for conditions that exist at the time of a specific flow event, for average conditions over a range in stage, or for anticipated conditions at the time of a future event. The flow behavior of a meandering channel is surprisingly more complex than that of a straight channel as reported by Sellin [2], Shiono *et al.* [3], Patra and Kar [4], Khatua [5], Patra and Khatua [6] and Khatua *et al.* [7]. For this reason, it is very difficult to select an appropriate value of roughness coefficient in calculating the sectional mean velocity. The equations [4] proposed by Chezy, Darcy-Weisbach or Manning are used in river hydraulics to compute the mean velocity. To compute the exact discharge in a meandering open-channel flow, the single most influencing important parameter is to select a suitable value of roughness coefficient in terms of Manning's n, Chezy's C or Darcy-Weisbach friction factor f. The basic approach proposed

by Cowan [8] was modified by Acrement and Schneider [9]. Waterways experiments were conducted by Army Corps of Engineers [10] in meandering channels at the Station to investigate the conveyance capacity of meandering channel. James [11] suggested a huge energy loss due to bending curvature in a meandering channel. The Soil Conservation Service method developed by USGS, Washington [12] is also used for selecting roughness coefficient values for meandering channels. These are found to be discontinuous at the certain limits of the defined sinuosity ranges with consequent ambiguity. The most sensitive parameter channel roughness of a natural river was calibrated by Parhi *et al.* [13] and Hameed and Ali [14] in developing a hydraulic model for flood forecasting and flood plain mapping. Maria and DaSilva [15] expressed the friction factor of rough turbulent meandering flows. Shiono *et al.* [3] reported the effect of bed slope and sinuosity on discharge of meandering channel. On the basis of dimensional analysis, Khatua *et al.* [7] proposed an equation to predict the roughness coefficient in terms of Chezy's *C* of a meandering channel.

From the literatures, it is found that reasonable effect of different hydraulic, geometry and surface parameters exists for estimating discharge in meandering channels. This resistance value is assigned an average value. For the proper estimation of discharge in an open-channel section, selection of a suitable value of Chezy's *C* is an important aspect that needs to be addressed properly. Under steady and uniform flow conditions, to compute the section mean discharge carried by a channel section is calculated with the help of Chezy's equation as

$$Q = CA \sqrt{RS} \tag{1}$$

where Q is the section mean discharge of the channel, R the hydraulic mean radius of the channel section, A the cross-sectional area of the channel, S the energy gradient slope of the channel and C the Chezy's channel coefficient. An easily implementable technique such as adaptive neuro-fuzzy inference system (ANFIS) can be adapted to develop a model for prediction of the Chezy's *C* of a meandering channel. This will be useful in accurate evaluation of discharge to solve many practical problems and to study the mechanism of flow in a better way. The results of the present model are compared well with other established methods.

2 DETERMINATION OF ROUGHNESS VALUE IN MEANDERING OPEN-CHANNEL FLOW

Presently, rapid development in intelligence computing not only lessens the tedious effort of experimentation but also eliminates tedious computations. Investigators adopted used back propagation algorithm of artificial neural network (ANN) for the prediction of roughness in a meandering open-channel flow [16]. ANFIS has been adopted in a variety of fields for accurate prediction of responses in situations where solution is imprecise and uncertain. Nowadays, discharge can be forecasted using the easily ANFIS techniques. When a relationship between input and output is difficult to establish using mathematical, analytical and numerical methods and it becomes cumbersome and time consuming, an easily implementable technique such as ANFIS can be adopted. Thus, an ANFIS model has been proposed in this work to predict a resistance factor in meandering open-channel flow. These ANFIS results are found to be quite comparable to that of the observed values of the published data.

3 EXPERIMENTAL SETUP AND PROCEDURE

Experiments on meandering channels of different sinuosity and flow conditions were conducted by utilizing the facility in the Fluid Mechanics and Hydraulics Engineering Laboratory of the Civil Engineering Department, at the National Institute of Technology, Rourkela, India. Inside the existing

tilting flumes, the meandering channels are fabricated for studying the influence of sinuosity in predicting roughness coefficients and stage-discharge relationships of a meandering channel. Here, the channels of trapezoidal in cross-section were casted using 6/10-mm thick Perspex sheets. The provision of tilting arrangements of the flumes has provided different longitudinal slopes for the experimental runs. The photograph of one of the run of a meandering channel with measuring equipments taken from the downstream side is now shown in Photo 1.

