AN APPROACH TO INVESTIGATE THE CHARACTERISTICS OF HIGH SPEED TURBO MACHINES

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ABSTRACT

The article proposes a method to investigate some of the essential characteristics of high speed rotating machinery. The underlying principle of power versus speed dependence of a rotating machine is defined by a second order differential equation with dependent and independent variables respectively as power and speed. The solution of such an equation together with the dynamic relation of flow of the medium would enable further to generate the essential characteristics specific power of the compressible medium of the turbo machine under consideration.

1. INTRODUCTION

The performance deterioration directly affects the profitability and operation of gas turbine power plants. Performance deterioration includes both recoverable and non-recoverable deterioration and various approaches monitoring the both steady and transient behavior of gas turbine deterioration models were studied by cyrus B.Meher Homji et.al **[1].** Gas turbine operates under large varying operating conditions and the manufacturers very rarely supply performance characteristics of each component to users hence a mathematical model based on analytical and neural networks for gas turbine off design was proposed by Andrea lazzaretto et.al. **[2]**. The fuel used was steam or patented gas and the calculation of the thermodynamic properties of the working medium was performed according to the theoretical relations in Moran and Shapiro **[3]** and Keating **[4],** while enthalpy and entropy values for each species were obtained from the Ulrich Grigull, Springer-Verlag, Berlin Heidelberg, New York**[5]**.

The performance degradation of multistage compressor operating under prolonged choked conditions were studied by Rainer Kurz et. al **[6]**. Gas turbine technology for industrial needs and other on shore power generation requirements is well established. The performance characteristics are well investigated. However as a prime mover in underwater medium it serves as a strategic tool with variable speed and different load conditions. Besides its performance coupled with compactness is also one of the primary considerations. The functionality of a high speed turbo machine is generally rated by specific power. To generate the necessary relationships between power and speed a conceptual relationship is assumed in differential form as follows:

$$A_o N^2 \left[\frac{d^2W}{dN^2}\right] + B_o N \left[\frac{dW}{dN}\right] + C_o W = 0 \qquad (1)$$

where W is the power and N is the speed. The constants A_o, B_o and C_o can be considered as the designers assigned constants which may be dependent on the specific design considerations of the unit. Subsequently the results employed for classification is specific power. Equation (1) satisfies dimensional homogeneity and it can be seen to be a functional relationship between W = F(N) where W is power and N is the speed in rpm. However, the design considerations assign the operating speed limits $N_1 \leq N \leq N_2$ for the corresponding ranges of power $W_1 \leq W \leq W_2$. Hence the solution of equation(1) must satisfy the boundary values for all practically realistic conditions of operation. The dependence between W and N must be monotonic in nature without discontinuities. The constants A_o, B_o, C_o may be considered as the designer's assigned values for the machine. The constants are dependent on the frictional power dissipation and other kinematic constraints of moving linkages. Hence, the objective of the investigation is to apply the concept to a specific case of high speed gas turbine utilized as an underwater weapon and thus generate necessary characteristics for different working media such as high pressure gas or steam. The approach suggested herein might be useful to establish workability of the machine under varying system conditions.

Solution of equation (1):

The boundary value solutions of equation (1) can be obtained subject to the boundary conditions for different sets of values of (A, B) with C=1 with the aid of finite difference computational method.

TABLE1:

TURBINE SPEED AND DYNAMOMETER LOADING DATA ON THE TEST BENCH.

S.NO	Turbine Speed (RPM)	Dyno speed (RPM)	Torque (N-m)	Brake Power absorbed by dynamo meter	Combustion gas pressure. (bar) inlet to turbine
1	35952	1618	810	137.24	220
2	31330	1410	810	119.60	180
3	35330	1590	1020	169.83	204
4	31775	1430	820	122.79	169
5	24220	1090	490	55.93	90
6	35774	1610	1020	171.97	216
7	31552	1420	820	121.94	150
8	24220	1090	510	58.21	88
9	35520	1600	1000	167.5	230
10	28860	1300	800	108.9	180
11	24420	1100	500	57.6	100

The entries in table 1 correspond to the load test on gas turbine of an underwater weapon.

