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Optimal Control Problem of Quarantined-Susceptible-Infected-Quarantined-Recovered Mathematical Model of the COVID-19 Epidemic with Fuzzy Parameter in Indonesia

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ABSTRACT

The COVID-19 pandemic started at the end of 2019 and spread fast around the world. This pandemic had a significant influence on daily life. In Indonesia, this pandemic has had significant effects, such as increasing cases and hard challenges to the health and economic system. In this research, we analyze the optimal control model of the COVID-19 epidemic in Indonesia by Considering Government Policies. We have two control parameters: a policy to prevent the spread of the disease among susceptible people and quarantine efforts accompanied by treatment to minimize infection or maximize recovery. We have applied Pontryagin's Maximum Principle and the cost-effectiveness analysis method to obtain the optimal solution. The cost-effectiveness analysis shows that the application of the two control actions at every temperature is significantly more cost-effective in preventing the spread of infection than when only a single control is applied.

1. INTRODUCTION

The COVID-19 pandemic that began in late 2019 has spread rapidly throughout the world, affecting various sectors of life, from health to the economy [1, 2]. The SARS-CoV-2 virus, which causes COVID-19, has resulted in millions of infections and deaths worldwide, requiring many countries to implement social restrictions and close public facilities to control its spread [3, 4]. In Indonesia, the pandemic has also had a significant impact with a spike in cases and major challenges for the health system and economy.

The Indonesian government has taken strategic steps to suppress the spread of COVID-19 by implementing various policies, ranging from large-scale social restrictions Pembatasan Sosial Berskala Besar (PSBB) [5] to the implementation of community activity restrictions Pemberlakuan Pembatasan Kegiatan Masyarakat (PPKM) [6]. In addition, the government has intensified the national vaccination campaign [7], encouraged the adaptation of health protocols in the community, and provided medical care for COVID-19 patients. These policies aim to suppress the rate of virus transmission and maintain economic stability amidst the challenges of the pandemic.

Mathematical models of optimal control are used to analyze and determine the best policies to control the spread of COVID-19 in Indonesia. The latest models developed now take into account factors such as vaccine effectiveness [8], new variants of SARS-CoV-2, and the impact of community behavioral adaptations to health protocols. Models such as the modified SEIR model [9-11], adaptive optimal control-based models, and machine learning-based models are used to predict the development of the pandemic and determine optimal steps in controlling COVID-19 in Indonesia. This approach aims to minimize the negative impact of the pandemic on public health and the economy, as well as support data-driven government policies.

Various studies on mathematical models of the dynamics of the spread of COVID-19 and its control that have existed are still incomplete, thus providing an opportunity for further research. Several studies have accommodated quarantine in the models developed, such as research conducted by Rois and Trisilowati [12] and Tiwari [13], although it only contains one quarantine class, namely quarantine for infected people. Abdy et al. [14] have developed a mathematical model for the COVID-19 outbreak in Indonesia with fuzzy parameters, but the model developed does not include quarantine classes and there is no optimal control analysis. Rois et al. [7, 15] have developed a mathematical model that includes quarantine classes and has also conducted optimal control analysis, but their model does not include quarantine classes from immigration. Several studies have stated that there is a link between weather, especially temperature and humidity, and the transmission of COVID-19. Tosepu et al. [16] stated that of the several weather components, only average temperature is significantly correlated with the COVID-19 pandemic. The same thing was also stated by Wang et al. [17], Anis [18], and Fang et al. [19]. Therefore, this article proposes a Quarantined-Susceptible-Infected-Quarantined-Recovered (QSIQR) optimal control model with fuzzy parameters that explicitly distinguish two types of quarantine subpopulations (quarantine from immigration and quarantine of infected people) and also associates temperature with some model parameters, as an attempt to fill the gap in the existing literature and provide a more realistic approach to the dynamics of COVID-19 spread in Indonesia.

The research on the mathematical model of optimal control in the COVID-19 outbreak that underlies this research is a study conducted by Rois et al. [20], which examines the optimal control of COVID-19 in Indonesia with comorbidity. Meanwhile, the research on the mathematical model of the COVID-19 outbreak with fuzzy parameters that underlies this research is a study conducted by Abdy et al. [14], which examines the SIR model with fuzzy parameters for the COVID-19 outbreak in Indonesia. The model developed in this study adds two control parameters as conducted by Rois et al. [20]. Abdy et al. [14] provided the basis for formulating fuzzy membership functions on several parameters in the model. The difference is only in the crisp variables used; if Abdy et al. [14] used the size of the virus in the body as the crisp variable, while in this study, the temperature of the area is used as the crisp variable. The novelty of this research is the addition of a quarantine class from immigration, the definition of parameters using a fuzzy membership function with temperature as the crisp variable, and the analysis of optimal control on the resulting model.

2. MATERIALS AND METHODS

2.1 Model formulation

The transfer diagram of the model can be seen in Figure 1 and the meaning of every parameter is given in Table 1. The definition of every variable is given. $S, I,$ and R are the number of susceptible, infected, and recovered people respectively. Q_1 is the number of quarantined persons from immigration. Q_T is the number of quarantined infected people.

