EFFECT DOUBLE STRATIFICATION ON MIXED CONVECTION IN A POWER-LAW FLUID SATURATED POROUS MEDIUM

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ABSTRACT

Mixed convection heat and mass transfer along a vertical plate embedded in a power-law fluid saturated Darcy porous medium with thermal and solutal stratification effects is studied. The governing partial differential equations are transformed into ordinary differential equations using similarity transformations and then solved numerically using shooting method. The non-dimensional heat and mass transfer coefficient are presented graphically for various values of power-law index, Lewis number, thermal and solutal stratification parameters.

1. Introduction

The mixed convection boundary layer flow along a vertical surface embedded in porous media has received considerable theoretical and practical interest. The mixed convection flow occurs in several industrial and technical applications such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. A number of studies have been reported in the literature focusing on the problem of mixed convection about different surface geometries in porous media. A review of convective heat transfer in porous medium is presented in the book by Nield and Bejan [1]. The majority of these studies dealt with the traditional Newtonian fluids. It is well known that most fluids which are encountered in chemical and allied processing applications do not satisfy the classical Newton's law and are accordingly known as non-Newtonian fluids. Due to the important applications of non-Newtonian fluids in biology, physiology, technology, and industry, considerable efforts have been directed towards the analysis and understanding of such fluids. A number of mathematical models have been proposed to explain the rheological behavior of non-Newtonian fluids. Among these, a model which has been most widely used for non-Newtonian fluids, and is frequently encountered in chemical engineering processes, is the power-law model. Although this model is merely an empirical relationship between the stress and velocity gradients, it has been successfully applied to non-Newtonian fluids experimentally.

The prediction of heat or mass transfer characteristics for mixed or natural convection of non-Newtonian fluids in porous media is very important due to its practical engineering applications, such as oil recovery and food processing. Abo-Eldahab and Salem [2] studied the problem of laminar mixed convection flow of non-Newtonian power-law fluids from a constantly rotating isothermal cone or disk in the presence of a uniform magnetic field. Kumari and Nath [3] considered the conjugate mixed convection conduction heat transfer of a non-Newtonian power-law fluid on a vertical heated plate which is moving in an ambient fluid. Degan et al. [4] presented an analytical method to investigate transient free convection boundary layer flow along a vertical surface embedded in an anisotropic porous medium saturated by a non-Newtonian fluid. Chamkha and Al-Humoud [5] studied mixed convection heat and mass transfer of non-Newtonian fluids from a permeable surface embedded in a porous medium under uniform surface temperature and concentration species. Chen [6] considered the problem of magneto hydrodynamic mixed convective flow and heat transfer of an electrically conducting, power-law fluid past a stretching surface in the presence of heat generation/absorption and thermal radiation. Elgazery and Abd Elazem [7] analyzed numerically a mathematical model to study the effects of a variable viscosity and thermal conductivity on unsteady heat and mass transfer in a non-Newtonian power-law fluid flow through a porous medium past a semi-infinite vertical plate with variable surface temperature in the presence of magnetic field and radiation. Effect of double dispersion on mixed convection heat and mass transfer in a non-Newtonian fluid-saturated non-Darcy porous medium has been investigated by by Kairi and Murthy [8]. Chamkha et al. [9] studied the effects of melting, thermal radiation and heat generation or absorption on steady mixed convection from a vertical wall embedded in a non-Newtonian power-law fluid saturated non-Darcy porous medium for aiding and opposing external flows. Hayat et al. [10] investigated the Magneto hydrodynamic (MHD) mixed convection stagnation-point flow and heat transfer of power-law fluids towards a stretching surface using the homotopy analysis method (HAM).

Stratification of fluid arises due to temperature variations, concentration differences, or the presence of different fluids.