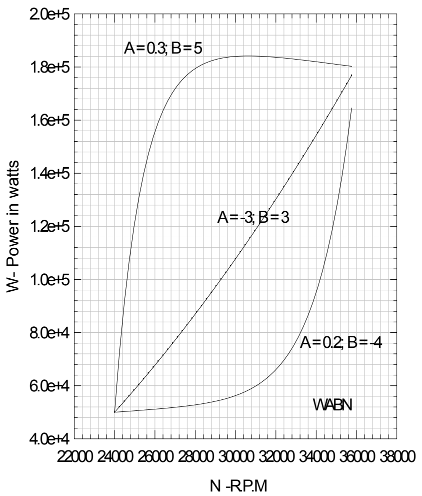

Fig1: EFFECT OF COEFFICIENTS 'A' -' B' ON THE SOLUTION

The variables both N and W are independent and dependent parameters of equation (1). Since the ranges of power and speed are fixed within the desired limits, boundary value solution of equation (1) can be tried through a finite difference technique as follows with the variable node **I.** At node (I) equation (1) holds good and it follows as:

$$C_o W(I) + B_o N(I) \frac{dw}{dN}\Big|_{[I]} + A_o N^2(I) \frac{d^2w}{dN^2}\Big|_{[I]} = 0 \quad (2)$$

Equation (2) can be solved by employing Taylor's series within the limits of the **1 ≤ (node)I ≤ J**+1

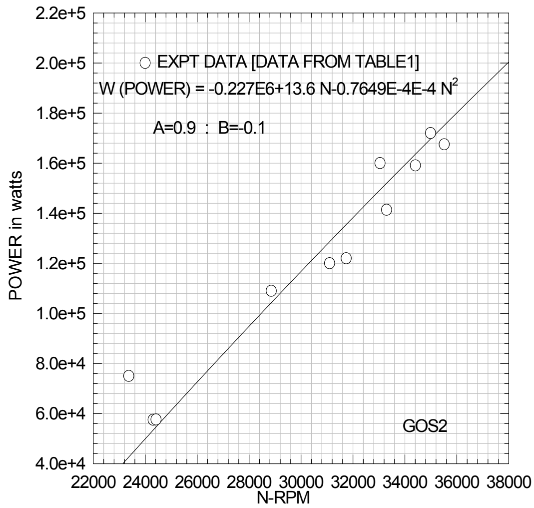

Fig 2: Experimental data on a test bench of turbo machine

$$W(I+1) = W(I) + \frac{dW}{dN}\Big|_{[J]} + \frac{N^2(I)}{2}\frac{d^2W}{dN^2}\Big|_{[J]} + \text{.........} \quad (3)$$

Substitution of W(I) from Eq.(3) into Eq.(2) with C_O tentatively assumed as unity yield:

$$W(I+1) = W(I) + (B_o - 1)\frac{dw}{dN}\Big|_{[I]} + N^2(I)(A_o - 0.5)\frac{d^2W}{dN^2}\Big|_{[I]} \quad (4)$$

Replacing the coefficients (B_o-1) and (A_o-0.5) in equation (4) by new set of constants B, A, C_o=1.