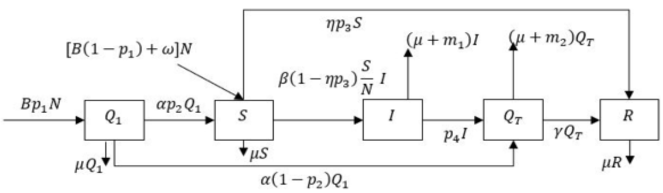


Figure 1. The transfer diagram of the QSIQR model

Table 1. The definition of the parameter in the model

Parameter	Definition
B	the rate of persons entering the population from immigration
p_1	the proportion of persons quarantined from immigration
p_2	the proportion of quarantined persons free from infection
ω	the birth rate
μ	the natural death rate
p_3	the vaccination rate of susceptible persons
α	the rate of persons out of quarantine
β	the infection rate of susceptible persons
η	the effectiveness of vaccination
m_1, m_2	the death rate because of infection
p_4	the rate of infected persons who get quarantine and treatments
γ	the recovery rate of the quarantined person

In this research, we assumed that the birth rate has the same value as the natural death rate, the death rate due to infection of the quarantine-infected group is too small, and all immigration persons must be quarantined.

Kementerian Kesehatan Republik Indonesia [21] stated that the majority of deaths due to COVID-19 in Indonesia occurred in patients with comorbidities, such as heart disease, diabetes, and respiratory disorders, while patients undergoing centralized isolation or quarantine with mild to moderate symptoms showed a very high recovery rate. The death rate of fully vaccinated COVID-19 patients, most of whom were included in the group undergoing centralized quarantine, was only 0.21% [22]. The results of the study by Bi et al. [23] showed that the rate of severe symptoms and death was very low, especially among those who were identified and quarantined early. There were no deaths among close contacts who were quarantined, indicating the effectiveness of quarantine in preventing the severity of infection. Based on these data, it can be concluded that the death rate in the subpopulation quarantined due to infection is very low, so it can be assumed to be zero in mathematical modeling. Meanwhile, the basis for choosing the value $p_1 = 1$ is the government policies that require everyone who comes from a country or region infected with an infectious disease to undergo health quarantine. The policies can be found in the studies [24-26]. Hence, $\omega = \mu$, $m_2 = 0$, and $p_1 = 1$, then based on Figure 1, we got System (1)

$$\begin{aligned}
\frac{dQ_1}{dt} &= BN - (\alpha+\mu)Q_1 \\
\frac{dS}{dt} &= \alpha p_2 Q_1 + \mu N - \beta(1-\eta p_3)\frac{S}{N} I - (\mu + \eta p_3)S \\
\frac{dI}{dt} &= \beta(1-\eta p_3)\frac{S}{N} I - (\mu + m_1 + p_4)I \\
\frac{dQ_T}{dt} &= \alpha(1-p_2)Q_1 + p_4 I - (\mu+\gamma)Q_T \\
\frac{dR}{dt} &= \gamma Q_T + \eta p_3 S - \mu R \\
N &= Q_1 + S + I + Q_T + R
\end{aligned} \tag{1}$$

We give the initial condition of every variable in the System (1) such that

$$Q_1(0) \geq 0, S(0) \geq 0, I(0) > 0, Q_T(0) \geq 0 \text{ and } R(0) \geq 0 \tag{2}$$

2.2 Parameter estimation and data fitting

In this subsection, we did parameter estimation and model validation. We used data from many kinds of sources. Data on the population of Indonesia based on the 2020 census was obtained from the reference [27], and we got 270,203,917 persons. We estimate the parameter B using immigration data from the study [28]. We got that the daily average of immigration from September 2020 to October 2021 is 4360.390588 persons. Using the total population of Indonesia based on census in 2020, we got $B = 0.00001614$. We got $\mu = 0.0000357$ from the study [29] (assuming that the life expectancy of Indonesian people is about 77 years). We used data about the COVID-19 epidemic from the study [30]. We used data about new deaths and newly recovered from June 13th, 2021, to October, 19th, 2021, and we obtained $m_1 = 0.001971553$ and $\gamma = 0.063230801$. The quarantine period for an immigration person is about 7 to 14 days, so we got the value of α is about 0.07142-0.14286. Nasir et al. [31] stated that the range of η is 62.1% - 95%. Nasir et al. [31] also stated that the average daily vaccination rate is 50,056 – 71,050 doses, so we got the range p_3 is 0,0001866 to 0,0002649. We estimated parameter β, p_2, and p_4 using the fourth-order Runge-Kutta Method. We also use data on the number of infected persons from June 13th, 2021 to October, 19th, 2021 (128 data). We have chosen data from that period because the peak of the outbreak occurred during that period. We got $\beta = 0.97, p_2 = 0.99$, and $p_4 = 0.9$. We also got MAPE= 0.098667. We used initial values of every parameters, i.e., $Q_{1_0} = 139433, I_0 = 113388$, $R_0 = 1745091$ and we assumed that $Q_{T_0} = I_0$ and $S_0 = N_0 - (Q_{1_0} + I_0 + Q_{T_0} = R_0)$ where $N_0 = 270{,}203{,}917$. A graphic of I data versus I estimation is given in Figure 2. The values of all parameters are given in Table 2.

