



Quantum Edge Detection in Digital Imaging: A Novel Approach Using Quantum Exponential Entropy

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ABSTRACT

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Quantum image processing is for the application of quantum computing and algorithms to the solution of various problems. Classical edge detection algorithms are generally effective but are prone to performing poorly in the handling of large sets of data and high-resolution pictures, which could degrade performance. Quantum computing can significantly increase efficiency and hasten breakthroughs in various sectors, thus making it an appealing solution for complex image processing problems. Automated methods for the processing and analysis of medical pictures can significantly benefit medics in the therapeutic and diagnostic process. Here, the difficult problem of edge detection in medical pictures is specifically addressed through the introduction of new multilevel solutions based on quantum image representations of the one-dimensional histograms of the distributions of gray levels. Our methods use the quantum exponential entropy approach to the determination of the quantum information in image histograms, allowing us to recognize and study subtle details within medical pictures on the quantum level. To establish the efficiency of the approaches presented, we approach comparative analysis in comparison to the classic techniques through the application of the varied sets of medical pictures. Experiment results show unequivocal proof for the efficiency of our approaches, illustrating the possibility for our new multilevel techniques based on quantum image representations and the quantum exponential entropy approach to the determination of the quantum information in image histograms to outclass the currently used techniques of edge detection in medical pictures. Through the enhancement of the precision and the efficiency of the detection of the edges, our work fosters the additional ongoing research in the topic of the automated techniques of quantum medical image analysis.

1. INTRODUCTION

Edge detection is the foundation of image processing, important in the detection of object boundaries and properties. It is of greatest value in medical image data, where proper identification of edges assists in the discrimination of anatomical structures and lesions in order to supplement object recognition, object separation, and three-dimensional reconstruction of body organs [1, 2]. Even though there is continued demand in properly detecting edges in noisy medical images, the process itself is still a significant barrier. The vast majority of medical image sets are marred by various types of noise—such as electronic sensor noise, speckles in ultrasound modalities, and low contrast—that mask true edges. Classical edge detection operators such as high-filter-based Sobel, Prewitt, Roberts, the Laplacian of Gaussian (LoG), and the Canny edge detector are typically noise-contaminated. They are susceptible to noise effects owing to the fact that noise contributes appreciable high-frequency variation in intensity, which is indistinguishable from true edges [3, 4]. For

the reasons outlined, the classical operators either overlook subtle edges or confuse noise artifacts for important boundaries, therefore being limited in noisy image situations. It is here where the emphasis on better edge detection operators able to discern true edges from noise has been called for [5, 6].

One of the successful methods for improving edge detection in noise-dominated conditions is entropy-based thresholding, where the detection of edges is viewed as a binary splitting issue—discernment of edges from non-edges—under the guidance of information theory axioms. Differently than conventional methods, which are largely based on the use of the image's neighborhood gradient information, entropy-based approaches utilize the spatial properties of the image intensity histogram to determine the optimum threshold to best split the image and maximize the information-based quantity between partitions. Shannon entropy, the very first measure of information quantity, has been used for broad-based global thresholding. An intriguing case is the algorithm proposed by Kapur et al. [7], which uses the joint entropy of the background

and foreground regions to select the threshold that best partitions the image with maximum information value. Along the same line of reasoning, researchers made generalized entropy models to better fit different image statistical properties. Tsallis entropy—a non-extensive measure of entropy subjected to the parameter q —has in particular been observed to possess benefits in the description of long-range intensity correlations and used to improve thresholding performance of images wherever the noise is of the textured/correlated type [8, 9].

Though generalized entropy measures are superior in edge detection for noisy images, they tend to involve the adjustment of a free parameter for every particular case, which could restrict their usefulness. Hill entropy—a generalized measure in its own right, which is based on Hill's diversity measure in ecology—falls into the same category. It is parameter-dependent, like other entropies, and is capable of approximating various entropy types, the Shannon entropy being the limit in particular circumstances [10, 11]. These entropy expressions have been investigated for various potential applications like image segmentation and edge detection. For example, Balochian and Baloochian [12] put forward a hybrid thresholding method which combines Shannon and Tsallis entropy to enhance the detection of edges in noisy medical pictures. Their algorithm proved to be less susceptible to noise than the common edge detectors [13].

Extending the concept further, in 2017, Elaraby and Moratal developed the two-stage threshold algorithm based on generalized Hill entropy for the specific application of edge detection in noisy medical imagery [1]. According to the first phase of the approach, the image is segmented into background and objects (foreground) through a maximization of the Hill entropy of the image histogram to reach a global threshold. Separating the large structures is helped by the initial step. Local thresholds are independently estimated in the background and the foreground by the entropy-based criterion in the second phase [1, 14].

The approach integrates the first global threshold and the two locally sharpened thresholds to generate two binary edge maps, symbolizing the object boundaries and the background transitions, respectively, which are summed to give the ultimate edge map. The approach allows the detection of fine edges which would otherwise be omitted by the single global threshold. From the experiment, they published that the approach exceeded the classic Canny detector and outperformed even a Tsallis entropy-based approach in noisy conditions, exhibiting superior continuity and accuracy of edges [15].

