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Buckling Analysis of Composites Plates Using Four Variable Refined Plate Theory

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ABSTRACT

This paper presents a combination of hyperbolic and polynomial four variable refined plate theory for first time to analyze buckling rectangular laminated plates with all simply supported edges. Parabolic variation of transverse shear stress over the thickness is presented to satisfy zero traction on the top and bottom surfaces of the plate. The governing equations solved for simply supported boundary conditions using Navier's functions are formulated and based on the total potential energy. Changing design effects (aspect, thickness, orthotropic) ratio and layers scheme on the buckling load of laminated plates under uniaxial and biaxial loading conditions are investigated in detail. This theory gives good results when compared other theories for buckling of both thick and thin plates but there were changing in mode number for some cases.

1. INTRODUCTION

Mechanical behavior of laminated plates is very important to allow safe structural design so that many researchers have searched using different methods such as theoretical (three and two) dimension elasticity theory, Numerical (finite element) solution with experimental methods. Akavci and Tanrikulu [1] used hyperbolic displacement models to investigate the buckling and frequency for simply supported [0/90] plate, the models used give good results when compared with other displacement. Aydogdu [2] improved a new higher order theory using three dimensions' elasticity bending solutions to study buckling of plate, this theory gives results accurate to three-dimension elasticity solutions. Djedid et al. [3] developed a new refined plate theory to obtain the buckling of functionally graded simply supported plates, Wankhade and Niyogi [4] used Reddy higher order (HOST) as a refined plate theory to analyze buckling of composite plates, Schreiber and Mittelstedt [5] improved the analytical stability analysis of antisymmetric laminated structures, using two methods, first method based on (classical theory with first and third deformation theory). The second method based on reduced bending method. Tounsi et al. [6] presented a novel hyperbolic higher-order shear deformation theory (HSDT) for buckling analysis of functionally graded plates, Belbachir et al. [7] employed a refined plate theory based on a hyperbolic function to obtain buckling of cross-ply composite plates, Srivastava et al. [8] developed for the initial buckling response of two-directional functionally graded material (TDFGM) plate using energy principle and discretized of radial basis function (RBF) based on considering the higher theory, Kettaf et al. [9] used different theories of thick plates to analyze mechanical and thermal buckling of laminated. Nguyen et al. [10] used Airy's stress function to obtain the buckling load of functionally graded composite plates and solved using (ABAQUS) for buckling response of laminated plate combined with geometric nonlinearity, Di Sciuva and Sorrenti [11] presented zigzag theory to analysis buckling of functionally graded plate of carbon nanotube reinforcement. Sorrenti et al. [12] investigate buckling of angle-ply multilayered and sandwich plates using the enhanced Refined Zigzag Theory (en RZT), Majeed and Abed [13] used Rayleigh-Ritz solution depending on classical laminated plate theory to investigate buckling a laminated thin plate for different boundary conditions, Hashim and Sadiq [14] used a polynomial refined plate theory (RPT) to obtain the thermal buckling analysis behavior of laminated simply supported plates, Yahea and Majeed [15] investigated vibration of laminated plates under thermal load using refined theory. Majeed and Sadiq [16] analyzed buckling and fundamental frequencies of [0/90] composite plates using new higher order theory. Singh and Chakrabarti [17] carried out buckling analysis of laminated composite plates using finite element method based on zigzag theory. Nguyen-Van et al. [18] investigated buckling load of composite plate and shell by developing the flat element.

For present work, a new RPT plate theory was employed to address the mechanical buckling analysis of simply supported plates modelled with four unknowns and no need to use correction factor. The present theory considers a parabolic distribution of transverse shear stress in the thickness, which gives exactly the boundary conditions on the free surfaces of the plate. The derived equations used in this work based on Hamilton Principle of total potential energy. Next, theoretical analysis of the buckling plates subjected to biaxial and uniaxial loading conditions has been found using the Navier's solution. The obtained results computed by the present model for critical buckling load are verified by comparing them with other results.