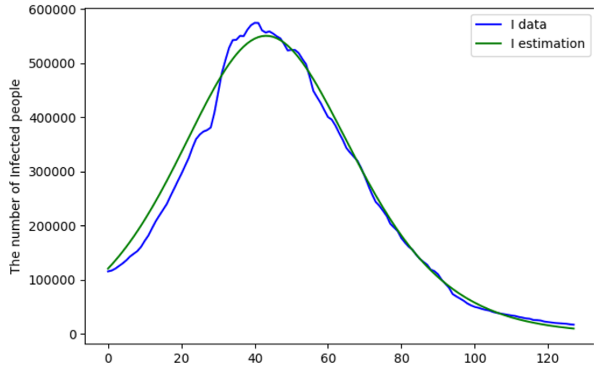


Figure 2. Graphic I data and I estimation

Table 2. The value of the parameter in the model

Parameter	Value	Parameter	Value
B	0.00001614	β	0.97
μ	0.0000357	η	0.95
α	0.071428571	p_3	0.000263158
m_1	0.001971553	p_4	0.9
p_2	0.99	γ	0.063230801

2.3 The membership function of the fuzzy parameter

In this research, we assumed that the humidity is constant. Using the membership function of fuzzy parameters, we defined β and p_4 as follows:

$$\beta(T) = \begin{cases} \beta_{min}, & \text{if } T < T_{min}; \\ \beta_{min} + \beta_1(1-\pi)(1-\theta).\dfrac{(T - T_{min})}{(T_{opt} - T_{min})}, & \text{if } T_{min} \le T < T_{opt}; \\ \beta_{min} + \beta_1(1-\pi)(1-\theta).\dfrac{(T_{max} - T)}{(T_{max} - T_{opt})}, & \text{if } T_{opt} \le T < T_{max}; \\ \beta_{min}, & \text{if } T \ge T_{max} \end{cases} \quad (3)$$

where,

$$\beta_{min} + \beta_1(1-\pi)(1-\theta) = 1 \Leftrightarrow \beta_{min} = 1 - \beta_1(1-\pi)(1-\theta),$$

$$p_4(\beta) = \begin{cases} p_4^0, & \text{if } 0 \le \beta < \beta_{min} \\ p_4^0 + c.\beta(T), & \text{if } \beta_{min} \le \beta < \beta(T_{opt}) \\ 1, & \text{if } \beta = \beta(T_{opt}) \end{cases}$$

and $p_4^0 + c.\beta(T_{opt}) = 1 \Leftrightarrow p_4^0 = 1 - c[\beta_{min} + \beta_1(1-\pi)(1-\theta)]$. Let $Y_{(\pi,\theta)} = [\beta_{min} + \beta_1(1-\pi)(1-\theta)]$. Hence, we get Eq. (4)

$$p_4(T) = \begin{cases} 1 - c.Y_{(\pi,\theta)}, & \text{if } T < T_{min} \\ 1 - c.Y_{(\pi,\theta)}\left[1 - \dfrac{(T - T_{min})}{(T_{opt} - T_{min})}\right], & \text{if } T_{min} \le T < T_{opt} \\ 1 - c.Y_{(\pi,\theta)}\left[1 - \dfrac{(T_{max} - T)}{(T_{max} - T_{opt})}\right], & \text{if } T_{opt} \le T < T_{max} \\ 1 - c.Y_{(\pi,\theta)}, & \text{if } T \ge T_{max} \end{cases} \quad (4)$$

where, β_1 is the standard virus transmission rate (based on the characteristics of the virus), π is the proportion of susceptible persons in implementing health protocols, θ is the effectiveness of government policies like vaccination and quarantine, and c is the weight of β for p_4. T_{min}, T_{opt}, and T_{max} successively are minimum, optimum, and maximum temperatures (°C). Anis [18] said that the optimal temperature for the spread of COVID-19 ranges from 13°C to 24°C, where cities with temperatures below 24°C are categorized as high-risk areas for transmission. Temperatures between 26°C–30°C with humidity above 60% do not have a significant impact on the spread of COVID-19 [32]. Let $\beta_1 = 0.99, T_{min} = 4, 13 \le T_{opt} \le 24$, and $T_{max} = 26$. Assumed that the value of $\beta = 0.97$ and $p_4 = 0.9$ (Table 2) occurred at $T = 25, \pi = 0.8$ and $\theta = 0.698$. Then we get $c = 0.2$. The value of β and p_4 based on T are given in Table 3.

The graphs of I with changing temperature (without control parameter) are given in Figure 3.

From Table 3 and Figure 3, the ratio between the rate of quarantine for infected people and the rate of infection is greater, causing the outbreak to disappear more quickly.

Table 3. The value of β and p_4 based on T where $\pi = 0.8$ and $\theta = 0.698$

T	β	p_4	$\frac{p_4}{\beta}$
7°C	0.960136	0.866667	0.90265
10°C	0.980068	0.933333	0.95231
22.5°C	1	1	1
25°C	0.970102	0.9	0.92774