With the success of this classical two-phase Hill entropy algorithm in dealing with noise, it is then to speculate on its promise in the quantum computing paradigms. It is possible to drastically speed up computations through the use of quantum parallelism and amplitude-based computations. In quantum image processing, whole images are encoded as quantum states so the whole image can be subject to parallel manipulation through quantum superposition [16, 17]. For instance, the Flexible Representation of Quantum Images (FRQI) embeds pixel intensity in the form of the amplitude of the quantum state while pixel locations are mapped to the basis states. This form allows the whole of the pixel values to be subject to the same quantum operation (unitary transform) simultaneously [18, 19].

Quantum edge detection algorithms already exhibit respectable speed advantages over classical counterparts. Yan

et al. [17] put forward the quantum Sobel operator, QSobel, in terms of the FRQI and attained exponential complexity reductions in the gradient calculation in all the pixels in parallel. It minimized the runtime from classical $O(N)$ to $O((\log N)^2)$ in the quantum scenario [17]. Even better still, Yao et al. [20] outlined the quantum approach of edge detection in using only the single-qubit operation, regardless of image dimensions, and brought out the promise of quantum computing for real-time large-set image handling [20]. It would seem to fall within the realm of possibility to map the two-phase Hill entropy measure to the quantum computational model. If then entropy-based thresholding and segmentations could be implemented in quantum regimes, the entire image intensity distribution could be subject to handling in parallel and the sets of threshold applied in logarithmic- or even constant-order time. It would prove of particular use in high-resolution 3D- or real-time medical imaging [21].

Coupling of Hill entropy and quantum computing is also possible due to the analogy between quantum probability amplitudes and information-theoretic quantities. A quantum entropy threshold algorithm would then be able to take advantage of quantum-state evaluation to simultaneously measure entropy or related cost functions for multiple threshold candidates, then accelerate optimal threshold choosing.

In summary, converting the classical Hill entropy edge detection method into a quantum model offers the potential to combine robust noise handling and theoretical rigor with the computational speed of quantum systems. This work proposes such a quantum algorithm, detailing the quantum image representation, formulation of Hill entropy in quantum terms, and the steps for implementing two-phase thresholding on a quantum computer. This approach is particularly well-suited for large-scale medical imaging and real-time analysis, where classical methods often struggle to balance noise resilience and processing speed [22].

Edge detection is a fundamental operation in classical image processing, aiming to identify significant discontinuities in pixel intensity that represent object boundaries. Traditional methods such as Sobel, Prewitt, and Canny rely on spatial derivatives to detect edges. However, these algorithms become computationally expensive when processing large-scale or high-resolution images. Quantum image processing (QIP) emerges as a promising paradigm by leveraging quantum mechanical principles to represent and manipulate images more efficiently. One of the earliest and most cited models is the Flexible Representation of Quantum Images (FRQI), where a grayscale image of size $2^n \times 2^n$ can be encoded into a quantum state using 2^n qubits and amplitude encoding [23].

Another extension of the above is the Novel Enhanced Quantum Representation (NEQR), which retains the pixel locations and grayscale values in the form of basis states, which offer a robust and direct image-retrieval mechanism [24]. Such representations allow quantum algorithms to perform image transformations, image enhancements, and edge detection in fewer computational steps than classical algorithms [25]. Moreover, quantum gates such as Hadamard, CNOT, and Pauli-X are used to manipulate qubits for the performance of pixel-wise operations, sharpening of edges, and measurements. The theoretical advantage also opens up the possibility of the use of classical optimization techniques, such as the hill climbing algorithm, in quantum circuits to find the optimum threshold values or boundary detection in the superposed state space.

2. RELATED WORKS

Numerous efforts have explored the application of quantum computing to edge detection. Researchers presented a method that applied the Flexible Representation of Quantum Images (FRQI) to perform edge detection in the quantum domain, showing that standard edge operators could be implemented using quantum gate operations on amplitude-encoded image data [23]. Another research gave the form of the quantum Sobel filter implemented by quantum Boolean logic, highlighting the advantage of the parallel calculation of edges in quantum computers. More recently, Zhang et al. [24] and Sundani et al. [25] detailed the design of the NEQR-based system for edge detection and image sharpening, showing improved fidelity and quantum noise robustness. Even if these efforts show the potential of quantum-enhanced image manipulation, they are inclined to act on static filters or rely heavily on binary logical gate sets. On the contrary, the contribution in the present paper advocates the merging of hill climbing optimization and quantum image representations for the dynamic searching of the optimum edge boundaries, hypothesizing the hybrid model for adaptation to the complexity of the image under consideration while being quantum-economical.

2.1 Classical edge detection methods

Edge detection is also among the earliest subjects of image-processing research for which numerous various algorithms and operators have been created. Perhaps two of the earliest are the Sobel and Prewitt operators of the late 1960s, which estimate the local intensity gradient by means of small convolutional masks. The Sobel operator, for example, employs two 3×3 kernels to approximate the horizontal and vertical derivatives of an image, with the gradient magnitude indicating edge presence. Although these gradient-based methods are efficient and straightforward, they tend to amplify high-frequency noise along with true edges. Similarly, the Roberts cross operator, which uses compact 2×2 kernels to detect diagonal changes, is lightweight in computation but highly susceptible to noise due to its limited neighborhood scope. The Prewitt operator, resembling the Sobel method but using uniform weights in its kernel, also estimates gradients and requires pre-processing steps such as smoothing to mitigate noise effects [26, 27].

