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Numerical Analysis of Water Exchange Dynamics Between Stream and Sloping Aquifer Using Finite Difference Method

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ABSTRACT

The interaction between surface water and groundwater is a continuous and significant aspect of watershed hydrology. This study presents a numerical approach for quantifying the interactions within a typical stream-aquifer geological system. The system consists of an unconfined aquifer above an impermeable base with a downward slope, interfacing with a stream where the water level increases exponentially over time. The exchange between the stream and aquifer is influenced by a vertical clogging streambank with defined hydraulic parameters. A numerical solution for a finite-length aquifer is developed using the Finite Element Method (FEM), which effectively simulates real-world hydrological scenarios. The results demonstrate that water flow varies with the slope angle and is significantly influenced by the clogging layer's characteristics. Numerical examples illustrate the model's applicability and a detailed sensitivity analysis of key hydraulic parameters is presented. The findings offer valuable insights for water resource management and streambank rehabilitation optimization. This model can be further enhanced to predict surface and groundwater exchanges under varying geological conditions.

1. INTRODUCTION

Water is an essential for all forms of life, supporting drinking, agriculture, industry, and sanitation. Groundwater is a crucial source of fresh water, especially in areas with limited surface water. It helps maintain the flow of rivers and wetlands during dry periods. Overuse and pollution of groundwater can lead to serious environmental and health problems. Conserving and managing water resources is vital for a sustainable future.

The interaction between surface water and groundwater plays a critical role in the sustainable management of water resources, making it a subject of great importance for scientists, engineers, and water resource managers. As population growth and climate variability increase the demand on water supplies, understanding this interaction becomes essential for the strategic allocation and efficient utilization of subsurface water resources. Groundwater, being a vital source for agricultural, industrial, and domestic needs, requires careful monitoring and regulation. A key aspect of groundwater management is the precise estimation of water table fluctuations within an aquifer system. These fluctuations are primarily influenced by anthropogenic pumping activities and natural recharge processes. Accurate predictions of these changes are essential for preventing overexploitation, maintaining ecological balance, and ensuring long-term availability.

The experimental studies in a groundwater hydrology are often time consuming, labor-intensive and expensive. Hence there has to be paradigm shift towards the use of mathematical modeling. Analytical and numerical models offer a powerful, cost-effective alternative for simulating aquifer behavior under varying conditions. These models provide initial insights into water table dynamics and are especially valuable during the preliminary stages of water management planning. Their ability to incorporate spatial and temporal variability, as well as their adaptability to changing hydrogeological parameters, has made them a preferred choice among researchers and decision-makers.

Modern groundwater models are designed to address complex subsurface interactions by integrating multiple factors that influence flow behavior. One such factor is the presence of a clogging layer—a low-permeability zone that can develop due to sediment deposition or biological activity at the interface between recharge sources and the aquifer. This layer significantly affects infiltration rates and alters the natural recharge patterns. Another important feature is a sloping aquifer base, which impacts the direction and velocity of groundwater flow, ultimately influencing the shape and behavior of the water table.

The objective of the present modeling effort is to investigate the combined effect of a clogging layer and a sloping base on water table variations. This study aims to enhance the predictive accuracy of water table models and contribute to the broader understanding of groundwater dynamics. By incorporating these features into the mathematical framework, the model can better simulate real-world aquifer conditions, thereby aiding in more informed and effective groundwater management strategies. Furthermore, beyond the physical and technical considerations, researchers are increasingly exploring the socioeconomic implications of groundwater resource management. Efficient groundwater modeling directly impacts policy-making, agricultural productivity, and community resilience—further underlining the importance of developing robust, reliable, and adaptable modeling tools.