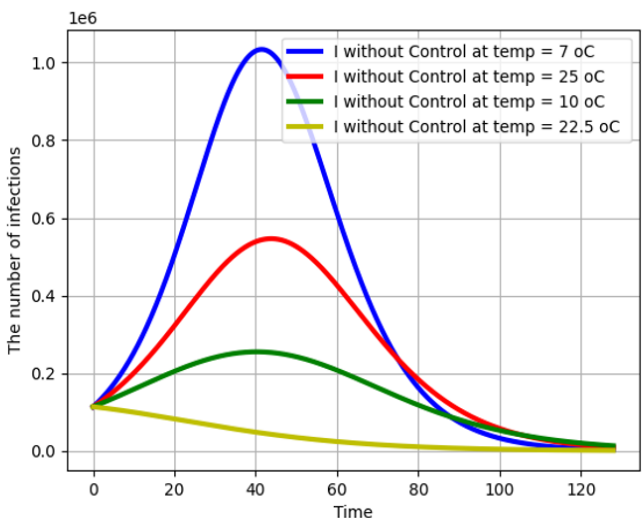


Figure 3. The graph of I by changing temperature

2.4 Control optimal model

We added two control parameters to System (1), and let $\beta(T) = \hat{\beta}$, $p_4(T) = \widehat{p_4}$, then we got System (5)

$$\begin{aligned}
\frac{dQ_1}{dt} &= BN - (\alpha + \mu)Q_1 \\
\frac{dS}{dt} &= \alpha p_2 Q_1 + \mu N - (1 - u_1)\hat{\beta}(1 - \eta p_3)\frac{S}{N}I \\
&\quad - (\mu + \eta p_3)S \\
\frac{dI}{dt} &= (1 - u_1)\hat{\beta}(1 - \eta p_3)\frac{S}{N}I - (\mu + m_1 + \widehat{p_4} + u_2)I \\
\frac{dQ_T}{dt} &= \alpha(1 - p_2)Q_1 + (\widehat{p_4} + u_2)I - (\mu + \gamma)Q_T \\
\frac{dR}{dt} &= \gamma Q_T + \eta p_3 S - \mu R \\
N &= Q_1 + S + I + Q_T + R
\end{aligned} \tag{5}$$

The meaning of two control parameters is:

u_1: a policy to prevent the spread of disease among susceptible people

u_2: quarantine and treatment efforts to minimize infection or maximize recovery.

Let $U = \{(u_1(t), u_2(t)): 0 \le u_1 < 1, 0 \le u_2 < 1, 0 < t < t_f\}$ is the set of receivable controls.

The goal is to find a control u that produces the lowest value for the objective function J without sacrificing the cost efficiency of implementation in System (5).

$$\text{Let } J = \min_{u_1,u_2} \int_0^{t_f} \left(I + \frac{1}{2}\sum_{i=1}^{2} w_i u_1^2 \right) dt \tag{6}$$

Subject to (5) where w_1 and w_2 are positive constants representing the relative cost weights for implementing control efforts u_1 and u_2 [15]. We assumed that the costs were non-linear. Therefore, the control variables in the objective function J are in the form of second-degree polynomials [33, 34]. Our main objective is to minimize the number of people exposed and affected by the disease while keeping the control costs as low as possible. Thus, we are going to find optimal controls (u_1^*, u_2^*), such that

$$J(u_1^*, u_2^*) = \min\{J(u_1, u_2) | (u_1, u_2) \in U\}$$

where, u_1 and u_2 are measurable with $0 \le u_i < 1, i = 1,2$ for $t \in [0, t_f]$.

3. RESULTS AND DISCUSSION

3.1 The Hamiltonian and optimality system

Here, we can formulate the necessary conditions for applying the Pontryagin Maximum Principle to obtain the optimal solution [12]. Therefore, this principle converts the model Eqs. (5), and (6) into a problem of minimizing a Hamiltonian, H, pointwise concerning u_1 and u_2, and we obtained a Hamiltonian (H) defined as:

$$H(t, x(t), u(t), \lambda(t)) = f(t, x(t), u(t)) + \lambda g(t, x(t), u(t))$$

where,

$$f(t, x(t), u(t)) = I + \frac{1}{2}w_1 u_1^2 + \frac{1}{2}w_2 u_2^2,$$
$$g(t, x(t), u(t)) = (g_1, g_2, g_3, g_4, g_5, g_6)^T,$$

and

$$\begin{aligned}
g_1 &= BN - (\alpha + \mu)Q_1 \\
g_2 &= \alpha p_2 Q_1 + \mu N - (1 - u_1)\hat{\beta}(1 - \eta p_3)\frac{S}{N}I - (\mu + \eta p_3)S \\
g_3 &= (1 - u_1)\hat{\beta}(1 - \eta p_3)\frac{S}{N}I - (\mu + m_1 + \widehat{p_4} + u_2)I \\
g_4 &= \alpha(1 - p_2)Q_1 + (\widehat{p_4} + u_2)I - (\mu + \gamma)Q_T \\
g_5 &= \gamma Q_T + \eta p_3 S - \mu R
\end{aligned}$$

where,

$$N = Q_1 + S + I + Q_T + R$$

Hence the Hamiltonian becomes $Q_1(0) \ge 0, S(0) \ge 0, I(0) > 0, Q_T(0) \ge 0$, and $R(0) \ge 0$.

$$\begin{aligned}
H(t, Q_1, S, I, Q_T, R) &= f(t, I, u_1, u_2) + \lambda_1 \frac{dQ_1}{dt} \\
&\quad + \lambda_2 \frac{dS}{dt} + \lambda_3 \frac{dI}{dt} + \lambda_4 \frac{dQ_T}{dt} + \lambda_5 \frac{dR}{dt} \\
H &= I + \frac{1}{2}w_1 u_1^2 + \frac{1}{2}w_2 u_2^2 + \lambda_1 g_1 \\
&\quad + \lambda_2 g_2 + \lambda_3 g_3 + \lambda_4 g_4 + \lambda_5 g_5
\end{aligned}$$

where, λ_i , $i = 1,2,\ldots,5$ are adjoint variables with transversality conditions $\lambda_i(t_f) = 0$, $i = 1,2,\ldots,5$ for an optimal control (u_1^*, u_2^*) that minimizes $J(u_1, u_2)$ and

$$\frac{d\lambda}{dt} = -\frac{\partial H}{\partial X}$$

where, $X = (Q_1, S, I, Q_T, R)^T$ and $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)^T$, $\lambda(t_f) = 0$ transcendentality condition.