All the runs of the experiments are carried out by a re-circulating water system. Water is lifted from underground sump to the overhead tank with the help of some parallel pump arrangement. From this overhead tank, water is led to the stilling tank that is located at the upstream of the channel. For reducing the turbulence of incoming water, a series of baffle walls are provided in between the stilling tank and channels. Water is allowed to flow through the channel and then passed through the tail gate of channel and finally collected in a masonry volumetric tank from where the flow it is turned back to the underground sump. From the sump, again water is pumped back to the overhead tank; thus, the setting covers a complete re-circulating system of water supply to the experimental channel. The tilting type of tailgate at the downstream end is there to establish uniform flow of water through the channel. Each experimental runs of the channel are carried out by maintaining the water surface slope parallel to the valley slope to achieve the steady and uniform flow conditions. This is explained in a better way with a sketch diagram (Fig. 1).

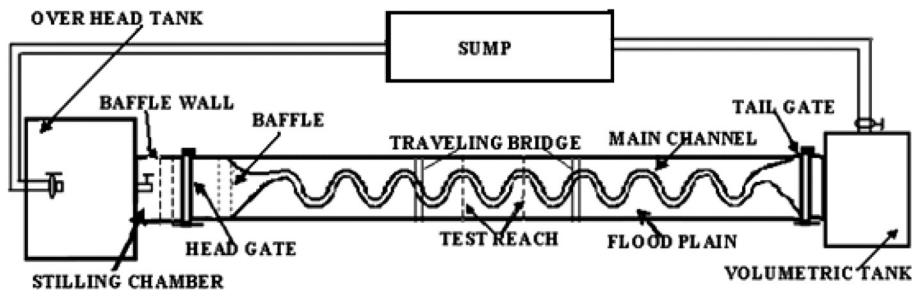Photo 1: Details of meandering channel with the measuring equipments.

Figure 1: Water supply to meandering channel with measuring equipments.

Table 1: Details of geometrical parameters of the experimental channels.

Item description	Present experimental channels	
Channel type	Meandering channel (Type I)	Meandering channel (Type II)
Flume size	4.0 m × 15 m × 0.5 m long	4.0 m × 15 m × 0.5 m long
Geometry of channel section	Trapezoidal (side slope 1:1)	Trapezoidal (side slope 1:1)
Nature of surface of bed	Smooth and rigid bed	Smooth and rigid bed
Channel width	33 cm at bottom and 46 cm at top	33 cm at bottom and 46 cm at top
Bank full depth of channel	6.5 cm	6.5 cm
Bed slope of the channel	0.0022	0.0022
Sinuosity	1.12	1.25
Amplitude	951 mm	1568 mm
Wave length in down valley direction	3972 mm	3944 mm

At the top of the experimental flume, main guide rails are provided on which a traveling bridge is moved in the longitudinal direction of the entire experimental channel. The point gauge and five micro-Pitot tubes each having 4.6-mm external diameter are attached to the traveling bridge with secondary guide rails allowing the equipments to move in both longitudinal and the transverse directions of the experimental channel. Water surface slope measurement is carried out using a pointer gauge fitted to the traveling bridge. It is having least count of 0.1 mm and operated manually. Slopes of all meandering channels are obtained by dividing the level difference of channel bed and water surface between the two points by the length of meander wave along the centerline.

There are five Pitot tubes that are in conjunction with suitable five vertical manometers used to measure point velocity and its direction of flow at the predefined points of the flow-grid. The Pitot tube is physically rotated normal to the main stream direction till it gives maximum deflection of manometer reading. The angle of limb of Pitot tube with longitudinal direction of the channel was noted by the circular scale and pointer arrangements attached to the flow direction meter. A rectangular notch has also been fabricated and placed at the u/s face, which is calibrated to find the continuous stage–discharge relationships of the channels. Velocity measurements using these Pitot tubes are taken at the bend apex in meandering channel where minimum curvature effect is there. Actual discharges of the flow of water through the meandering channels were measured by the time rise method in the measuring tanks located at the downstream end of the experimental channel. All the observations are performed only under steady and uniform conditions. The geometrical parameters and hydraulic details of the experimental runs are given in Table 1.