In practical situations where the heat and mass transfer mechanisms run parallel, it is interesting to analyze the effect of double stratification (stratification of medium with respect to thermal and concentration fields) on the convective transport in power-law fluid. The analysis of natural convection in a doubly stratified medium is a fundamentally interesting and important problem because of its broad range of engineering applications. The applications include heat rejection into the environment such as lakes, rivers and the seas; thermal energy storage systems such as solar ponds and heat transfer from thermal sources such as the condensers of power plants. Although the effect of stratification of the medium on the heat removal process in a fluid is important, very little work has been reported in the literature. Jumah and Mujumdar [11] studied the free convection heat and mass transfer of non-Newtonian power law fluids with yield stress from a vertical flat plate in saturated porous media. Murthy et al.[12] discussed the effect of double stratification on free convection heat and mass transfer in a Darcian fluid saturated porous medium using the similarity solution technique for the case of uniform wall heat and mass flux conditions. Lakshmi Narayana and Murthy [13] analyzed the free convection heat and mass transfer from a vertical flat plate in a doubly stratified non-Darcy porous medium using series solution technique. Cheng [14] discussed the combined heat and mass transfer in natural convection flow from a vertical wavy surface in a power-law fluid saturated porous medium with thermal and mass stratification. Recently, Postelnicu et al. [15] analyzed the free convection heat and mass transfer in a doubly stratified porous medium saturated with a power-law fluid.

Motivated by the investigations mentioned above, the purpose of the present work is to investigate the thermal and solutal stratification effects on mixed convection heat and mass transfer from vertical plate in Darcy porous media saturated with power-law fluid with variable surface temperature and concentration conditions.

2. MATHEMATICAL FORMULATION

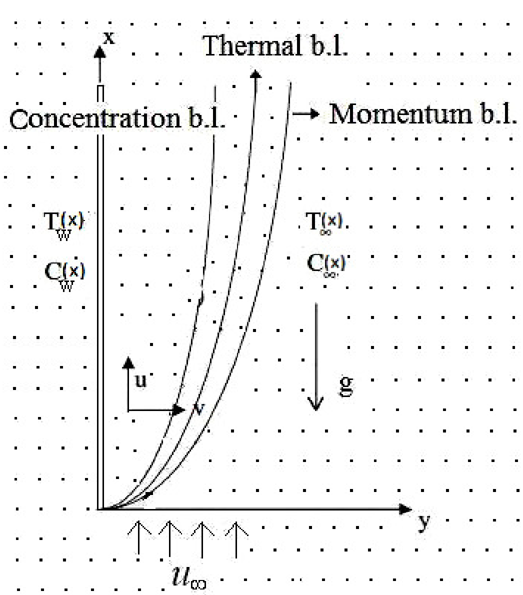

Figure1: Flow model and physical coordinate system.

Consider the mixed convection heat and mass transfer along a vertical plate in a non-Newtonian power-law fluid saturated Darcy porous medium. Choose the coordinate system such that *x*- axis is along the vertical plate and *y*-axis normal to the plate. The plate is maintained at variable temperature and concentration, $T_w(x)$ and $C_w(x)$ respectively. The temperature and concentration of the ambient medium are $T_\infty(x)$ and $C_\infty(x)$ respectively as shown in Fig.1. Assume that the fluid and the porous medium have constant physical properties except for the density variation required by the Boussinesq approximation. The flow is steady, laminar, two dimensional. The porous medium is isotropic and homogeneous. The fluid and the porous medium are in local thermo dynamical equilibrium. In addition the thermal and solutal stratification effects are taken in to consideration. The ambient medium is assumed to be vertically non-linearly stratified with respect to both temperature and concentration in the form $T_\infty(x)= T_{\infty,0}+G\, x^l$ and $C_\infty(x)= C_{\infty,0}+H\, x^m$ respectively, where G and H are constants and varied to alter the intensity of stratification in the medium.

Using the Boussinesq and boundary layer approximations, the governing equations for the power law fluid are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u^n=u_\infty^n+\frac{gK}{\upsilon}\left(\beta_T\left(T-T_\infty\right)+\beta_C\left(C-C_\infty\right)\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where u and v are the Darcian velocity components along x and y directions, T is the temperature, C is the concentration, υ is the kinematic viscosity, K is the permeability, g is the acceleration due to gravity, β_T is the coefficient of thermal expansion, β_C is the coefficient of concentration expansion, α_m is the thermal diffusivity, D_m is the mass diffusivity of the porous medium, n is the power-law index. When $n = 1$, the Eq. (2) represents a Newtonian fluid. Therefore, deviation of n from a unity indicates the degree of deviation from Newtonian behavior. For $n < 1$, the fluid is shear thinning and for $n > 1$, the fluid is shear thickening.