Hence

$$\begin{aligned}
\frac{d\lambda_1}{dt} &= -\frac{\partial H}{\partial Q_1} = -\lambda_1[B - (\alpha + \mu)] + (\lambda_3 - \lambda_2)(1 - u_1) \\
&\quad \hat{\beta}(1 - \eta p_3)\frac{SI}{N^2} - \lambda_2(\alpha p_2 + \mu) - \lambda_4 \alpha(1 - p_2) \\
\frac{d\lambda_2}{dt} &= -\frac{\partial H}{\partial S} = -\lambda_1 B - (\lambda_3 - \lambda_2)(1 - u_1) \\
&\quad \hat{\beta}(1 - \eta p_3)\frac{I(Q_1 + I + Q_T + R)}{N^2} + \lambda_2 \eta p_3 - \lambda_5 \eta p_3 \\
\frac{d\lambda_3}{dt} &= -\frac{\partial H}{\partial I} = -1 - \lambda_1 B - \lambda_2 \mu \\
&\quad -(\lambda_3 - \lambda_2)(1 - u_1)\hat{\beta}(1 - \eta p_3)\frac{S(Q_1 + S + Q_T + R)}{N^2} \\
&\quad + \lambda_3(\mu + m_1 + \widehat{p_4} + u_2) - \lambda_4(\widehat{p_4} + u_2) \\
\frac{d\lambda_4}{dt} &= -\frac{\partial H}{\partial Q_T} = -\lambda_1 B - \lambda_2 \mu + (\lambda_3 - \lambda_2)(1 - u_1) \\
&\quad \hat{\beta}(1 - \eta p_3)\frac{SI}{N^2} + \lambda_4(\mu + \gamma) - \lambda_5 \gamma \\
\frac{d\lambda_5}{dt} &= -\frac{\partial H}{\partial R} = -\lambda_1 B - \lambda_2 \mu + (\lambda_3 - \lambda_2)(1 - u_1) \\
&\quad \hat{\beta}(1 - \eta p_3)\frac{SI}{N^2} + \lambda_5 \mu
\end{aligned} \tag{7}$$

Similarly, we obtained the controls by solving the equation $\frac{\partial H}{\partial u_1} = 0$ at u_i^*, for $i = 1,2$ following Pontryagin's methods and obtained:

$$\frac{\partial H}{\partial u_1} = 0 \Leftrightarrow u_1 = \frac{(\lambda_3 - \lambda_2)\hat{\beta}\,(1 - \eta p_3)}{w_1}\frac{SI}{N}$$

$$\frac{\partial H}{\partial u_2} = 0 \Leftrightarrow u_2 = \frac{(\lambda_3 - \lambda_4)}{w_2} I$$

Hence,

$$u_1^* = \max\left\{0, \min\left\{1, \frac{(\lambda_3 - \lambda_2)\hat{\beta}(1 - \eta p_3)}{w_1}\frac{SI}{N}\right\}\right\} = \begin{cases} 0, \dfrac{(\lambda_3 - \lambda_2)\hat{\beta}(1 - \eta p_3)}{w_1}\dfrac{SI}{N} \leq 0 \\ \dfrac{(\lambda_3 - \lambda_2)\hat{\beta}(1 - \eta p_3)}{w_1}\dfrac{SI}{N}, \\ 0 < \dfrac{(\lambda_3 - \lambda_2)\hat{\beta}(1 - \eta p_3)}{w_1}\dfrac{SI}{N} < 1 \\ 1, \dfrac{(\lambda_3 - \lambda_2)\beta(1 - \eta p_3)}{w_1}\dfrac{SI}{N} \geq 1 \end{cases} \quad (8)$$

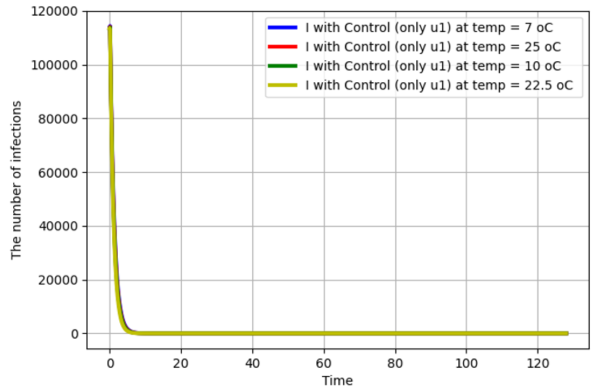
$$u_2^* = \max\left\{0, \min\left\{1, \frac{(\lambda_3 - \lambda_4)}{w_2} I\right\}\right\} = \begin{cases} 0, \dfrac{(\lambda_3 - \lambda_4)}{w_2} I \leq 0 \\ \dfrac{(\lambda_3 - \lambda_4)}{w_2} I, 0 < \dfrac{(\lambda_3 - \lambda_4)}{w_2} I < 1 \\ 1, \dfrac{(\lambda_3 - \lambda_4)}{w_2} I \geq 1 \end{cases} \quad (9)$$