2. DISPLACEMENT FIELD

With the same mathematical model for displacement components of the plate, in present work, the shape function is chosen as a combination of hyperbolic and polynomial functions to satisfy the zero strain on the inner and outer surfaces of the plate is taken as [19]:

$$u\,(x,y,z) = u\,(x,y) - z\left(\frac{\partial w_b}{\partial x}\right) - F(z)\left(\frac{\partial w_s}{\partial x}\right) \tag{1}$$

$$v\,(x,y,z) = v\,(x,y) - z\left(\frac{\partial w_b}{\partial x}\right) - F(z)\left(\frac{\partial w_s}{\partial x}\right) \tag{2}$$

$$w(x,y,z) = w_b(x,y) + w_s\,(x,y) \tag{3}$$

where,

$$F\,(z) = z - h * \sinh\left(\frac{z}{h}\right) - \left(\frac{4}{3} * \frac{z^3}{h^2}\right)\cosh\,(0.5) \tag{4}$$

And u, v, w_b, w_s the four unknown functions.

The strain-displacement relations are:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \tag{5}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} \tag{6}$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \tag{7}$$

$$\varepsilon_{xy} = \frac{1}{2}\gamma_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{8}$$

$$\varepsilon_{xz} = \frac{1}{2}\gamma_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \tag{9}$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{1}{2}\gamma_{yz} \tag{10}$$

Substituting Eqs. (1)-(3) into Eqs. (5)-(10) to give:

$$\varepsilon_{xx} = \varepsilon_{xx}^0 - z\,\varepsilon_{xx}^1 - F(z)\varepsilon_{xx}^2 \tag{11}$$

$$\varepsilon_{yy} = \varepsilon_{yy}^0 - z\,\varepsilon_{yy}^1 - F(z)\varepsilon_{yy}^2 \tag{12}$$

$$\varepsilon_{xy} = \varepsilon_{xy}^0 - z\,\varepsilon_{xy}^1 - F(z)\varepsilon_{xy}^2 \tag{13}$$

$$\gamma_{xz} = \varepsilon_{xz}^0 - g\,(z)\,\varepsilon_{xz}^3 \tag{14}$$

$$\gamma_{yz} = \varepsilon_{yz}^0 - g\,(z)\,\varepsilon_{yz}^3 \tag{15}$$

where,

$$\begin{Bmatrix}\varepsilon_{xx}^0\\ \varepsilon_{yy}^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial u}{\partial x}\\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\end{Bmatrix} \tag{16}$$

$$\begin{Bmatrix}\varepsilon_{xx}^1\\ \varepsilon_{yy}^1\\ \gamma_{xy}^1\end{Bmatrix} = \begin{Bmatrix}\frac{\partial^2 w_b}{\partial x^2}\\ \frac{\partial^2 w_b}{\partial y^2}\\ 2\frac{\partial^2 w_b}{\partial x\,\partial y}\end{Bmatrix} \tag{17}$$

$$\begin{Bmatrix}\varepsilon_{xx}^2\\ \varepsilon_{yy}^2\\ \gamma_{xy}^2\end{Bmatrix} = \begin{Bmatrix}\frac{\partial^2 w_s}{\partial x^2}\\ \frac{\partial^2 w_s}{\partial y^2}\\ 2\frac{\partial^2 w_s}{\partial x\,\partial y}\end{Bmatrix} \tag{18}$$

$$\begin{Bmatrix}\gamma_{xz}^0\\ \gamma_{yz}^0\end{Bmatrix} = \begin{Bmatrix}\frac{\delta w_s}{\partial x}\\ \frac{\delta w_s}{\partial y}\end{Bmatrix} \tag{19}$$

$$g\,(z) = 1 - F'\,(z) \tag{20}$$

3. HAMILTONS PRINCIPLES

Using refined theory to derive equations of motion with the principle of virtual displacements [20]:

$$0 = \int_0^v (\delta U + \delta V)\,dv \tag{21}$$

where,

δU: virtual strain energy and

δV: virtual work done by applied forces.