It follows

$$W(I+1) = W(I) + BN(I)\frac{dW}{dN}\Big|_{[I]} + AN^2(I)\frac{d^2W}{dN^2}\Big|_{[I]} \quad (5)$$

The boundary conditions are as follows between the nodes (1< I< J+1):

at N(I=1): W(1) = W1 (PRESCRIBED)

at N(I=J): W(J+1) = WJ (PRESCRIBED) -----(6)

Expressing the differentials of equation (5) in finite difference format with the aid of equations (7) and (8) and with the variable node I between the limits 1 and J+1

$$\frac{dW}{dN}\Big|_{[J]} = \frac{W(J+2) - W(J)}{2H} \quad (7)$$

$$\frac{d^2W}{dN^2}\Big|_{[J+1]} = \frac{W(I+2) - 2W(I+1) + W(I)}{H^2} \quad (8)$$

It follows:

$$W(I+2) = W(I+1)\left[2 - \frac{B\,\Delta N}{A\,N(I)} - \left(\frac{\Delta N}{N(I)}\right)^2 \frac{1}{A}\right] + W(I)\left[1 - \frac{B}{A}\left(\frac{\Delta N}{N(I)}\right)\right]H \quad (9)$$

Solution of equation (9):

The formulation is mathematically complete to generate the relationship between W = F(N) between the limits

at N = N(I=1): W = W(I=1)

at N = N(I=J): W = W(I=J+1) (10)

with $\Delta N = \frac{N(J+1) - N(J)}{J}$ and 1<I<(J+1). However, the numerical method employed requires an initial guess of W(2). If the initial guess W(2) is accurate enough, the computations yield W(J+1) corresponding to the pre assigned value of the power. However to achieve the desired accuracy between the computed value and the prefixed value at the final node a linear interpolation technique is employed. However, following the procedure outlined for arbitrary values of A and B, different solutions can be generated as shown in figure 1 still satisfying the prescribed boundary values. For example, the correct choice for A and B by trial and error can be ascertained with the experimental data plotted on the same plot .With the choice of A = 0.9 and B = -0.1 shown in figure 2 depicts a fair agreement of the theoretical line passing through the experimental data on the test bench of the gas turbine. Besides, the best fit of the line through the data shown in figure (2) is represented by the equation as follows:

$$W=-2.52\times10^{5}+13.795N-4.915\times10^{-5}N^{2} \qquad (11)$$

Equation (11) is applicable for the range of data shown in Table1.

From the operating point of view of the turbine under various load conditions and pressure variations, the discharge of the gas can be obtained from the principles of thermodynamics as follows:

$$W=\dot{m}C_P\left[\frac{P_1}{\rho_1 R}\right]\left[1-\left(\frac{P_E}{P_1}\right)^{\left(\frac{n-1}{n}\right)}\right] \qquad (12)$$

Thus, from equations (11) and (12)

$$\dot{m}=\left\{C_P\left[\frac{P_1}{\rho_1 R}\right]\left[1-\left(\frac{P_e}{P_1}\right)^{\left(\frac{n-1}{n}\right)}\right]\right\}^{-1}\left(-2.52\times10^{5}+13.795\mathrm{N}-4.915\times10\text{-}5\,\mathrm{N}^{2}\right)\ \text{(Kg/S)} \qquad (13)$$

ESTIMATION OF FRICTIONAL POWER:

For the ranges of speeds and inlet pressures of the gas under operating conditions $\dot{m}$- the gas discharge can be estimated from equation (13). Further the corresponding brake power can be computed from equation (12). These values when plotted in figure will enable us to estimate the frictional power. The intercept of the line passing through the computed values with abscissa will give the frictional power.

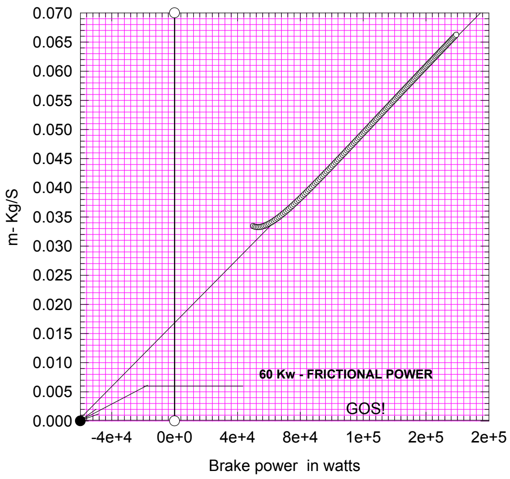

Fig3: Discharge versus brake power

Mechanical Efficiency:

By definition the mechanical efficiency is given by the relationship

$$\eta=\left[\frac{Brake\ Power}{Brake\ Power+Friction\ Power}\right] \qquad (14)$$

The variation of mechanical efficiency with brake power is shown plotted in figure (4) for the unit under consideration.