4 ANALYSIS OF EXPERIMENTAL RESULTS

4.1 Variation of Chezy's C

The experimental results shown in Tables 2 and 3 belong to meandering channels Type I and Type II, respectively. The experiments are conducted in meandering channels of different sinuosities namely $S_r = 1.12$ corresponds to Type I and $S_r = 1.25$ corresponds to Type II channel. The experimental obser-

Table 2: Details of experimental observations (Type I).

Run no.	Q (m³/s)	H (m)	A (m²)	C
MT1	0.0013	0.018	0.0063	58.809
MT2	0.0016	0.019	0.0066	58.076
MT3	0.0019	0.021	0.0072	56.033
MT4	0.0023	0.024	0.0083	54.844
MT5	0.0026	0.025	0.009	53.947
MT6	0.0032	0.029	0.0104	51.214
MT7	0.0041	0.034	0.0122	50.686
MT8	0.0051	0.038	0.0141	49.047
MT9	0.0056	0.041	0.0151	48.196
MT10	0.0062	0.043	0.0162	47.867
MT11	0.0064	0.044	0.0166	47.283
MT12	0.0078	0.050	0.0190	46.282
MT13	0.0085	0.053	0.0201	43.108
MT14	0.0108	0.061	0.0239	36.451
MT15	0.0110	0.062	0.0244	35.266
MT16	0.0110	0.063	0.0247	35.183
MT17	0.0118	0.065	0.0254	35.059

Table 3: Details of experimental observations (Type II).

| Run no.	Q (m³/s)	H (m)	A (m²)	C
MT1	0.0014	0.016	0.0055	56.356
MT2	0.0022	0.021	0.0073	55.286
MT3	0.0032	0.027	0.0096	53.762
MT4	0.0033	0.028	0.0100	52.963
MT5	0.0039	0.031	0.0112	52.012
MT6	0.0044	0.033	0.0120	51.554
MT7	0.0052	0.037	0.0135	48.761
MT8	0.0054	0.038	0.0138	48.319
MT9	0.0064	0.042	0.0157	46.473
MT10	0.0062	0.045	0.0155	45.629
MT11	0.0082	0.050	0.0189	38.964
MT12	0.0090	0.053	0.0201	37.257
MT13	0.0093	0.054	0.0206	36.690
MT14	0.0099	0.056	0.0216	35.614
MT15	0.0103	0.057	0.0221	34.525
MT16	0.0117	0.062	0.0243	33.914
MT17	0.0124	0.063	0.0254	33.457
MT18	0.0121	0.066	0.0251	32.400
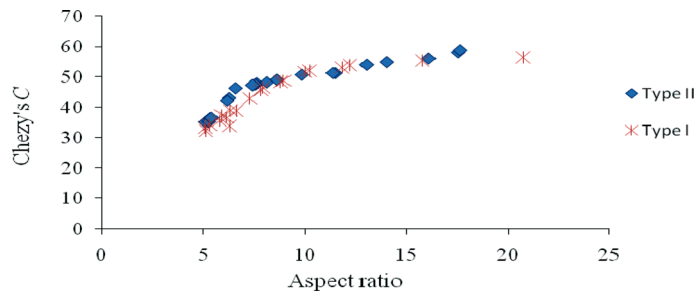
Figure 2: Variation in Chezy's *C* with aspect ratio.

vations for both the meandering channels are recorded and presented here in Tables 2 and 3, respectively. Variation in Chezy's *C* with depth of flow (*H*) for these channels was investigated as shown in Fig. 2. The value of *C* for both the experimental meandering channels exhibits an increasing trend with aspect ratio (ratio of width of the channel to the depth of flow). It confirmed that meandering channel losses more energy during its flow process. Variation in friction factor with a depth of flow (H) for the same channels was also investigated, which has already been reported in Moharana *et al.* [17].