The boundary conditions are

$$v=0,\ T=T_W(x),\ C=C_w(x) \quad \text{at } y=0 \tag{5a}$$

$$u=u_\infty,\ T\to T_\infty(x),\ C\to C_\infty(x) \quad \text{as } y\to\infty \tag{5b}$$

where the subscripts w and ∞ indicate the conditions at the wall and at the outer edge of the boundary layer respectively.

It is noticed that the similarity transformations are possible only when the variation in the temperature and concentration of the plate are in the form $(T_w(x) - T_{\infty,0}) = E\,x^{n/3}$ and $(C_w(x) - C_{\infty,0}) = F\,x^{n/3}$ respectively and the temperature and concentration stratifications are in the form $T_\infty(x) = T_{\infty,0} + Gx^{n/3}$ and $C_\infty(x) = C_{\infty,0} + Hx^{n/3}$ respectively.

In view of the continuity eq.(1), we introduce the stream function ψ by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

Substituting eq.(6) in eqs.(2),(3) and (4) and then using the following similarity transformations

$$\left.\begin{aligned} &\eta = B\,y\,x^{-1/3}, \quad \psi = A\,x^{2/3} f(\eta), \\ &\theta(\eta) = \frac{T - T_\infty(x)}{T_w(x) - T_{\infty,0}}, \; T_w(x) - T_{\infty,0} = E\,x^{n/3} \\ &\phi(\eta) = \frac{C - C_\infty(x)}{C_w(x) - C_{\infty,0}}, \; C_w(x) - C_{\infty,0} = F\,x^{n/3} \end{aligned}\right\} \tag{7}$$

we get the following nonlinear system of differential equations.

$$(f')^n = (1 + \theta + N\phi) \tag{8}$$

$$\theta'' = \frac{1}{3}\left(n\,f'\theta - 2\,f\,\theta' + \varepsilon_1 f'\right) \tag{9}$$

$$\phi'' = \frac{Le}{3}\left(n\,f'\phi - 2\,f\,\phi' + \varepsilon_2 f'\right) \tag{10}$$

where primes denote differentiation with respect to η alone.

$\varepsilon_1 = \frac{nG}{E}$ is the thermal stratification parameter ,

$\varepsilon_2 = \frac{nH}{F}$ is the solutal stratification parameter,

$N = \frac{\beta_C F}{\beta_T E}$ is the buoyancy ratio,

$Le = \frac{\alpha_m}{D_m}$ is the Lewis number.

Making use of dimensional analysis, we get

$$A = \left(\frac{\rho g E K \beta_T \alpha_m^n}{\mu}\right)^{1/2n} \quad \text{and} \quad B = \left(\frac{\rho g E K \beta_T}{\mu \alpha_m^n}\right)^{1/2n}$$

The boundary conditions (5) in terms of *f, θ,* and *ϕ* become

$$f = 0, \; \theta = 1, \; \phi = 1 \quad \text{at} \quad \eta = 0 \tag{11a}$$

$$f^1 = 1, \; \theta = 0, \; \phi = 0 \quad \text{as} \quad \eta \to \infty \tag{11b}$$

The parameters of engineering interest for the present problem are the Nusselt and Sherwood numbers, which are given by the expressions

$$\frac{Nu_x}{Bx^{2/3}} = -\theta'(0) \quad \text{and} \quad \frac{Sh_x}{Bx^{2/3}} = -\phi'(0) \tag{12}$$

3. SHOOTING METHOD NUMERICALPROCEDURE

The flow Eq.(8) coupled with the energy and concentration Eqs.(9) and (10) constitute a set of nonlinear non-homogeneous differential equation for which closed-form solution cannot be obtained. Hence the problem is solved numerically using shooting technique along with fourth order Runge-Kutta integration. The basic idea of shooting method for solving boundary value problem is to try to find appropriate initial condition for which the computed solution “hit the target” so that the boundary conditions at other points are satisfied. Furthermore, the higher order non-linear differential equations are converted into simultaneous linear differential equations of first order and they are further transformed into initial valued problem applying the shooting method incorporating fourth order Runge-Kutta method. The iterative solution procedure was carried out until the error in the solution became less than a predefined tolerance level.