To overcome the limitations of first-order methods, second-order derivative techniques were introduced, notably the Laplacian of Gaussian (LoG) detector, also known as the Marr-Hildreth algorithm. This method begins with Gaussian smoothing to suppress noise, followed by the application of a Laplacian filter to detect rapid intensity changes. Edges are identified at the zero-crossings of the LoG response. While more robust to noise than basic gradient operators due to the smoothing step, LoG often produces thick edge lines and relies on subsequent thresholding and thinning. The effectiveness of this method heavily depends on selecting an appropriate standard deviation for the Gaussian kernel: insufficient smoothing retains noise, whereas excessive smoothing can obscure important details [28].

Among the classic approaches, the Canny edge detector is arguably the gold standard. In 1986, John Canny proposed the algorithm, which made key constraints for edge detection—to best maximize the reliability of detection, to accurately localize edges, and to inhibit repeated edge output—using the

calculus of variations to find the best smoothing filter. The Canny detector algorithm encompasses multiple steps: the application of Gaussian smoothing, the calculation of the gradient (more typically through Sobel operators), the application of non-maximum suppression to thin the lines of edges, and double-threshold hysteresis to join the edge segments while removing noise artifacts. For the entire pipeline, the method of the Canny thus produces smooth, continuous edges while coping decently with noise robustness. However, the method is still susceptible to underperformance in extremely noisy or low-contrast conditions and is vulnerable to proper parameter adjustment, particularly for smoothing settings and thresholds [5].

In short, traditional edge detection operators such as Sobel, Prewitt, Roberts, and LoG are quick in strengthening edges but tend to interpret abrupt intensity changes due to noise as genuine edges, and thus they behave like high-pass filters. Even the Canny algorithm, which is better than the above techniques in terms of the addition of extra smoothing and adaptive thresholding, is also tied to the paradigm of linear filtering. Under heavy noise conditions, these techniques tend to either neglect edges in order to prevent false alarms or make errors in noise being interpreted as edge information. To overcome these drawbacks, there are some attempts to look into data-driven and adaptive techniques such as entropy-based and fuzzy logic-based techniques, which are discussed in the following sections.

2.2 Entropy-based edge detection and thresholding

Entropy-based approaches to edge detection are image-segmentation- and information-theory-based, where the aim is to maximize the value of information gained through an image. Kapur et al. [7] contributed to the literature by introducing a new entropy thresholding based on Shannon entropy. Here, in the approach, the histogram of grayscale image is divided by the potential threshold T in the two disjoint sets—evidently observable by the user as background and foreground. The entropy sum, $H_{\text{total}(T)} = H_{\text{back}} + H_{\text{fore}}$, of the two subregions is then obtained, and the threshold maximizing the sum is adopted to be the optimum. Although originally motivated for binary partition of images, the approach is portable to edge detection by allotting the first of the two sets (typically the minority) to the edge pixels and the second to the non-edge pixels. Shannon entropy-based thresholding is extremely successful when the image histogram is bimodal but may fail if the distributions of edges and the non-edges are severely overlapping, or if higher-order statistical interrelations are significant [29, 30].

To counter the lack of Shannon entropy in adequately representing complex image properties, generalized entropy expressions have also been employed by researchers. Tsallis entropy is another popular representative, which is Shannon's measure generalized by the addition of a tunable parameter $q > 0$. When the value of q approaches 1, Tsallis entropy is close to Shannon entropy. The parameter q adjusts the sensitivity to different parts of the probability distribution to better respond to rare events or noise in the form of outliers—a good attribute when handling long-range dependent images or non-Gaussian noise. For the purpose of thresholding, maximizing Tsallis entropy in the grayscale histogram can give better segmentations, particularly in cases where Shannon entropy is of little use for segment discrimination between different classes. However, the main issue here is in picking the appropriate q for each image [31].

Researchers have had to resort to generalized entropy measures in order to address the complexity of image data, capturing finer statistical details which are overlooked by the simpler schemes. Tsallis entropy is such an entropy measure which is an extension of Shannon entropy and is regulated by an adjustable parameter $q > 0$. For $q \rightarrow 1$, Tsallis entropy approaches Shannon entropy. The parameter q regulates the weight which is assigned to different parts of the probability distribution; e.g., for some values of q , Tsallis entropy is particularly sensitive to the tail probabilities and is therefore suited best for image data which includes spikes/long-range dependencies. When applied to image thresholding, maximizing the Tsallis entropy over the image histogram in some cases obtains a better split between object and background where the assumptions of Shannon's entropy are violated. However, the great challenge which is presented in the use of Tsallis entropy is determining the optimum value of q for the specific image in all instances [32].