5 MODEL DEVELOPMENT

In this present work, an efficient technique such as ANFIS is proposed for the prediction of Chezy's *C*. An adaptive fuzzy system named as ANFIS stands for adaptive neuro fuzzy inference system. It is the combination of ANNs and fuzzy system [18]. Adaptive-network-based fuzzy inference system is a novel architecture that develops fuzzy if–then rules with suitable membership functions to have the required input–output pairs. This system is a fuzzy Sugeno by a forwarding network structure. This fuzzy model was first suggested systematically by Sugeno and Takagi [19]. This popular approach has been found in many practical applications including decision analysis, forecasting, operation management and control prediction system.

5.1 ANFIS architecture

The first-order Sugeno system optimizes the fuzzy system by using a hybrid learning rule. An example ANFIS system having two inputs *x* and *y* and one output is taken in this system. Its rule is given by Tashnehlab and Menhaj [20]

$$\text{If } x \text{ is the } A_1 \text{ and } y \text{ is the } B_1, \text{ then } f = p_1 x + q_1 y + r_1$$
$$\text{If } x \text{ is the } A_2 \text{ and } y \text{ is the } B_2, \text{ then } f = p_2 x + q_2 y + r_2 \tag{2}$$

where *C* is the output and p_i, q_i and r_i are the consequent parameters of the *i*th rule. A_i and B_i are the linguistic labels that are represented by fuzzy sets whose membership function parameters are premise parameters [21]. From ANFIS architecture, it is observed that the given values of the premise parameters and the overall output can be expressed as a linear combination of the consequent parameters. Basically, a fuzzy inference system is composed of five functional blocks as shown in Fig. 3.
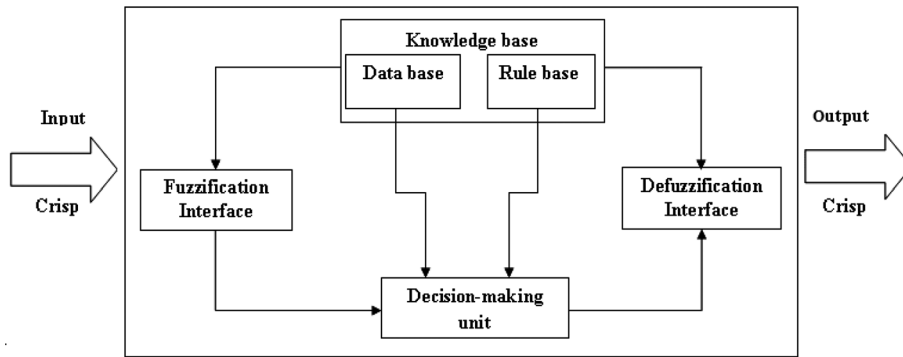
Figure 3: Block diagram of an ANFIS system.

6 SOURCE OF DATA AND SELECTION OF PARAMETERS

For simulation of discharge in meander river, a proper value of roughness coefficient is very essential. In the present study, taking different geometric and hydraulic conditions into our consideration the influencing non-dimensional parameters such as bed slope of the channel (S), sinuosity (S_r), aspect ratio (a), Reynolds Number (Re), roughness ratio (γ) and side slope (tangent of angle θ with vertical) are taken as inputs for the model and roughness coefficient is taken as the output. Sinuosity (S_r) is the ratio of channel length to valley length, which refers to sinuous path of a channel. Aspect ratio (a) is the ratio of the channel bottom width (b) to flow depth (h) in the channel. The data set was collected from several references such as Shiono *et al.* [3], Patra and Kar [4], Khatua *et al.* [7], Channel of Willets and Hardwick [22], Al-Romaih [23] and FCF channel [24]. Along with the present experimental results, some literatures are also reviewed and finally a data set is prepared as given in Table 4.