$$\delta U = [\int_{\frac{-h}{2}}^{\frac{h}{2}}\{\int_{\Omega}^{k}\sigma_{xx}\delta\,\varepsilon_{xx}^k + \sigma_{yy}\delta\varepsilon_{yy}^k + \sigma_{xy}\delta\varepsilon_{xy}^k + \sigma_{yz}\delta\varepsilon_{yz}^k + \sigma_{xz}\delta\varepsilon_{xz}^k]\,dxdy\}dxdy] = 0 \tag{22}$$

$$\delta U = \int (N_1\delta\,\varepsilon_{xx}^0 + M_1^b\delta\varepsilon_{xx}^1 + M_1^s\delta\varepsilon_{xx}^2 + N_2\delta\varepsilon_{yy}^0 + M_2^b\delta\varepsilon_{yy}^1 + M_2^s\delta\varepsilon_{yy}^2 + N_6\delta\varepsilon_{Xy}^0 + M_6^b\delta\varepsilon_{Xy}^1 + M_6^s\delta\varepsilon_{Xy}^2 + Q_5\delta\varepsilon_{yz}^0 + Q_4\delta\varepsilon_{xz}^0)dxdy \tag{23}$$

where,

$$(Ni, Mi, Pi) = \sum_{k=1}^{N}\int_{z^{k-1}}^{z^k}\sigma_i^k(1, z, F(z))dz, \qquad i = (1,2,6)$$

$$(Qi) = \sum_{k=1}^{N}\int_{z^{k-1}}^{z^k}\sigma_i^k(g^2)\,dz \qquad (i = 4,5)$$

$$\delta V = \int_{\Omega}\left[N_{xx}\delta\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial x^2}\right) + N_{yy}\delta\left(\frac{\partial^2 w_b}{\partial y^2} + \frac{\partial^2 w_s}{\partial y^2}\right)\right]dx\,dy \tag{24}$$

Substituting Eqs. (11)-(15) into Eqs. (23) and (24) and integrating by parts to get energy equation in form of displacement components and resultant forces.

4. EQUATIONS OF MOTION

Substituting Eqs. (23) and (24) in Eq. (21) and to give four equations of motion as follows:

$$\delta u: \frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0 \tag{25}$$

$$\delta v: \frac{\partial N_2}{\partial y} + \frac{\partial N_6}{\partial x} = 0 \tag{26}$$

$$\delta wb: \frac{\partial^2 M_1^b}{\partial x^2} + \frac{\partial^2 M_2^b}{\partial y^2} + 2\frac{\partial^2 M_6^b}{\partial x\,\partial y} + \left(N_{xx}\frac{\partial^2 w_b}{\partial x^2} + N_{yy}\frac{\partial^2 w_b}{\partial y^2}\right) = 0 \tag{27}$$

$$\delta ws: \frac{\partial^2 M_1^s}{\partial x^2} + \frac{\partial^2 M_2^s}{\partial y^2} + 2\frac{\partial^2 M_6^s}{\partial x\,\partial y} + \left(N_{xx}\frac{\partial^2 w_s}{\partial x^2} + N_{yy}\frac{\partial^2 w_s}{\partial y^2}\right) = 0 \tag{28}$$

The result forces are given by:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \sum_{k=1}^{N} \int_{z^k}^{z^{k+1}} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} dz, \tag{29}$$

$$\begin{Bmatrix} M_1^b \\ M_2^b \\ M_6^b \end{Bmatrix} = \sum_{k=1}^{N} \int_{z^k}^{z^{k+1}} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} z\, dz, \tag{30}$$

$$\begin{Bmatrix} M_1^s \\ M_2^s \\ M_6^s \end{Bmatrix} \sum_{k=1}^{N} \int_{z^k}^{z^{k+1}} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} F(z) dz \tag{31}$$

$$\begin{Bmatrix} Q_4 \\ Q_5 \end{Bmatrix} = \sum_{k=1}^{n} \begin{Bmatrix} \sigma_5 \\ \sigma_4 \end{Bmatrix} (g^2) dz \tag{32}$$

The plane stress reduced stiffnes Q_{ij} is:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}},$$
$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13} \tag{33}$$

where, G (shear modulus), E (Young's modulus) and υ (poison's ratio) of plate.