These other generalized measures of entropy also gained application in image segmentation, some of which are Renyi entropy, Kapur's extended entropy, fuzzy entropy, and Kaniadakis entropy. All of these also bring along the parameter to be adjusted in order to obtain the trade-off between the information release and noise removal. Hill entropy, which is based on Hill's measure of diversity, is another family of measures of entropy to interpolate between different statistics descriptors. It brings the intensity count of simple number-based measure to the continuum up to Shannon entropy and measures of concentration like Simpson's index. Even though less frequent in the use in the past in image processing literature, Hill entropy also gained application recently to treat the drawback of the consideration of the dominant and rare intensity values. The common pitfall of the entropy models is the needed parameter choice, but if the appropriate optimization method like cross-validation or auxiliary criterion is employed, they can significantly enhance the performance of the threshold-based edge detection [33, 34].

There are various empirical research which have pointed out the effectiveness of entropy-based thresholding for finding edges in noisy conditions. For example, Heshmat et al. proposed the hybrid approach of Shannon and Tsallis entropy. By the merging of the classical and generalized measures of entropy, their method corrects for bias between rare and common pixel intensities, yielding better robustness to salt-and-pepper noise in medical scans, such as scans of blood cells. Their method also keeps computational overhead low by minimizing space for best-possible thresholds to search. Other studies looked at multi-phase entropy models to determine multiple thresholds for multi-level segmentations. The methods give broader edge bands, which are further improvable. Furthermore, fuzzy entropy methods, such as the fuzzy divergence approach of Chaira and Ray, correct for the intrinsic uncertainty of pixel classification in the vicinity of edges. The methods first transform the image to the fuzzy space and then apply an entropy-based condition to threshold. The fuzzy entropy models are particularly excellent in dealing with fuzzy edge regions where membership of the pixels is in doubt [35, 36].

The paper of Elaraby and Moratal [1] is of specific interest for the work herein, for they had outlined a two-stage thresholding method utilizing generalized Hill entropy. It starts off its approach by choosing an optimum value of the global threshold T_1 in maximizing the Hill entropy of the whole image histogram, thus generally partitioning the image

into object and background regions. Local background and object area thresholds T_2 and T_3 are then determined by maximizing Hill entropy in the background and object subregions, respectively. The resulting edge map is then generated by integrating the edges thus obtained across all three threshold values, therefore encompassing broad-based, in addition to fine details-based, changes in intensity. The hierarchical method is seen to be particularly well-adapted to the detection of faint edges, which are otherwise generally neglected by the classical methods.

Comparative evaluations demonstrated that this method outperformed both Tsallis-based thresholding and the Canny edge detector, particularly under noisy conditions, thereby highlighting the potential of multi-level, adaptive entropy frameworks for robust edge detection in complex imagery [37, 38].

2.3 Quantum image processing and quantum edge detection

Quantum Image Processing (QIP) has emerged from the convergence of quantum computing and classical image processing, driven by the goal of harnessing quantum computational speed for enhanced efficiency in image data manipulation [39-43]. Central to QIP is the encoding of images into quantum memory, with multiple quantum image representation (QIR) frameworks proposed to optimize processing in quantum systems:

- Flexible Representation of Quantum Images (FRQI):** This method encodes a $2^n \times 2^n$ -sized image into a quantum state using $2n$ qubits for spatial coordinates and one qubit for pixel intensity. Each basis state in this model corresponds to a pixel's position, with its amplitude encoding grayscale or color values. By exploiting quantum superposition, FRQI stores all pixel values simultaneously in a single quantum state $|I\rangle$. A key strength lies in quantum parallelism: operations such as global rotations can act on all pixels concurrently, enabling tasks like bulk intensity adjustments. However, retrieving individual pixel values necessitates measurement, collapsing the quantum state. Consequently, FRQI-based algorithms must prioritize operations within the quantum phase space to avoid state collapse [19].
- Novel Enhanced Quantum Representation (NEQR):** It refines FRQI by encoding pixel *intensities* as discrete binary values within qubit basis states. For an 8-bit grayscale image, NEQR allocates 8 qubits for intensity and $2n$ qubits for coordinates. The resultant quantum state $|y, x, I(y, x)\rangle$ represents a superposition of basis states, where $I(y, x)$ denotes the binary intensity at coordinates (y, x) . Unlike FRQI's amplitude-dependent encoding, NEQR stores intensity values in orthogonal states, enabling unambiguous pixel retrieval through direct measurement of intensity qubits. This explicit encoding simplifies intensity comparisons and algorithmic design. While NEQR requires more qubits than FRQI for equivalent images, it achieves quadratic acceleration in state preparation and circumvents the complexities of amplitude-based encoding [44, 45].

The advancement of quantum algorithms for image processing tasks, particularly edge detection, has been propelled by innovations in quantum image representations. Another work pioneered this domain by demonstrating edge detection on a quantum processor through their Quantum

Probability Image Encoding (QPIE) method. Their approach employed quantum operations to identify abrupt intensity transitions, achieving edge extraction in constant time irrespective of image dimensions. Subsequent work by Heo et al. [46] introduced QSobel, a quantum Sobel operator leveraging the Flexible Representation of Quantum Images (FRQI). By exploiting quantum parallelism, QSobel computes gradients across all pixel positions simultaneously via quantum arithmetic, reducing computational complexity to $O(n^2)$ for $2^n \times 2^n$ images, a significant improvement over the classical $O(4^n)$ complexity. Further developments extended this paradigm to Prewitt, Robinson, and Haar wavelet-based quantum edge detectors, where convolution and frequency-domain operations are executed through tailored quantum circuits. For instance, Zhang et al. [47] implemented an eight-directional Robinson compass mask on quantum-encoded images, utilizing superposition to evaluate all orientations concurrently and applying quantum thresholding gates to classify edges. These methodologies universally capitalize on quantum state representations to perform filtering and differentiation operations with enhanced efficiency, harnessing the inherent parallelism of quantum systems [17, 48, 49].