The non-linear differential equations (10) - (12) are converted into in to the system of first order linear differential equations and then integrated using the fourth order Runge - Kutta method from η = 0 to $\eta = \eta_{max}$ over successive steps Δη by giving appropriate initial guess values for for $f^I(0)$, $\theta^I(0)$ and $\phi^I(0)$ as they are not specified in (13). Here η_{max} is the value of η at ∞ and chosen large enough so that the solution shows little further change for η larger than η_{max}. The accuracy of the assumed initial values $f^I(0)$, $\theta^I(0)$ and $\phi^I(0)$ is then checked by comparing the calculated values of $f^I(0)$, $\theta^I(0)$ and $\phi^I(0)$ at $\eta = \eta_{max}$ with their given value in (11). If a difference exists, another set of initial values for $f^I(0)$, $\theta^I(0)$ and $\phi^I(0)$ are assumed and the process is repeated until the agreement between the calculated and the given condition at $\eta = \eta_{max}$ is within the specified degree of accuracy. In order to see the effects of step size (Δη) we ran the code for our model with three different step sizes as Δη = 0.01, Δη = 0.001 and Δη = 0.005 and in each case we found very good agreement between them. A step size of Δη = 0.01 is selected to be satisfactory for a convergence criterion of 10^{-6} in all cases. In the present study, the boundary conditions for η at ∞ vary with parameter values and it is suitably chosen at each time such that the velocity approach to one and temperature and concentration approach zero at the outer edge of the boundary layer. Extensive calculations are performed to obtain the wall velocity, temperature and concentration fields for a wide range of parameters.

4. RESULTS AND DISCUSSION

The non-dimensional heat and mass transfer coefficients (Nu_x and Sh_x)) is plotted against power-law index (*n*) for different values of thermal and solutal stratification parameters in Figs.2-5 with *N* = *1, Le* = 1. It is observed from Fig.2 that the increasing the power -law index (*n*), increases the Nusselt number (Nu_x), but increasing the value of thermal stratification parameter (ε_1), Nusselt number is decreased. It is noted from Fig.3 that the increasing the power-law index (*n*), increases the Sherwood number (Sh_x), but increasing the value of thermal stratification parameter decreases Sh_x. It is noted from Fig.4 that the Nusselt number (Nu_x) is increased with increasing the power -law index (*n*), but increasing the value of solutal stratification parameter (ε_2) decreases Nu_x. Fig.5 demonstrates that increasing the power-law index (*n*), increases the Sherwood number (Sh_x), but increasing the value of solutal stratification parameter Sherwood number is decreased.

Figure-6 illustrates the variation of heat transfer coefficient (Nusselt number, Nu_x) with Lewis number (*Le*) for different values of power-law index (*n*) and $N = 1$, $\varepsilon_1=0.5$ and $\varepsilon_2=0.5$. It is observed from the figure that increasing Lewis number decreases the Nusselt number, but increasing the values of power law index increases Nusselt number (Nu_x).

Fig.7 depict the variation of mass transfer coefficient (Sherwood number, Sh_x) with Lewis number (*Le*) for different values of power -law index (*n*) and $N = 1$, $\varepsilon_1=0.5$ and $\varepsilon_2=0.5$. It is observed from the figure that increasing Lewis number increases the Sherwood number. Also, increasing the power -law index increases Sh_x.

5. CONCLUSIONS

In this paper, mixed convection heat and mass transfer along a vertical plate embedded in a power-law fluid saturated Darcy porous medium in presence thermal and solutal stratification has been considered. The wall is maintained at variable temperature and concentration $T_w(x)$ and $C_w(x)$ respectively. It can be concluded from the present analysis that the increasing the of thermal stratification parameter decreases the Nusselt number and Sherwood number. The same trend is observed in case of solutal stratification parameter increases the Nusselt and Sherwood numbers decreases. An increase in the values of the power-law index parameter Nusselt and Sherwood numbers increased. Also, the higher value of Lewis number Nusselt number decreases, but Sherwood number increases. It is also observed that as Lewis number increases, that is, the thermal boundary layer thickness increases and the concentration boundary layer thickness decreases rapidly. The Lewis number has a more pronounced effect on the concentration field than on the temperature field.