The transformed stress-strain relation is:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} Q_{12} Q_{16} \\ Q_{12} Q_{22} Q_{26} \\ Q_{16} Q_{26} Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{34}$$

The force results are:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{bmatrix} A_{11} A_{12} A_{16} \\ A_{12} A_{22} A_{26} \\ A_{16} A_{26} A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} B_{12} B_{16} \\ B_{12} B_{22} B_{26} \\ B_{16} B_{26} B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^1 \\ \varepsilon_{yy}^1 \\ \gamma_{xy}^1 \end{Bmatrix} + \begin{bmatrix} E_{11} E_{12} E_{16} \\ E_{12} E_{22} E_{26} \\ E_{16} E_{26} E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^2 \\ \varepsilon_{yy}^2 \\ \gamma_{xy}^2 \end{Bmatrix} \tag{35}$$

$$\begin{Bmatrix} M_1^b \\ M_2^b \\ M_6^b \end{Bmatrix} = \begin{bmatrix} B_{11} B_{12} B_{16} \\ B_{12} B_{22} B_{26} \\ B_{16} B_{26} B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} D_{12} D_{16} \\ D_{12} D_{22} D_{26} \\ D_{16} D_{26} D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^1 \\ \varepsilon_{yy}^1 \\ \gamma_{xy}^1 \end{Bmatrix} + \begin{bmatrix} F_{11} F_{12} F_{16} \\ F_{12} F_{22} F_{26} \\ F_{16} F_{26} F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^2 \\ \varepsilon_{yy}^2 \\ \gamma_{xy}^2 \end{Bmatrix} \tag{36}$$

$$\begin{Bmatrix} M_1^s \\ M_2^s \\ M_6^s \end{Bmatrix} = \begin{bmatrix} E_{11} E_{12} E_{16} \\ E_{12} E_{22} E_{26} \\ E_{16} E_{26} E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} F_{11} F_{12} F_{16} \\ F_{12} F_{22} F_{26} \\ F_{16} F_{26} F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^1 \\ \varepsilon_{yy}^1 \\ \gamma_{xy}^1 \end{Bmatrix} + \begin{bmatrix} H_{11} H_{12} H_{16} \\ H_{12} H_{22} H_{26} \\ H_{16} H_{26} H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^2 \\ \varepsilon_{yy}^2 \\ \gamma_{xy}^2 \end{Bmatrix} \tag{37}$$

$$\begin{Bmatrix} Q_4 \\ Q_5 \end{Bmatrix} = \begin{bmatrix} L_{44} & L_{45} \\ L_{45} & L_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{38}$$

where,

$$A_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}\, dz \quad i = (1,2,4,5,6) \tag{39}$$

$$B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}(z, z^2, F(z), z * F(z), (F(z))^2)\, dz \quad i = (1,2,6) \tag{40}$$

$$L_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}\,(g^2) dz \quad i,j = (4,5) \tag{41}$$

5. BUCKLING ANALYSIS

Substituting Navier's equations which satisfied simply supported boundary conditions [20], for (cross – angle) ply with the force and moment resultants from Eqs. (29)-(32) into equations of motion Eqs. (25)-(28), the following eigenvalue equation is obtained:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ - & C_{22} & C_{23} & C_{24} \\ - & - & C_{33}-(\alpha^2 * N_x + k*\beta^2 * N_y) & C_{34} \\ - & - & - & C_{44}-(\alpha^2 * N_x + k*\beta^2 * N_y) \end{bmatrix} \{d_{ij}\} = 0$$

where,

$$\{d_{ij}\} = \{U_{mn}, V_{mn}, W_{bmn,} W_{smn}\}$$

C_{ij}= stiffness element which given in appendix.

6. RESULTS AND DISCUSSIONS

A comparison of buckling load for laminated plates using a combination of hyperbolic and polynomial displacement function [19], with other plate theories and solving techniques are investigated also plates with different thickness (a/h) ratio, orthotropic ratio, number of plies, loading conditions and aspect (a/b) ratio are studied and solved by Matlab22 program.

Different theories and solution methods used by other researchers are compared to present theory which takes little efforts than other analytical or numerical methods based on five variables refined or third order plate theories as shown in Table 1, which present a comparison between present four variable refined theory, TSDT and finite element method for [0/90] square plate for different (a/h), the results give good agreement and mode number not changed.

Table 1. Effect of thickness ratios (a/h) on nondimentional critical uni-axial buckling loads (Ncr) for simply supported [0/90] square plate

Source	(a/h)			
	(10)	(20)	(50)	(100)
Our Work	11.126	12.452	12.884	12.948
Ref. [16]	11.616	12.602	12.910	12.955
Ref. [17]	11.310	12.427	12.800	12.873
Ref. [18]	11.360	12.551	12.906	13.039
Ref. [21]	11.349	12.510	12.879	12.934
Ref. [22]	11.563	12.577	12.895	12.942

Notes: Using material 1. Mode for all: (q=s=1) accept when written.