Beyond gradient-based methods, quantum thresholding and segmentation techniques integrated with entropy principles have gained traction. Pramanik et al. [50] addressed breast tumor edge detection as a multilevel thresholding optimization problem, deploying a quantum-inspired genetic algorithm grounded in Tsallis entropy maximization. Their hybrid quantum-classical framework identified optimal thresholds to segment mammogram images, yielding superior PSNR/SSIM metrics compared to conventional approaches. Similarly, Yao et al. [20] had created a quantum edge detection mechanism

incorporating Hill entropy in the new enhanced quantum representation (NEQR) scheme. The method they employed constituted the application of hybrid neural-quantum filter to suppress noise in encoded medical data and then entropy-based thresholding for the dissection of the background and foreground regions. The method achieved 97.5% accuracy in edge detection and enhanced the PSNR, which establishes the efficiency of the entropy measure generalizations in quantum image processing. Such findings indicate the paradigm-shifting potential of the integration of quantum computing and entropy-based measures for medical image use cases [49-51].

Subsequent quantum image processing advancements looked into adaptive and hybrid solutions for maximizing the performance of edge detection. Shukla and Vedula [52] put forward an adaptive quantum edge detection approach evolved based on the usage of Quantum Pixel Imaging (QPI), which encompasses the application of dynamic threshold optimization techniques for maximizing the detection accuracy in various conditions of an image. The approach outperformed the static quantum filters in adaptability but required heavy quantum computing resources. In the ensuing direction [52], Shannon [53] developed a hybrid classical-quantum framework using an enhanced version of the FRQI representation. Their model successfully integrated classical preprocessing with quantum logic to achieve effective edge extraction in grayscale images. These two contributions underscore a shift in the field towards more flexible, optimization-driven, and hardware-aware quantum image processing frameworks, which aligns closely with the objectives of the current study [53]. Table 1 shows a comparison between different quantum image representation techniques.

Table 1. Comparison between different quantum image representation techniques

Study	Representation	Technique	Strengths	Limitations
[19]	FRQI (Flexible Representation of Quantum Images)	General quantum image processing	Foundational flexible encoding model-basis for further developments	Not suitable for high-resolution grayscale images
[23]	NEQR (Novel Enhanced Quantum Representation)	Quantum Boolean Edge Detection	Direct pixel value encoding-more accurate than FRQI	Does not support color images-sensitive to noise
[24]	NEQR+Quantum Gates	Quantum Sobel+ Edge Filtering	High fidelity-improved noise resistance	Based on static filters-lacks adaptability
[44]	QPI (Quantum Pixel Imaging)	Adaptive Quantum Edge Detection	Dynamic edge adaptation- optimization-based detection	Computationally complex-requires advanced quantum hardware
[46]	Enhanced FRQI	Hybrid Classical-Quantum Model	Integrates classical and quantum processing in simulation	Experimental only-not tested on real quantum systems

3. PROPOSED METHODOLOGY

Entropy is the measure of uncertainty proposed by Shannon in information theory to measure information in a source which obeys the law of probability [54]. It is subsequent to Shannon introducing information theory that there is vast literature on invoking the concept in some of its applications which is why Pal and Pal propose another measure in the form of exponential entropy [55]. The traditional exponential entropy is for the histogram of the gray-levels of an image which carries the information of the distribution of the frequency of the pixel intensities [55, 56]. The quantum Exponential entropy, however, requires an appropriate quantum representation of the image to reflect the information quantum in nature. Shannon entropy is defined as:

$$H(p) = - \sum_{i=1}^k p_i \ln(p_i) \quad (1)$$

Exponential entropy given by:

$$eH(p) = \sum_{i=1}^n p_i e^{(1-p_i)} \quad (2)$$

As there are some merits put forward by for handling exponential entropy in lieu of Shannon's entropy, which is extremely renowned, we observe the measure of the self-info of an event with probability p_i being thought of as $\log(1/p_i)$, which is a function decreasing in p_i . The same decreasing character alternately can be maintained by taking the function

to be in terms of $(1-p_i)$ in place of being in terms of $(1/p_i)$. Also, for the uniform probability distribution $P = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})$ exponential entropy has a fixed upper bound

$$\lim_{n \rightarrow \infty} H\left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right) = e - 1 \quad (3)$$

which is not the case for Shannon's entropy.

The additive characteristic, central in Shannon's theory, of the function for self-information for independent events may not be of significant practical effect (impact) in some instances. Otherwise, such as in the case of the probability law, the double function for self-information would be product rather than sum of the function for the self-information of two independent instances.

The above thoughts suggest the self-information in the exponential form of $(1-p_i)$ and introduce another measure in exponential entropy form. The measure of entropy is of great significance, particularly for image processing, since an image can be considered an information source with the probability law being the image histogram.