7 RESULTS

An equivalent ANFIS system is presented in Fig. 4a and a membership function used is given in Fig. 4b. In train epoch, the outputs of nodes (Fig. 4c) is calculated at the fourth layer, moving forward and then in next step moving backward, the ratio of error is propagated over the parameters based on error descent gradient method according to Rumelhart *et al.* [26]. In this section, to verify the accuracy of the results further regression analysis is also carried out. Regression curves are plotted showing the difference between actual Chezy's *C* and predicted Chezy's *C* through ANFIS model for training data and testing data, respectively. By using the testing data set, roughness value is calculated. It can be observed that the data are well fitted because a high degree of coefficient of determination (R^2) as 0.913 for training (Fig. 5) and as 0.881 for testing data (Fig. 6) is obtained. Comparison of actual Chezy's *C* and predicted Chezy's *C* with different methods is shown in Fig. 7. From the regression analysis, it can be observed that ANFIS method showed maximum accuracy.

The performance of developed ANFIS model results for the training and testing phase is shown in Figs 7 and 8. Model results show a very good agreement with the actual results in predicting roughness values. As the predicted data pattern follows actual data with little or no exception, it demonstrates that the model predicts the pattern of the data distribution with adequate accuracy because a high degree of coefficient of determination is obtained for training and testing data.

Table 4: Geometrical parameters of data set.

| Sl. no.	Source of data	Bed slope of channel (S)	Sinuosity (S_r)	Aspect ratio (α)	Reynolds No (Re)	Roughness ratio (γ)	Side slope (θ)
				Parameters			
1	Present experimental channels [17]	0.0022	1.12–1.25	5.09–20.75	21,302–353,493	1	45
2	Hydraulics Lab. at NIT Rkl [7]	0.0019–0.021	1–1.91	1.04–10.91	23,665–349,516	1	45–90
3	Hydraulics Lab. at IIT Kgp [4]	0.001–0.0061	1.043–1.438	0.49–5.12	56,759–779,959	1–2	90
4	University of Bradford, UK [23]	0.0005–0.002	1.092–1.571	2.89–24.48	41,549–527,297	1	45
5	FCF channel (Series A) [24]	1.92817–0.000996	1–1.21	4.47–22.62	62,704–802,467	1–1.8	45
6	SERC Flood Facility at HR Wallingford, UK [3]	0.00073	1.37	6.82–15.25	323,225–425,255	1	45
7	Channel of Willets and Hardwick [22]	0.000621	2.04	2.78	203,477–654,231	1	45
8	Series B extension, Glasgow [25]	0.00073	1.374	2.67–15.69	564,532–652,387	1–2	30–90
9	Natural River Channel (non-cohesive sand) [24]	0.000996–0.001021	1.3–1.5	6–15	238,452–673,284	2.8	45
10	Natural River Channel (cohesive sand) [24]	0.000996–0.001021	1.4–1.8	6–15	218,752–773,514	2	45

(a)

(b)