3.2 Numerical simulation

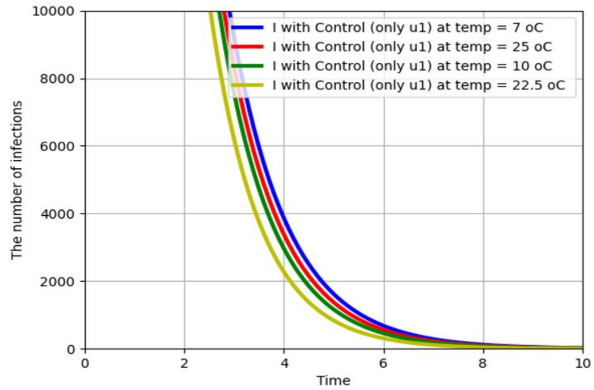
We use the parameter value in Table 1 for simulation. We use the following data to calculate the values of w_1 and w_2. The total cost for handling the COVID-19 outbreak is IDR 1895.5 trillion [35]. The total cost of vaccination is IDR 57.84 trillion [36] and counseling is IDR 0.75 trillion [37], so the total cost for vaccination and counseling is IDR 58.59 trillion. Hence, we get $w_1 = \frac{58.59}{1895.5} \approx 0.03$. The total cost of quarantine and treatment in 2020 is IDR 62.7 trillion [38], in 2021 and 2022 respectively IDR 100 trillion and IDR 122.5 trillion [39], so the total cost for quarantine and treatment is IDR 285.2 trillion. Hence, we get $w_2 = 0.15$. Based on Rois et al. [15], we take three strategies, i.e., Strategy 1 uses only u_1, Strategy 2 uses only u_2, and Strategy 3 uses both u_1 and u_2. The simulation graphics of the optimal control problem related to temperature changes can be seen in Figure 4.

Figures 4 (b) and (d) show that if only a single control action (only u_1 or u_2) is applied, then the number of infected people becomes zero starting at day 8 for every temperature. Figure 4(f) shows that if both control actions (u_1 and u_2) are applied, then the number of infected people becomes zero starting at day 4 for every temperature. Hence, the application of both control actions (u_1 and u_2) caused the epidemic will be extinct faster than the effect of the application of a single control action.

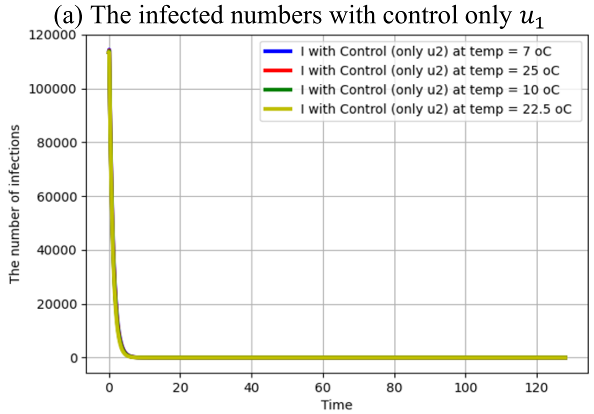
From Figures 4(g) and 4(h), we see that to minimize the objective function J in (4), the value of u_1 and u_2 (only one of them is applied) are maintained at the maximum level of 100% for about 19 days and 17 days respectively before relaxing to the minimum in final time. As expected, the number of infected people is reduced when control is applied. Further, in Figures 4(i) and 4(j) the value of u_1 and u_2 (both are applied) are maintained at the maximum level of 100% for about 10 days and 7 days, respectively before relaxing to the minimum in final time. Hence, the application of both control actions will shorten the duration of the maximum level of every control action. Hence, the application of the two control actions is significantly more effective in preventing the spread of the infection than when only a single control is applied. Information about confidence interval of I can be seen in Table 4.



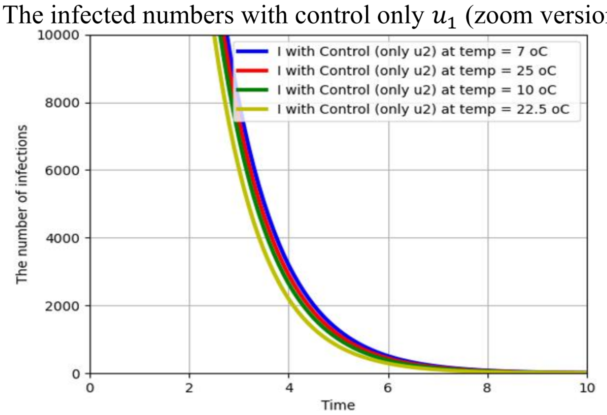
(a) The infected numbers with control only u_1