Based on the definition of exponential entropy, the entropy of Object pixels and the entropy of Back ground pixels can respectively be presented by the following definitions:

$$eH^O(p) = \sum_{i=1}^t \frac{p_i}{P_A} e^{(1-\frac{p_i}{P_A})} \quad (4)$$

$$eH^B(p) = \sum_{i=t+1}^k \frac{p_i}{P_B} e^{(1-\frac{p_i}{P_B})} \quad (5)$$

The exponential entropy $eH(t)$ is also parametric in the threshold value (t) of background and object. It is presented in the form of the sum of the individual entropy such that the pseudo-additive property is attainable for statistically independent setups. We try to optimize the information measure between two classes (background and object). For the function $eH(t)$ being in the maximum value, the value of the luminance level t maximizing the function is considered to be the optimum threshold value. It is attainable for an inexpensive computational cost.

$$t^{\text{opt}} = \text{Arg max } [eH^O(t) + eH^B(t)] \quad (6)$$

In quantum image processing, there are different quantum image representations. Here, we specifically use the flexible representation of quantum images (FRQI). The representation of the FRQI is able to provide an adaptable and flexible form of representing classical images in the form of quantum systems. It is possible to map pixel intensity and space to a quantum state so that we can perform quantum operations and measurements over image information. We can exploit quantum advantages in information processing and take advantage of tasks like thresholding and edge detection using the FRQI representation. Apart from this, employing FRQI with quantum exponential entropy allows us to exploit the quantum representation of the information contained in images as well as the qualitative measure based on entropy. Through the use of the two in combination, we are able to benefit from a different type of approach of image thresholding and edge detection, which could benefit in the form of better performance and higher accuracy. The approach of the FRQI

is capable of representing the digital image in the form of the quantum system. In particular, the image histogram is capable of being presented in the form of the entangled state of the composite quantum system. For a grayscale image which contains 256 gray levels, we are in need of 256 angles θ_i to encode the i^{th} level of the intensity. These are used in creating the vector through the use of the principle of superposition providing in the following equation:

$$|I(\theta_i)\rangle = \cos\theta_i|0\rangle + \sin\theta_i|1\rangle, \quad (7)$$

where, $\theta_i = \frac{\pi}{2} p_i$ represents the probability of the i^{th} intensity level, $|0\rangle$ and $|1\rangle$ are the spin down and up. The quantum state of C_j is.

$$|I(C_j)\rangle = \sum_{t_{j+1} \leq i < t_j} |I(\phi_i)\rangle \quad (8)$$

with

$$\phi_i = \frac{\theta_i}{\sum_{t_{j+1} \leq i < t_j} p_i} \quad (9)$$

The density matrix (density operator) corresponding to the quantum state of class C_j can be written as:

$$\begin{aligned} \rho_j |I(C_j)\rangle \langle I(C_j)| \\ = \left(\sum_{t_{j-1} \leq i < t_j} \cos\phi_i, \sum_{t_{j-1} \leq i < t_j} \sin\phi_i \right) \end{aligned} \quad (10)$$

The density matrix ρ_j contains valuable information about differences and probability distributions among intensity levels within class C_j , specified by thresholds t_{j-1} and t_j . This is very useful information for thresholding operations.

Quantum exponential entropy quantifies how much quantum information is present in class C_j . It is given by:

$$QE_a(C_j) = \text{trace } \rho_j e^{(1-\rho_j)} \quad (11)$$

Here, α is an arbitrarily real parameter never equal to 1. The quantum exponential entropy is an extension of classical exponential entropy, and it is itself an extension of Shannon entropy. Due to the additivity principle, we can calculate total information contained in system S (the image), including information from each subsystem and inter-subsystem quantum correlations $C_j, j=1, \dots, k+1$, which is calculated by:

$$QE_a(S) = \sum_{j=1}^{k+1} QE_a(C_j) \quad (12)$$

The objective of the thresholding approach is to identify the optimal subsystems, $T^* = T_1^* < \dots < T_k^*$, that maximizes the total entropy, thereby producing the highest-quality image segmentation into $k+1$ subsystems $C_1, \dots, C_{k+1}$.

$$T^* = \text{Argmax } QE_a(S), T \in \{g_{\min}, \dots, g_{\max}\}^k \quad (13)$$

Edge detection identifies the boundaries between regions in an image that have distinct levels of brightness or color. To carry out the operation of edge detection, the spatial filter mask

is specified by the definition of the matrix w of order $m \times n$. Spatial filtering is attained by the simple shifting of the filter mask w of order $m \times n$ point to point in an image. If we suppose that $m=2a+1$ and $n=2b+1$, where the nonnegative integers a and b specify the mask size, then the minimum size of the meaningful mask is 3×3 . Moving the window through the whole binary image, the entropy of the probability of each central pixel of the window is computed. If the probability of central pixel $p_c = 1$ then the entropy of the pixel is zeros. Thus, if the level of the gray of all the pixels underneath the mask are homogenous then $p_c = 1$ and $H = 0$. In such case, if the difference between the gray levels of pixels in the local mask is minimal, then the central pixel belongs to a homogeneous area and as such will not be digitized as an edge pixel. But if the difference between the gray levels in the neighborhood window is significant, it can be assumed that the central pixel is on an edge area of the image.