(c)
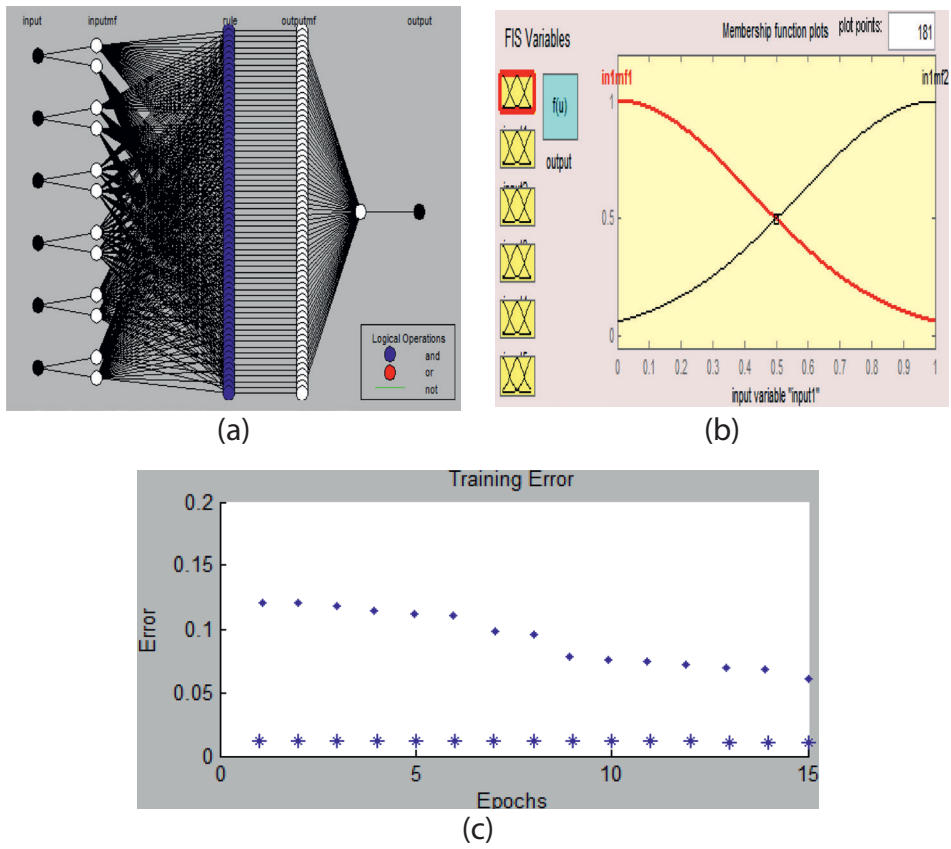
Figure 4: (a) Architecture of developed model; (b) membership function for ANFIS output; and (c) training error with epochs.
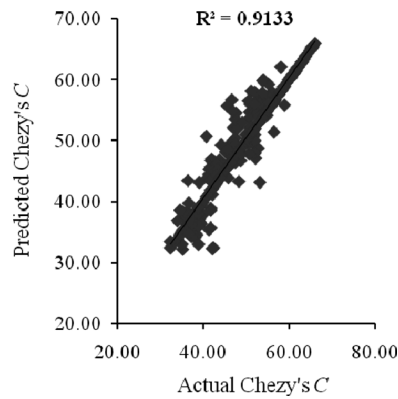

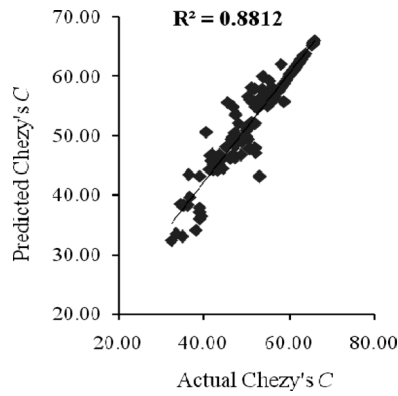Figure 5: Distribution of training data.
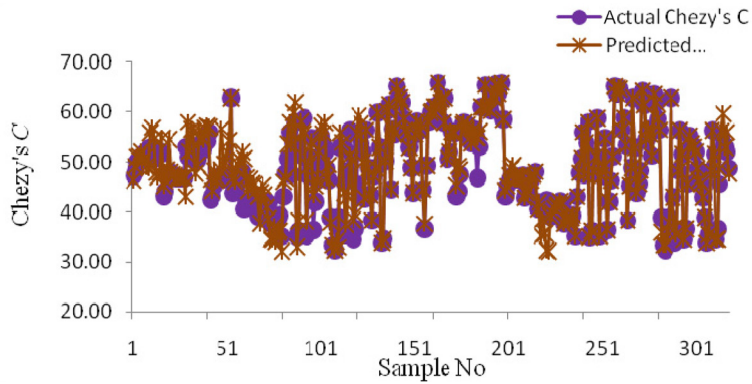
Figure 6: Distribution of testing data.

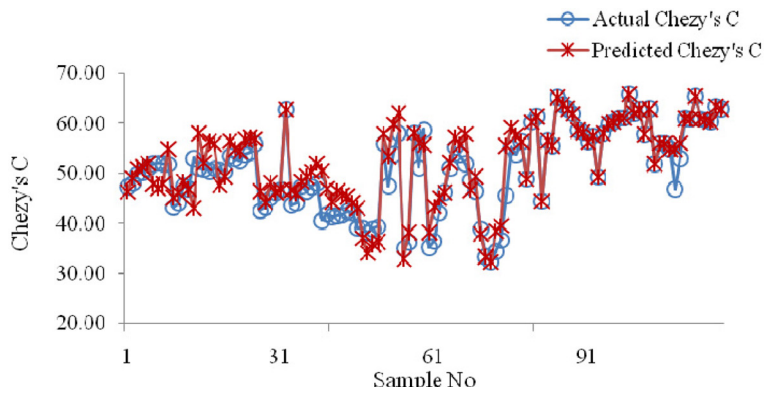Figure 7: Matching figure of actual and ANFIS model results for *C* in training step.