The majority of models that study the interface between streams and unconfined aquifers below the subsurface are built upon the Boussinesq equation which was introduced by French mathematician and physicist Joseph Valentin Boussinesq in the late 19th century. Bear [1] made significant contributions to the field of groundwater hydrology and the understanding of fluid flow in porous media. Bear's contributions included the development and analysis of various equations governing fluid flow and transport in porous media, building upon the foundations laid by earlier scientists like Boussinesq. Boussinesq equation is a complex second-order nonlinear partial differential equation, that handles many analytical challenges as mentioned by Mishra and Kuhlman [2]. Despite this, approximated analytical solutions derived from it are widely accepted. For comprehension flow processes across different latitudinal and time-based scales are explained by Moench and Barlow [3] and Mohyud-Din et al. [4]. These solutions assist in analyzing the transient performance of the water table under precise activities like withdrawal from wells or recharging a basin artificially studied by Rai et al. [5, 6] and Mahdavi [7]. An analytical model is been developed by Lin and Lin [8]. The equation is solved to evaluate groundwater flow in an aquifer-Fault-aquifer system. An impact of fault zone -crucial geological structure is studied. Antangana and Botha [9] utilized the homotopy decomposition method to solve the groundwater flow equations. Furthermore, researchers [10-14] presented an analytical solution for variation in the water table and presented the effect of seepage and recharge on the aquifer. The main focus of their work is the effect of sloping beds. Wang et al. [15] developed an analytical solution for water table variation under variables boundaries and recharge condition. Ma et al. [16] developed a mathematical model in the fracture costal aquifers and studied the hydraulic variation induced by tidal waves. induced by into the dynamic analysis of tide-induced variations in water tables within an unconfined aquifer system. Additionally. Saxena et al. [17] developed a mathematical model by considering various cases to examine growth or fall in the water table in the sloping aquifer. The solution is developed analytically and used to simulate hydraulic head distribution. Also, a relation between various aquifer parameters has been presented. An exhaustive review in the area of hydrological modeling in the surface-groundwater interaction is presented by Lande et al. [18]. An analytical method is used to determine the extent of pumped freshwater by Kurylyk et al. [19]. An excellent field work is done by Yanes et al. [20] which analyzes the temperature of surface water and ground water at various depths and identifies the trends. An excellent review is been presented by Yeh and Chang [21], which addresses the mathematical modelling, methods to solve the mathematical modelling and various geological situations and parameters. Similarly, a review water table fluctuation due to recharge is presented by Becke et al. [22].

After conducting a broad literature review, it is clear that the majority of existing models work under an essential suspicion, assuming the spring is lying underneath a perfectly impenetrable bed. In a real field, aquifers are often irregular in shape, sloping, and many times resting on an impervious base. So main limitation of these models is the researcher's assumption about aquifer geology. Therefore, the formulations in the previous literature may not be suitable for various natural systems that involve leaky aquifers and multilayered aquifers. Recharge and withdrawal mechanisms within profound sedimentary basins influence the hydrological properties of the aquifers or aquitards present in the system. Consequently, employing finding from existing studies might result in either underestimation or overestimation of the actual outcomes.

In this study, a novel numerical solution for the one-dimensional linearized Boussinesq equation is developed to address the main concern- sloping aquifer and sedimentary layers. The hydrological model setup contains an unconfined isotropic aquifer, overlaying on an inclined base, adjacent to an ascending stream. By applying the finite difference method, the linearized Boussinesq equation is solved to predict the variation in hydraulic head. The inflow is observed for different bed slopes. The model can be used to explain the sensitivity of water heads concerning variations in a boundary.

2. MATHEMATICAL FORMULATION

Figure 1 depicts a water table in an aquifer that is overlying on a fully impervious sloping bed with slope angle *β*. The homogeneous, isotropic finite aquifer is assumed to be dry initially and interfaces with an adjacent stream whose water level gradually changes from initial level zero to final level h_0. The interaction between the stream and the aquifer occurs through the vertical clogging layer with low hydraulic conductivity as compared to that of the aquifer. Such low-permeable layers are generally formed due to sediment loading on the riverbank or accumulation of materials during periods of low discharge [23-25]. A no-flow condition is imposed at the right boundary, i.e., at $x=L$.