After finding out global optimal threshold, set each pixel from the image as background or object. Then, traverse a small window of size five by five (or three by three) over the image. For each central pixel within the window:

- Construct a local histogram that models the distribution for pixel intensities within that window and normalize it for achieving the local probability distribution.
- Split this local distribution across the world threshold into two distributions, and calculate each local's exponential entropy by utilizing the same formula utilized in the global step.
- Flag the central pixel as an edge if there is both background and object pixels in the window, and if the overall local entropy is beyond a tiny threshold value (reflecting a mixed or ambiguous neighborhood), or if both local probabilities of classes are non-zero and significant instead of being nearly zero.
- This process links global QEE thresholding with local spatial consistency so that the technique can pick up on thin edges that a global decision itself may overlook.

Executable pseudo-code:

Input:

- $I \leftarrow$ grayscale image
- $t_{opt} \leftarrow$ global threshold obtained from QEE
- $w \leftarrow$ window size (default = 3)

Output:

- $E \leftarrow$ binary edge map (0 for non-edge, 1 for edge)

Procedure:

1. Initialize $E \leftarrow \text{zeros}_{\text{like}(I)}$
2. For each pixel (r, c) in I :
3. Extract $W \leftarrow$ local window around (r, c) of size $w \times w$
4. Compute local histogram h_{loc} from W
5. Normalize to obtain $p_{loc}[i] \leftarrow h_{loc}[i] / (w \times w)$
6. Compute probabilities:
 - $P_B \leftarrow \text{sum of } p_{loc}[i] \text{ for } i \leq t_{opt}$
 - $P_O \leftarrow 1 - P_B$
7. If $(P_B == 0)$ or $(P_O == 0)$:
 - $E[r, c] \leftarrow 0$ # no edge
- Else:
 8. Normalize local distributions:
 - $p_{B_{loc}}[i] \leftarrow p_{loc}[i] / P_B$, for $i \leq t_{opt}$
 - $p_{O_{loc}}[i] \leftarrow p_{loc}[i] / P_O$, for $i > t_{opt}$
 9. Compute local entropies:
 - $eH_B \leftarrow \text{EXP_ENTROPY}(p_{B_{loc}})$
 - $eH_O \leftarrow \text{EXP_ENTROPY}(p_{O_{loc}})$

10. If $(eH_B + eH_O) \geq \tau$: # τ : small threshold
 - $E[r, c] \leftarrow 1$ # mark as edge
- Else:
 - $E[r, c] \leftarrow 0$
11. Return E

4. EXPERIMENTAL RESULTS ANALYSIS

To investigate the effectiveness of new algorithms, we validate them using different sets of benchmark images. This section also describes the image sets.

4.1 Images sets

The images sets include natural images, aerial images and medical images which are commonly used as benchmarks for edge detection.

4.2 Experimental results

In this section, four algorithms' results are shown for images sets natural images, aerial images and medical images that used experimental which are commonly used as benchmarks for edge detection.

Figure 1 shows various images of dataset including natural, aerial and medical images. Figure 2 investigates the capacity of quantum exponential entropy edge detection of tested images. Figure 3 investigates the capacity of Canny edge detection of tested images. Figure 4 investigates the capacity of LoG edge detection of tested images. Figure 5 investigates the capacity of Sobel edge detection of tested images.

The resulting images indicate that the proposed approach is better than others on the different image sets based on suggestive compression as object boundary for every image is clearer.

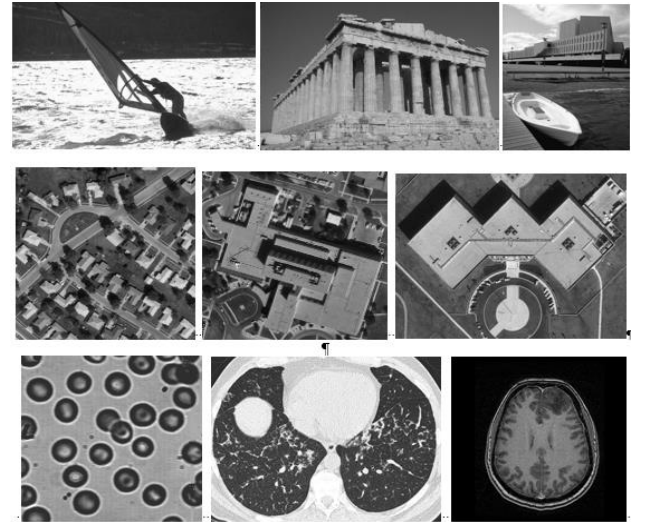


Figure 1. Examples of natural, aerial and medical images

To evaluate the performance of the proposed methods, we utilized samples of natural, aerial, and medical images. The comparison involved our quantum-based approach alongside three benchmark methods: Canny, LoG, and Sobel. In total, four edge detection methods were assessed to determine their effectiveness and performance. Numerical experiments were conducted using MATLAB, revealing that the Hill entropy

threshold effectively localizes specific objects within the edge maps.

Figures 2-5 present the edge detection results from the competing methods across nine selected images for visual examination and comparison. The results indicate that the quantum-based method produces the most defined contours, while the Sobel method yields the least satisfactory results. Additionally, the Canny and LoG methods provide contours that are approximately similar in quality.