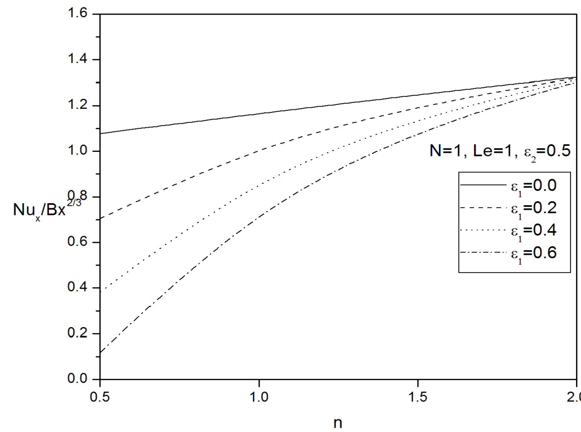

Figure 2. Variation of non-dimensional heat transfer coefficient with *n* for varying ε_1 for $N=1$, $Le=1$, $\varepsilon_2=0.5$.

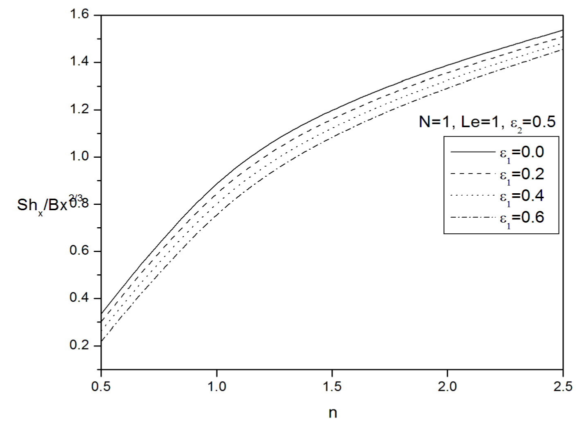

Figure 3. Variation of non-dimensional mass transfer coefficient with *n* for varying ε_1 for $N=1$, $Le=1$, $\varepsilon_2=0.5$.

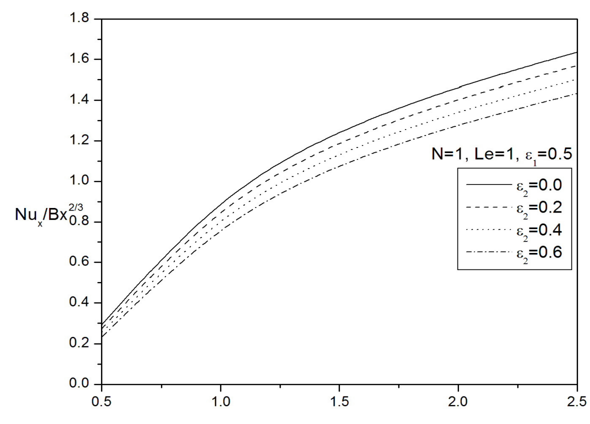

Figure 4. Variation of non-dimensional heat transfer coefficient with *n* for varying ε_2 for $N=1$, $Le=0.5$, $\varepsilon_1=0.5$.

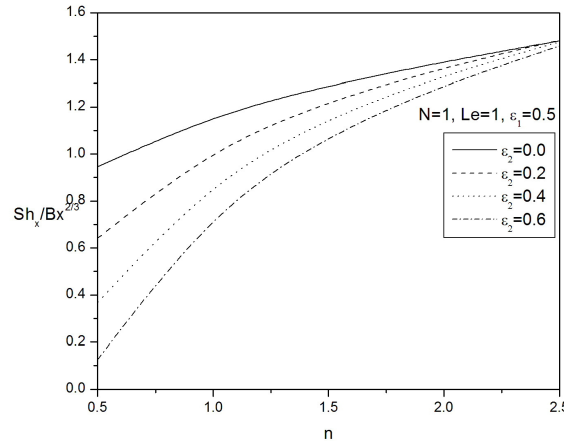

Figure 5. Variation of non-dimensional mass transfer coefficient with *n* for varying ε_2 for $N = 1$, $Le = 1$, $\varepsilon_1 = 0.5$.