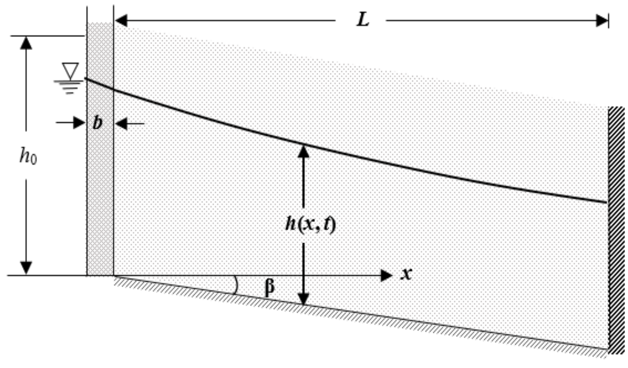

Figure 1. Definition sketch of an unconfined sloping aquifer of length L interacting with a rising stream in the presence of a vertical streambank

If x represents the spatial coordinate and $h(x,t)$ signifies the height of vertical water head height measured above the inclined bed as shown in Figure 1. The subsurface seepage flow follows a Boussinesq equation [23] which is a nonlinear partial differential equation and parabolic in nature.

The equation is given below:

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)-\tan\beta\frac{\partial h}{\partial x}=\frac{S}{K\cos^2\beta}\frac{\partial h}{\partial t} \tag{1}$$

Here the parameters in the equation are as follows:
$h(x,t)$ – variable height of water table
S – specific yield
K – hydraulic conductivity of the aquifer
β – sloping angle

Because of the non-linearity of the equation, an analytical solution is not feasible. The approximate analytical solutions derived by Werners and Brutsaert using linearization techniques are widely recognized. Generally, Eq. (1) is linearized by substituting the coefficient h of $\partial h/\partial x$ as average saturated depth, denoted by h_{avg}.

Although, there are several techniques to determine the average saturated depth; however, the most efficient iterative method is to choose h_{avg} as $h_{\text{avg}}=(h_i+h_t)/2$ where h_i is the initial depth considered to be zero, and h_t represents time-dependent height. The iterative process calculates h_{avg}, and this approach is articulated by Marino [26]. The linearized Boussinesq equation is expressed as:

$$\frac{\partial^2 h}{\partial x^2}-\frac{\tan\beta}{h_{avg}}\frac{\partial h}{\partial x}=\frac{S}{K\,h_{avg}\cos^2\beta}\frac{\partial h}{\partial t} \tag{2}$$

Since the aquifer is initially dry, we can prescribe the condition

$$h(x,\,t=0)=0 \tag{3}$$

Simulation of subsurface seepage flow through vertical streambank in a sloping aquifer is derived by Bansal et al. [10] and is considered as the boundary condition as under:

$$-K\left(\frac{\partial h}{\partial x}\right)_{x=0^+}\cos^2\beta=k\frac{h_s(t)-h(x=0^+,\,t)}{b} \tag{4}$$

The symbols b and k are the width of the streambank and hydraulic conductivity of the sedimentary layer, and $h(x=0^+,t)$ denotes the height of free surface at the interface between the stream bank and the aquifer. $h_s(t)$ signifies the fluctuating stream phase at time t given by

$$h_s(t)=h_0\left(1-e^{-\lambda t}\right) \tag{5}$$

The stream water elevations fluctuate at the rate determined by a parameter λ. The instantaneous increase is observed in the limiting condition $\lambda\to\infty$. The simulation corresponding to the no-flow situation at the right boundary of the aquifer is achieved by imposing the following conditions:

$$\left(\frac{\partial h}{\partial x}\right)_{x=L}=0 \tag{6}$$

3. NUMERICAL SOLUTION

The governing equation of flow is a nonlinear partial differential equation known as Bosussineq equation given in Eq. (1) is

$$\frac{\partial h}{\partial t}=A_1\left(\frac{\partial^2 h^2}{\partial x^2}\right)-A_2\frac{\partial h}{\partial x} \tag{7}$$

where, $A_1=(K\cos^2\beta)/2S$ and $A_2=(K\sin 2\beta)/2S$.