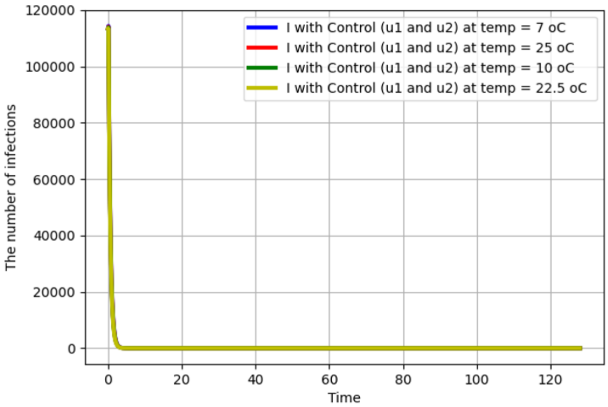
(b) The infected numbers with control only u_1 (zoom version)



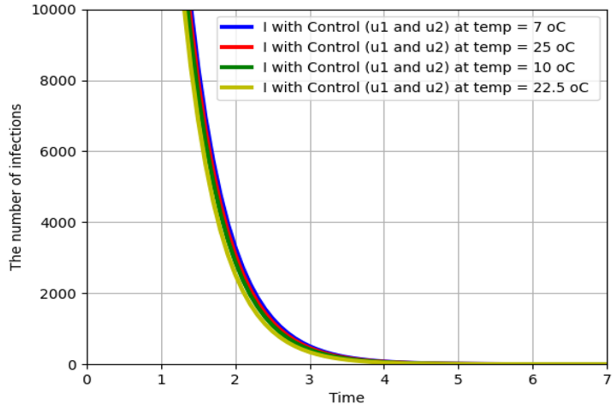
(c) The infected numbers with control only u_2



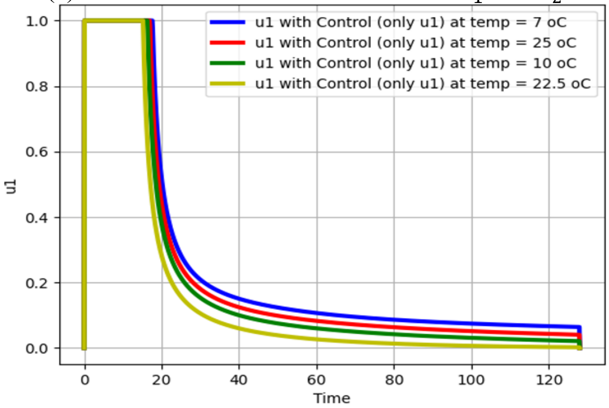
(d) The infected numbers with control only u_2 (zoom version)



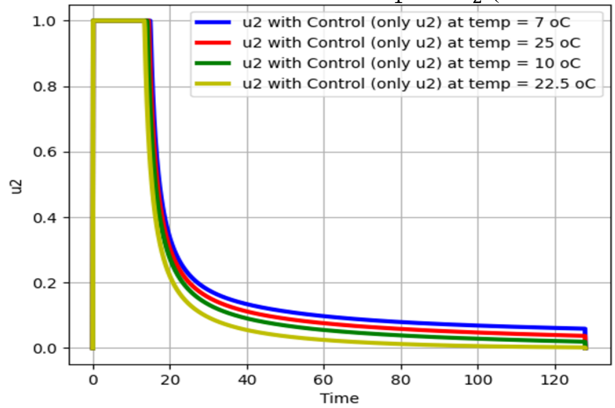
(e) The infected numbers with control u_1 and u_2



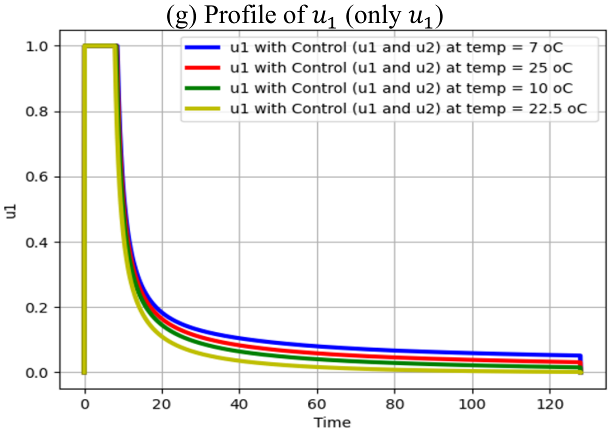
(f) The infected numbers with control u_1 and u_2 (zoom version)



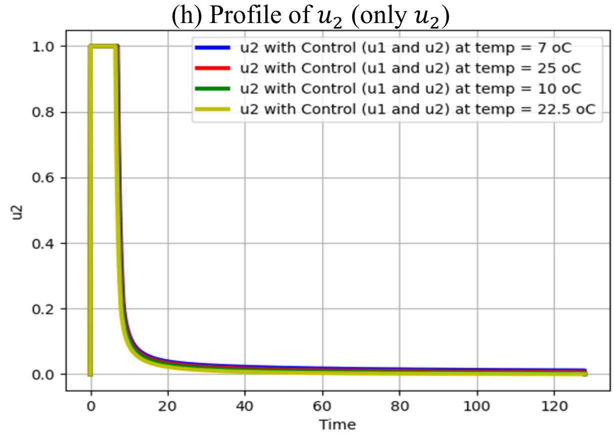
(g) Profile of u_1 (only u_1)



(h) Profile of u_2 (only u_2)