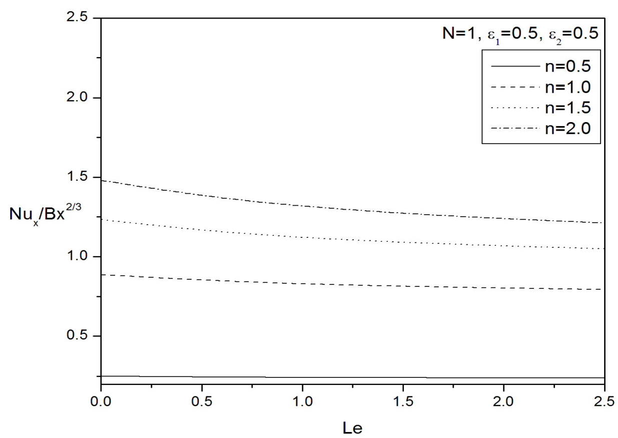

Figure 6. Variation of non-dimensional heat transfer coefficient with *Le* for varying *n* for $N=1$, $\varepsilon_1=0.5$ and $\varepsilon_2=0.5$.

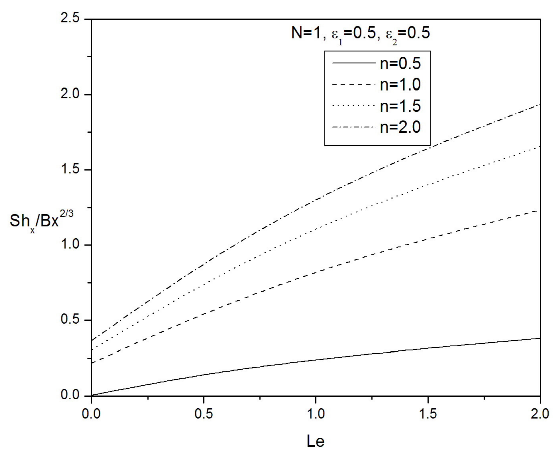

Figure 7. Variation of non-dimensional mass transfer coefficient with *Le* for varying *n* for $N=1$, $\varepsilon_1=0.5$ and $\varepsilon_2=0.5$

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Nomenclature

A	dimensional constant. (-)
B	dimensional constant. (-)
G	Slope of ambient temperature.(-)
H	Slope of ambient concentration.(-)
C	Concentration. ($kmolm^{-3}$)
C_w	Wall concentration. ($kmolm^{-3}$)
$C_{\infty,0}$	Ambient concentration. ($kmolm^{-3}$)
D_m	Mass diffusivity. ($m^2 s^{-1}$)
N	Buoyancy ratio. (-)
g	Gravitational acceleration. (ms^{-2})
K	Darcy Permeability. (m^{n+1})
k_T	Thermal diffusion ratio. (-)
Le	Lewis number. (-)
Nu_x	Local Nusselt number. (-)
n	Power-law index. (-)
Sh_x	Local Sherwood number. (-)
k	Thermal conductivity. (-)
K_p	Permeability of porous medium.
Nu_x	Local Nusselt number. (-)
T	Temperature. (K)
T_m	Mean temperature. (K)
T_w	Wall temperature. (K)
T_∞	Ambient temperature. (K)
u, v	Velocity components in x and y directions. (ms^{-1})

Greek Letters

α_m	Thermal diffusivity. ($m^2 s^{-1}$)
β_T, β_C	Coefficients of thermal and solutal expansion. (K^{-1})
η	Similarity variable. (m)
θ	Dimensionless temperature. (-)
ϕ	Dimensionless concentration. (-)
μ	Dynamic viscosity. ($kgm^{-1}s^{-1}$)
υ	Kinematic viscosity. ($m^2 s^{-1}$)
ρ	Density of the fluid. (kgm^{-3})
ψ	Stream function. (-)
ε_1	Thermal stratification parameter.(-)
ε_2	solutal stratification parameter.(-)

Subscripts

w	Wall condition
∞	Ambient condition
C	Concentration
T	Temperature

Superscript

$'$	Differentiation with respect to η