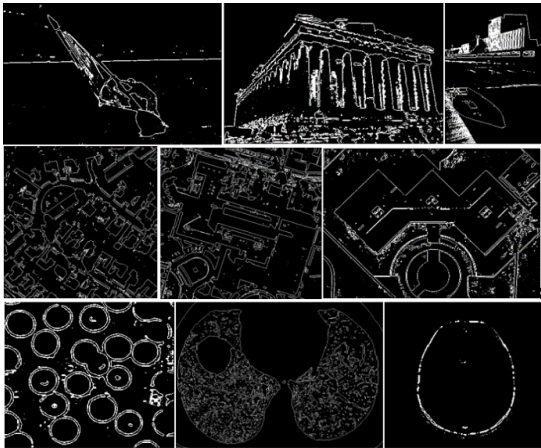


Figure 2. Results of proposed quantum exponential entropy approach

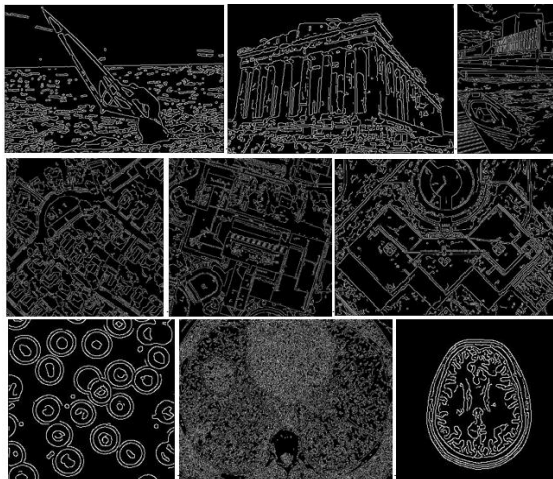


Figure 3. Results of canny algorithm

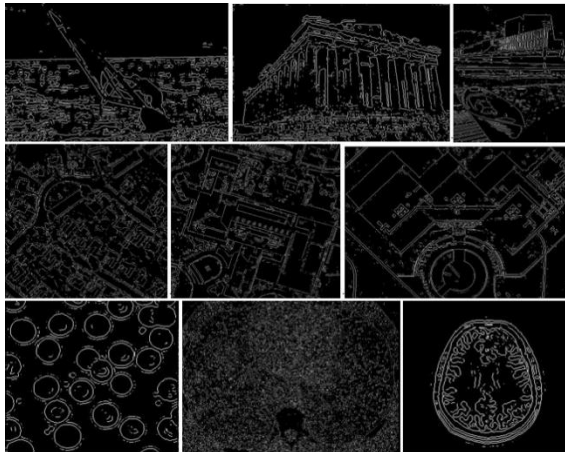


Figure 4. Results of LoG algorithm

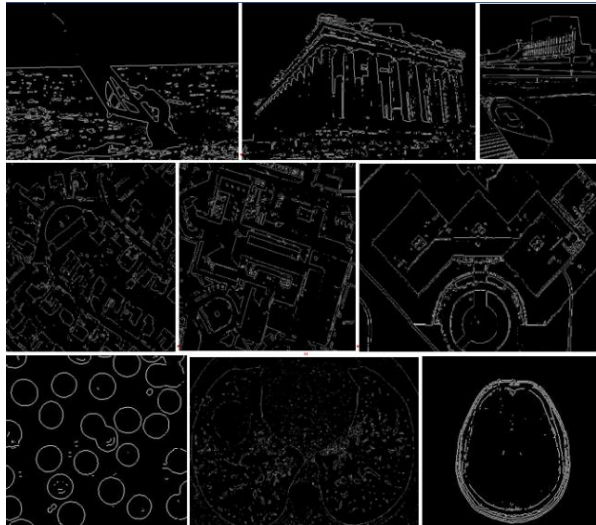


Figure 5. Results of sobel algorithm

5. CONCLUSION

In the present paper, we present the first original approach to quantum edge detection in quantum image representations. Considering the image's one-dimensional histogram as a quantum system along with the proper quantum representations, we suggest an original approach to the detection of edges. Quantum exponential entropy is a key measure of the quantity of quantum information in histogram representations. For the verification level of our approach, we conduct the first comparison study, comparing our approach to established methods like the Canny, LoG, and Sobel approaches. Based on a set of ten representative images, our numerical findings clearly indicate the benefits of our quantum-based approach. Our findings indicate the promise of quantum image representations for the further evolution of edge detection approaches. The application of quantum principles and of the quantum exponential entropy adds novel insight and novel dimensions to the detection of edges in medical images. Through the exploration of new avenues, our related approaches exhibit excellent efficiency and contribute to the further evolution of image analysis and image processing. It is the first pioneering study of quantum image representations for the detection of edges, and there is immense potential for further research. Future research studies could examine further types of quantum image representations and entropies, and also apply our approach in combination with further quantum methodologies, such as qubit-based representations, quantum Fourier transformation. The application of the use of the integration of our approach in combination with further quantum methodologies could yield further possibilities for the detection of edges and for the analysis of medical images, and adding novel dimensions to the field. The avenues for research we propose have countless possibilities for further evolution, expanding the scope of use of these approaches, both theoretically and practically.

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