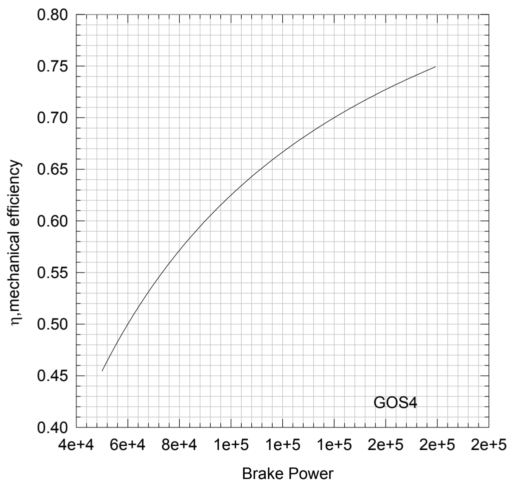

Fig4. Variation of η with brake power

Working medium:

Further choice of working medium can be either commonly used steam or any patented gas with known thermo physical properties. Hence to make use of equations (11), (12) and (13) variation of C_P, ρ and T_S must be known as functions of pressure for the two media under consideration.

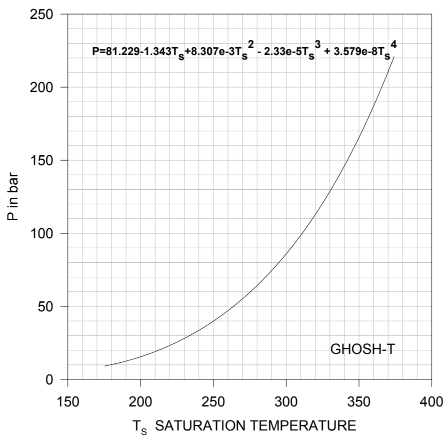

Fig 5: Variation of Saturation temperature with Pressure for steam

To achieve generality in the approach and extend the thermodynamic ranges the gas/steam properties are defined functionally in terms of pressure and temperature. The properties of steam are taken from Eq.(1).

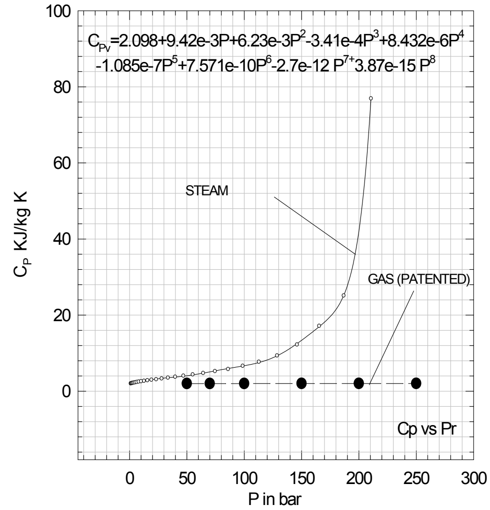

Fig6: Variation of C_P with pressure for steam and gas

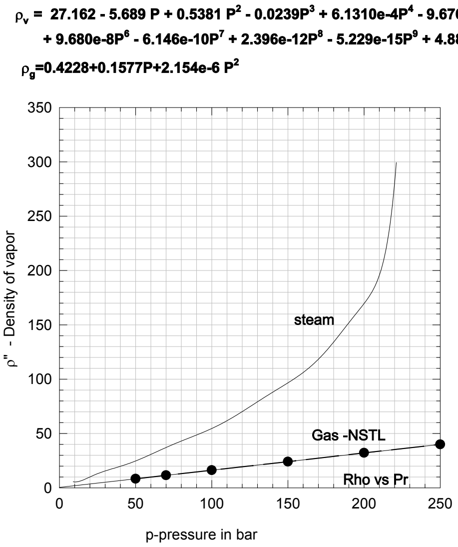

Fig7: Variation of Density with Pressure

For steam (16 bar < P < 220 bar) the variation of saturation temperature T_S with system pressure P is given by the equation