(i) Profile of u_1(both non-zero)



(j) Profile of u_2 (both non-zero)

Figure 4. The graphics of the simulation at temperature 7℃, 10℃, 22.5℃, and 25℃

Table 4. Confidence interval of *I*

Temp. (℃)	Control Options	Mean	Std. Dev.	95% CI Lower	95% CI Upper
7	no control	377564.6364	354948.8208	358119.5963	397009.6765
	only u1	1073.6478	7933.5812	639.0251	1508.2704
	only u2	1019.7155	7745.0977	595.4184	1444.0125
	u1 and u2	524.0944	5701.2965	211.7622	836.4266
10	no control	143560.9293	81991.6163	139069.2106	148052.6481
	only u1	995.7220	7639.4728	577.2114	1414.2326
	only u2	967.7934	7539.2015	554.7759	1380.8109
	u1 and u2	505.3025	5594.2811	198.8329	811.7721
22.5	no control	34944.7091	35018.8495	33026.2834	36863.1348
	only u1	928.1639	7374.3248	524.1788	1332.1491
	only u2	920.6920	7347.1677	518.1946	1323.1894
	u1 and u2	487.7614	5492.4407	186.8709	788.6519
25	no control	251510.8476	183987.2562	241431.5365	261590.1587
	only u1	1033.2452	7782.5808	606.8948	1459.5957
	only u2	993.1068	7640.3117	574.5502	1411.6634
	u1 and u2	514.5339	5647.1140	205.1700	823.8978

Table 5. ICER calculation of every Strategy at different temperatures

Temp.	Control Measure	Total Infections Prevented	Total Costs	ICER
7℃	S1	481908465	107925.2121	0.000223954
	S2	481977499	136194.3264	0.409499455
	S3	482611894	72754.2135	-0.100000972
10℃	S1	182483465	101735.2461	0.000557504
	S2	182519214	129548.1752	0.778013404
	S3	183111202	70348.8096	-0.100000904
22.5℃	S1	43541178	96288.8787	0.002211444
	S2	43550742	123519.1158	2.847137673
	S3	44104893	68103.5239	-0.100000865
25℃	S1	320611331	104727.5580	0.000326650
	S2	320662708	132788.3466	0.546171509
	S3	321275282	71530.4446	-0.100000934

Table 6. ICER calculation of Strategy 1 and Strategy 3 at different temperatures

Temp.	Control Measure	Total Infections Prevented	Total Costs	ICER
7℃	S1	481908465	107925.2121	0.0002240
	S3	482611894	72754.2135	-0.0499994
10℃	S1	182483465	101735.2461	0.0005575
	S3	183111202	70348.8096	-0.0499993
22.5℃	S1	43541178	96288.8787	0.0022114
	S3	44104893	68103.5239	-0.0499993
25℃	S1	320611331	104727.5580	0.0003266
	S3	321275282	71530.4446	-0.0499994

3.3 Cost-effectiveness analysis

Cost-effectiveness analysis is used to determine which COVID-19 control measures are most effective and efficient, either by a single application or a combination of two given measures. The goal is to optimally reduce the spread of COVID-19 at the lowest possible cost. Hence, we used the incremental cost-effectiveness ratio (ICER) to determine the most effective optimal control measure, and the ICER formula is given by

$$\text{ICER} = \frac{TC}{TN} \tag{10}$$

where, TC is the change in total costs between control measures and TN is the change in the total number of infections averted by control measures [40].

The total cost is obtained from the objective function (6), and the total number of infections averted is obtained by calculating the difference between infectious individuals without and with control measures. Let S1, S2, and S3 represent Strategy 1, Strategy 2, and Strategy 3, respectively. The results of the ICER calculation of each Strategy by using Eq. (10) are given in Table 5.

From Table 5, we see that ICER(S1) is less than ICER(S2) for every temperature. This means that Strategy 2 is more costly and less effective than Strategy 1. In other words, Strategy 1 dominates Strategy 2. Thus, a single implementation of u_2 control is removed from the list. Therefore, ICER (S1) and ICER (S3) are calculated again in Table 6.

From Table 6, we see that Strategy 3 dominates Strategy 1 since ICER(S3) is less than ICER(S1). This means Strategy 3 is less costly and more effective than Strategy 1. Hence, a single implementation of preventive measures is excluded from the list. Hence, the application of the two control actions is significantly more cost-effective than when only a single control is applied.

This result is similar to the result obtained by Rois et al. [12], namely, Strategy 3 (combined controls) is more cost-effective than single controls. The added value of this study is that the optimal control analysis is carried out at several different temperature conditions, and also the process of determining the weight value for each control action is also given.

4. CONCLUSIONS

Using Pontryagin's maximum principle, the application of the two control actions at every temperature is significantly more effective in preventing the spread of the infection than when only a single control is applied. From the result of the Cost-effectiveness analysis, the application of the two control actions at every temperature is significantly more cost-effective than when only a single control is applied.

This study still has several limitations, one of which is the lack of empirical data on the relationship between temperature and transmission rate used in validating the fuzzy parameter formulation. Another limitation is that data on the total cost of implementing control measures cannot yet be obtained from scientific journal article sources. The results of this study can be used as one of the supporters of policy making in dealing with future outbreaks that have similar behavior to the COVID-19 outbreak.

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