Table 2. Effect of thickness ratios (a/h) on normalized critical uni-axial buckling loads (Ncr) for angle-ply square plate, using material 2. Mode for all: (q=s=1) accept when written

a/h	Source	Layers Type		
		[5 /-5]6	[30/ -30]6	[45/ -45]6
5	Ref. [20]	11.082	13.546	12.169
	Present work	13.572 q=2, s=1	13.357 q=2, s =1	12.7635 q=3, s =1
10	Ref. [20]	22.592	33.701	32.405
	Present work	26.399	35.806	34.680 q=2, s =1
20	Ref. [20]	31.577	47.643	53.198
	Present work	33.628	50.572	56.166
50	Ref. [20]	35.657	53.951	60.760
	Present work	36.434	57.227	64.496

Notes: Using material 2. Mode for all: (q=s=1) accept when written.

Table 2 presents buckling load for different angle plates and give good agreement with those obtained by other researchers used TSDT, but with changing mode number for some thick and moderately thick plates. Table 3 shows the effect of (b/a) for the laminated plate on buckling load which give the same behavior to those obtained by other researchers, also changing orthotropic ratio (E1/E2) shown in Table 4, but under biaxial in plane loading and give results close to those obtained by studies [22].

Table 3. Nondimentional uni-axial buckling loads (Ncr) for different (a/h) and (a/b) for [0/90]s square plate

a/b	Source	a/h				
		5	10	20	50	100
0.5	Ref. [16]	8.848	18.488	25.856	29.151	29.693
	Ref. [17]	8.739	18.347	25.746	29.087	29.657
	Present work	9.113	18.931	26.092	29.200	29.706
1	Ref. [16]	12.029	23.394	31.716	35.400	36.005
	Ref. [17]	11.858	23.134	31.517	35.278	35.923
	Present work	11.614	23.519	31.888	35.442	36.016
2	Ref. [16]	16.681 (q=3, s=1)	48.119 (q=2, s=1)	93.579 (q=2, s=1)	113.21 (q=1, s=1)	115.335 (q=1, s=1)
	Ref. [17]	15.000	47.368	92.847	112.81	115.029
	Present work	15.631 q=3, s=1	46.458 q=2, s=1	92.572	111.52	114.891

Notes: Using material 1. Mode for all: (q=s=1) accept when written.

Table 4. Normalized critical biaxial buckling load for different thickness and orthotropic ratios for different angle-lamination square plate

a/h	Source	E1/E2=10	
		[45/ -45]	[45/ -45]4
10	Ref. [22]	3.923	6.771
	Present work	4.043	6.994
100	Ref. [22]	4.526	8.792
	Present work	4.542	8.844
a/h	**Source**	**E1/E2=25**	
		[45/ -45]	**[45/ -45]4**
10	Ref. [22]	6.115	12.067
	Present work	6.468	12.715
100	Ref. [22]	7.717	20.437
	Present work	7.735	20.502

Notes: Using material 1. Mode for all: (q=s=1) accept when written.

Buckling load for symmetric cross and angle ply plate with antisymmetric ones are listed in Table 5, from which it is noted that symmetric plies have larger buckling load than antisymmetric and angle plied plate has larger buckling load than cross plied plate same behavior given by other theories. As expected, critical buckling load for laminated plate under uniaxial load are larger than those under biaxial load as shown in Table 6.

Material1: the first one used in present work is: E1/E2 = 40, G12 = G13 = 0.6 E2 (Gpa), G23 = 0.2 E2 (Gpa), υ12 = υ13= 0.25.

Material 2: the second one used in present work is: E1/E2 =40, G12=G13=0.6E2 (Gpa), G23=0.5E2 (Gpa), υ12= υ13=0.25 and Ncr = (N*a2/E2*H3).

Table 5. Comparison of nondimentional uni-axial buckling loads (Ncr) with different orthotropic ratio for different cross and angle-ply square plate

Layers	E1/E2				
	5	10	20	30	40
[0/ 90]s	6.887	10.256	16.267	21.459	25.992
[0/ 90]2	6.481	9.267	14.284	18.693	22.602
[45 /-45]s	9.100	15.269	25.232	31.102 q=2, s=1	35.086 q=2, s=1
[45/ -45]2	8.387	13.565	22.065	27.894 q=2, s=1	31.584 q=2, s=1

Notes: Using material 2. Mode for all: (q=s=1) accept when written.