Figure 8: Matching figure of actual and ANFIS model results for *C* in testing step.

8 DISCUSSION

Roughness coefficient is calculated with appropriate hydraulic parameters using standard methods such as Shiono and Knight method (1999), Khatua *et al.* method (2011), LSCS method (1992), and comparison with ANFIS model is done by plotting the correlation plots. By using the testing data set, Chezy's *C* value is calculated for different standard methods. Comparison of actual Chezy's *C* with predicted Chezy's *C* for different methods is shown in Fig. 9. In addition to this, to check the efficiency of the newly developed model different statistical analyses are conducted by the global data set. The results of statistical analysis are given in Table 5.

From Fig. 9, it is clear that Khatua *et al.* method (1999) gives good results showing coefficient of determination value 0.802 as compared with the other two established methods LSCS method (1992) and Shiono and Knight method (1999) having $\underline{R}^2$ 0.742 and 0.685, respectively. Here also ANFIS model shows good performance having the coefficient of determination value as 0.881. From comparison of different methods, it is definitely concluded that ANFIS model shows very good results among all the existing methods for the prediction of roughness coefficients in terms of Chezy's *C*.
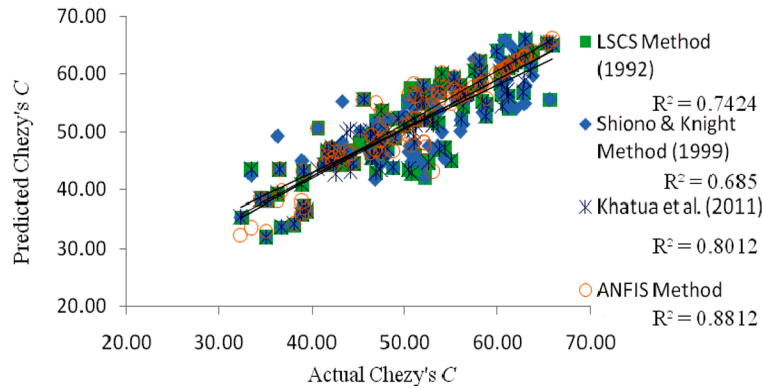

Figure 9: Comparison with different methods.

Table 5: Error analysis of different methods in prediction of *C*.

Statistical parameters	LSCS method (1992)	Shiono and Knight method (1999)	Khatua *et al.* method (2011)	ANFIS method
MAE	0.00783	0.00995	0.00648	0.00538
MSE	0.0000653	0.0000872	0.00003841	0.00002649
RMSE	0.008081	0.009338	0.006198	0.005147
MAPE	26.824	32.261	23.11	16.801
R^2	0.742	0.685	0.802	0.881

9 CONCLUSIONS

An experimental investigation was carried out to study the influence of sinuosity in determination of roughness coefficients of a meandering channel. An ANFIS approach is applied for proper estimation of roughness value in terms of Chezy's *C* in a meandering channel. Different standard methods to predict the roughness coefficients of a meandering channel were also tested and verified with the laboratory findings and global data as well. The present ANFIS model performed better than other discussed methods. The reason may be that the traditional methods cannot establish nonlinear relationship between geometrical and hydraulic input parameters associated with roughness coefficients, whereas the ANFIS method could establish. Among the discussed methods, the Khatua *et al.* method (2011) and LSCS method (1992) overestimate roughness value, whereas Shiono and Knight method (1999) underestimates it. Statistical error analysis is carried out from which it is obvious that ANFIS model performed more accurately giving results by less error than different existing strategies. The analysis shows a high level of accuracy with regard to the ANFIS-based model developed for predicting the Chezy's *C* especially coefficients of determination found to be 0.881.

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