Eq. (7) can be discretized using finite difference scheme. The temporal derivative on LHS is discretized using forward difference whereas spatial derivatives are discretized using central difference.

$$\begin{aligned}\frac{h_m^{n+1}-h_m^n}{\Delta t}&\\ =A_1&\left[\frac{\left(h_{m+1}^{n+1}\right)^2-2\left(h_m^{n+1}\right)^2+\left(h_{m-1}^{n+1}\right)^2}{(\Delta x)^2}\right]\\ &-A_2\left[\frac{h_{m+1}^{n+1}-h_{m-1}^{n+1}}{2\Delta x}\right]\end{aligned} \tag{8}$$

Subscript m denotes the variable in space grid and superscript n denotes the variable in time grid. To solve the system of equation we set the substitution

$$h_m^{n+1}=h_m^n+v_m^n \tag{9}$$

Substituting this in Eq. (8),

$$\begin{aligned}v_m^n=B_1&\left[\begin{array}{l}\left(h_{m+1}^n\right)^2+2\,h_{m+1}^n\,v_{m+1}^n-2\left(h_m^n\right)^2\\ -4\,h_m^n\,v_m^n+\left(h_{m-1}^n\right)^2+2h_{m-1}^n\,v_{m-1}^n\end{array}\right]\\ &-B_2\left[h_{m+1}^n+v_{m+1}^n-h_{m-1}^n-v_{m-1}^n\right]\end{aligned} \tag{10}$$

where,

$$B_1=\frac{A_1\,\Delta t}{(\Delta x)^2}\quad\text{and}\quad B_2=\frac{A_2\,\Delta t}{2\Delta x} \tag{11}$$

Neglecting the higher powers of v_m, v_{m-1} and v_{m+1} and rearranging the terms equation obtain is

$$\begin{aligned}&\left[2B_1\,h_{m-1}^n+B_2\right]v_{m-1}^n+\left[-1-4\,h_m^n\,B_1\right]v_m^n\\ &\quad+\left[2B_1h_{m+1}^n\right]v_{m+1}^n\\ &=-B_1\left[\left(h_{m-1}^n\right)^2-2\left(h_m^n\right)^2+\left(h_{m+1}^n\right)^2\right]\\ &\quad+B_2\left[h_{m+1}^n-h_{m-1}^n\right]\end{aligned} \tag{12}$$

Rewrite Eq. (10) as

$$P_{m-1}\,v_{m-1}^n+Q_{m-1}v_m^n+S_{m-1}\,v_{m+1}^n=R_{m-1} \tag{13}$$

where,

$$P_{m-1} = 2B_1\, h_{m-1}^n + B_2\ ,Q_{m-1} = -1 - 4h_m^n\, B_1,\ S_{m-1} = 2B_1 h_{m+1}^n$$

$$R_{m-1} = -B_1\left[\left(h_{m-1}^n\right)^2 - 2\left(h_m^n\right)^2 + \left(h_{m+1}^n\right)^2\right] + B_2\left[h_{m+1}^n - h_{m-1}^n\right]$$

For computing the value of v_{m-1}, v_m and v_{m+1} by putting the different value of m in Eq. (13) the system of algebraic equations in the form of the tridiagonal matrix for a given time step is solved. This system can be solved using many techniques or algorithms available in various texts of numerical analysis and then h_{m-1}, h_m, and h_{m+1} can be computed using Eq. (3). Thus, the values of n+1-time steps can be calculated.

4. RESULT AND DISCUSSION

To demonstrate the applicability of the closed-form solution obtained in this paper, we take an aquifer with K=2.5 m/s, S=0.2, hi=2 m, b=1 and b=3 m, λ=0.2 per hr. and k=0.25, angle β=5 degrees. Considering a dry aquifer initially, a transient profile of water head height $h(x,t)$ are plotted in Figure 2 and Figure 3 for different time periods and for different stream bank widths.