Table 6. Comparison of nondimentional critical biaxial buckling loads (Ncr) with various orthotropic ratio for different cross and angle-ply square plate, (a/h = 10)

Layers	E1/E2			
	Uniaxial buckling		Biaxial buckling	
	20	40	20	40
[0 /90]s	16.267	25.992	8.133	12.996
[0/ 90]2	14.284	22.602	7.142	11.301
[30/ -30]s	21.201	27.856 q=2, s=1	10.600	15.839
[30/ -30]2	18.783	25.575 q=2, s=1	9.391	14.061

Notes: Using material 1. Mode for all: (q=s=1) accept when written.

7. CONCLUSIONS

Buckling analysis of cross and angle laminated simply supported plates is studied by using refined hyperbolic shear theory for first time, under two types of mechanical loadings. The displacement field of the proposed theory contains four unknowns, and involves a hyperbolic shape function to account for more acceptable distribution of the transverse shear strains through the thickness; with no need for shear correction factor. The equations are derived by using the Hamilton's principle and the analytical solutions are obtained using the Navier's solution method. The reliability of the present approach is checked by comparing it with various shear deformation theories. The numerical results show that the proposed refined plate theory is in excellent agreement with respect to other higher-order shear deformation theories for the evaluation of critical buckling of laminated plates but changing mode number.

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NOMENCLATURE

a	Plate dimension in x-direction (m)
b	Plate dimension in y-direction (m)
h	Plate thickness
E1, E2, E3	Elastic modulus components (GPa)
A_{ij}, B_{ij}, D_{ij}, B^s_{ij}, D^s_{ij}, H^s_{ij}	Extension, bending, extension coupling (N/m)
k=0 or 1	In plane load factor
n	Total number of plate layers
N_x, N_y, N_{xy}	In-plan force per unit length (N/m)
M^b_x, M^b_y, M^b_{xy}	Bending moment result per unit length (N.m/m)
M^s_x, M^s_y, M^s_{xy}	Force per unit length due to shear moment (N/m)
Q_{xz}, Q_{yz}	Transverse shear force (N)
TSDT	Third shear deformation theory
x, y, z	Cartesian Coordinate system
W_b, W_s	Displacement in, bending and shear respectively
u_s, v_s	Displacement in x ans y direction due to shear respectively

Greek symbols

ε_x, ε_y, ε_z	Strain components (m/m)
γ_{xz}, γ_{yz}	Transverse shear strain (m/m)
σ_x σ_y σ_{xy} σ_{yz} σ_{xz}	Stress components (Gpa)
v_{12} v_{21}	Poisson's ratio

APPENDIX

$$C_{11} = - A11 * (\alpha^2) - A66 * (\beta^2);$$

$$C_{12} = - A12 * \alpha * \beta - A66 * \alpha * \beta;$$

$$C_{13} = (B11 * \alpha^3) + (B12 * \alpha * \beta^2) + (2 * B66 * \alpha * \beta^2);$$

$$C_{14} = (E12 * \alpha * \beta^2) + (E11* \alpha^3) + (2 * E66 * \alpha * \beta^2);$$

$$C_{22} = (-A66 * \alpha^2) - (A22 * \beta^2);$$

$$C_{23} = (2 * B66 * \beta * \alpha^2) + (B12 * \beta * \alpha^2) + (B22 * \beta^3);$$

$$C_{24} = (E12 + (2 * E66)) *(\beta * \alpha^2) + (\beta^3) * (E22);$$

$$C_{33} = (-D22 * \beta^4) - (D11 * \alpha^4) - (2 * D12 * (\alpha^2) * (\beta^2)) - (4 * D66 * (\beta^2) * (\alpha^2));$$

$$C_{34} = (\alpha^4) * (-F11) + ((\alpha^2) * \beta^2) * (-2 * F12 - 4 * F66) + (\beta^4) * (-F22);$$

$$C_{44} = (-H11) * (\alpha^4) + ((\alpha^2) * \beta^2) * (-2 * H12 - 4 * H66) + (\beta^4 * (-H22) -(L11) * (\alpha^2) - (L22) * (\beta^2).$$