$P = 81.229 - 1.343T_S + 8.307e\text{-}3T_S^2 - 2.33e\text{-}5T_S^3 + 3.57e\text{-}8T_S^4$

$\rho_v = F_1[P]$ is given by equation (15)

$$\rho_v = 27.162 – 5.689 P + 0.5381 P^2 – 0.0239 P^3 + 6.1310e\text{-}4 P^4 -9.676e\text{-}6 P^5 + 9.680e\text{-}8 P^6 – 6.146e\text{-}12 P^7 + 2.396e\text{-}12 P^8 - 5.229e\text{-}15 P^9 + 4.88e\text{-}18 P^{10}. \quad \text{-----} \ (15)$$

For gases: 16 bar < P < 220 bar.
$\rho_v = F_2[P]$ the density of the gas is given by Eq.(16)

$$\rho_g = 0.4228 + 0.1577 P + 2.154e\text{-}6 P^2 \quad (16)$$

Figure (6) gives the variation of specific heat at different pressures.

For steam: 16 < P < 220

$$C_{Pv} = 2.098 + 9.42e\text{-}3 P + 6.23e\text{-}3 P^2 -3.41e\text{-}4 P^3 +8.432e6 P^4 - 1.085e\text{-}7 P^5 + 7.57e\text{-}10 P^6 - 2.7e\text{-}12 P^7 + 3.87e\text{-}5 P^8 \quad (17)$$

For gas: 16 < P < 220

$$C_{Pg} = 1977.344 + 0.388 P - 6.38e\text{-}4 P^2 \quad (18)$$

Specific Power: Making use of these properties the specific power can be computed.
It is defined as the power generated per unit discharge of the working medium. Thus,

$$\lambda = [W/\dot{m}] = \left\{ C_P \left[\frac{P_1}{\rho_1 R} \right] \left[1 - \left(\frac{P_e}{P_1}\right)^{\left(\frac{n-1}{n}\right)} \right] \right\} \quad (19)$$

Where W = Power in Watts
$\dot{m}$ = flow rate in kg/sec
n = index of expansion
P_1 = inlet pressure at the entry to the blade in N/m² of gas/steam.
P_e = pressure at the exit to the blade in N/m² of gas/steam.
R = Gas constant

The variations of λ as functions of, brake power speed of rotation and pressure are shown plotted in figures (8), (9) and (10). The figures indicate that steam can be chosen as the working medium since λ is substantially quite high in comparison to the patented gas studied in the analysis.

The data from the study when subjected to regression yields the following relationships for λ both for steam and gas as follows:

For Gas, the specific power:

$$\lambda = 0.113X10^7 \ N^{0.962} P^{-1.06} \quad (20)$$

For steam, the specific power is

$$\lambda = 0.1622X10^{-26} \ N^{-4.881} P^{9.076} \quad (21)$$

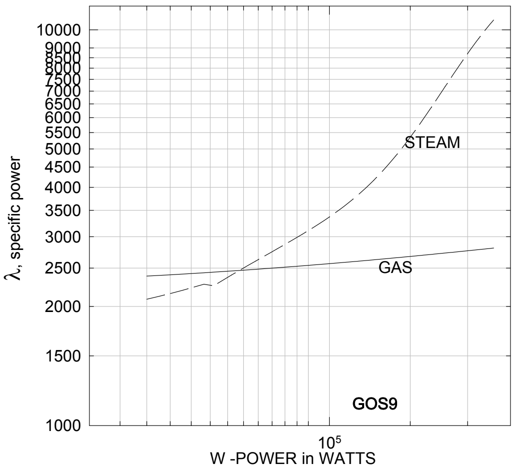

Fig8 Variation of λ with W

Conclusions:

1. The present theoretical study is developed based on the random experimental data on a gas turbine working on a test bench. The solution of the differential equation gives the best fit relationship as follows for the ranges under consideration:
   $\mathbf{W = -2.52x10^5 + 13.79N - 4.91x10^{-5} N^2.}$
   This equation is the best fit solution with the choice A=0.9: B=-0.1: C=1

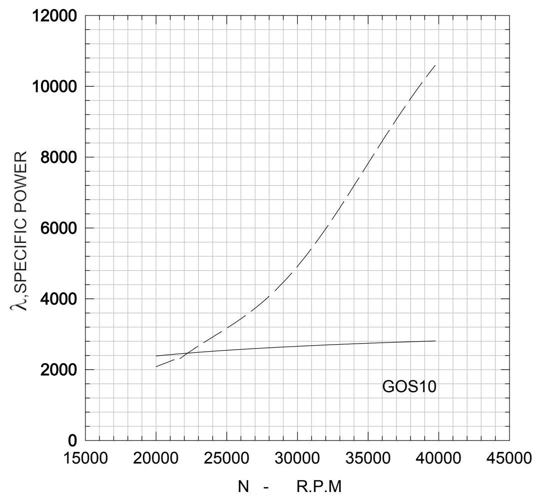

Fig 9 Variation of Specific power with R.P.M

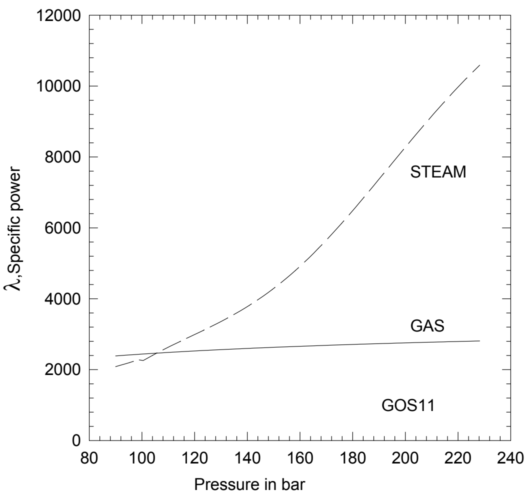

Fig 10 λ versus pressure

2. The specific power for the medium steam increases both with R.P.M and pressure. The study indicates that conceptually the power versus –rpm data of a turbine can be initialized in developing the specific mass flow relationship as function of speed and pressure. For a specific machine under consideration and for the data available the regression equations can be provided by equations (20) and (21)
3. Such derived equations are essential to control the performance of the turbine under varying speed and load conditions. The derived characteristics are from the hypothesis that W=F(N) is defined by the differential equation(1). A method is generated to derive the specific power of the turbo unit with the aid of the relationship W=F(N).

NOMENCLATURE

A_0 = Constant
B_0 = Constant
C_0 = Constant
C_p specific heat, (J/kgK)
h Enthalpy, kJ/kg
K thermal conductivity, (W/mK)
$\dot{m}$ mass flow rate, (kg/sec)
N Speed of the Rotor, RPM
n Index of expansion
P_1 InletPressure, (N/m^2) of gas/steam
P_e Exit Pressure, (N/m^2) of gas/steam
Q heat, (Watts)
R Gas constant (J/kgK)
t Time, (s)
δt time increment, (s)
T temperature, (C)
T_s Saturation temperature (C)
W Power (Watts)

Greek symbols

ρ Density, (kg/m^3)
η Efficiency
λ Specific power

6. REFERENCES

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