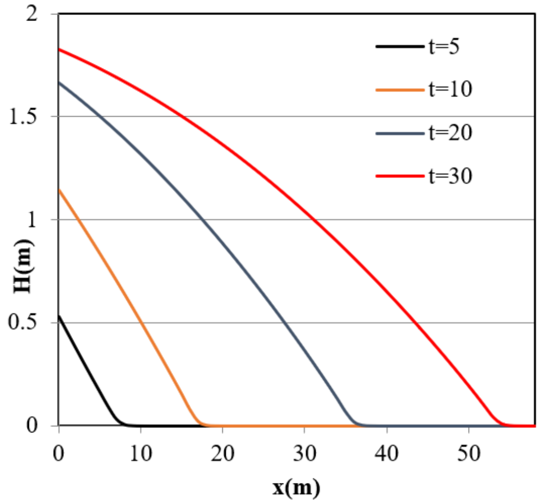

Figure 2. Water head profile for b=1 and hi=2

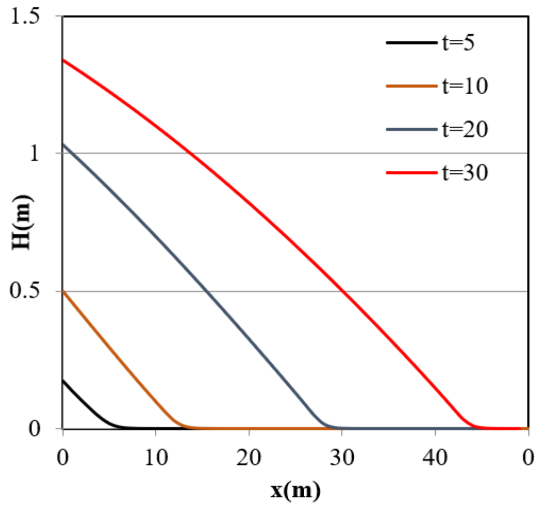

Figure 3. Water head profile for b=3 and hi=2

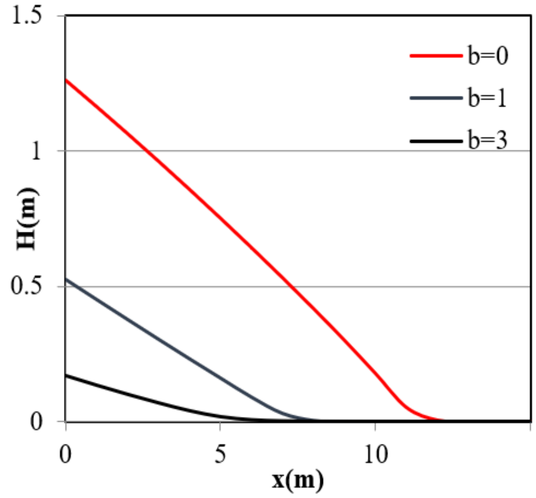

Figure 4. Water head profile for t=5 day

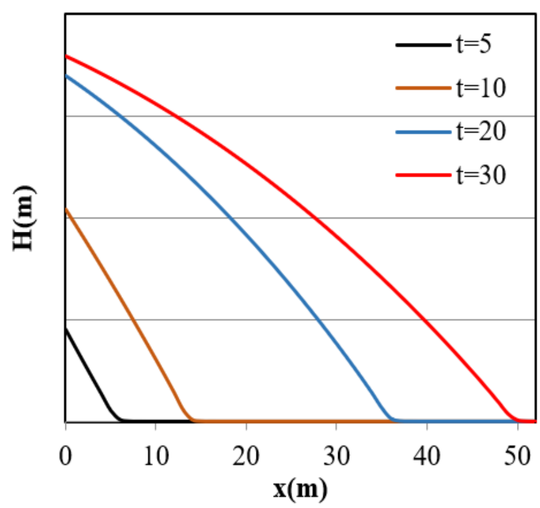

Figure 5. Water head profile for sudden rise (λ=2)

Implementation of the new solutions is illustrated to simulate surface–groundwater interaction between stream and aquifer under the combined influence of bed slope, vertical sedimentary layer, and stream stage variations. It is observed from Figure 2 and Figure 3 that the stream bank width plays a significant role in controlling the rate at which water flows into the aquifer. Specifically, when the stream bank is wider, the infiltration of water into the aquifer is relatively limited, whereas a narrower stream bank allows a greater volume of water to enter the aquifer. This behavior can be attributed to the contrast in permeability between the stream bank material and the aquifer itself. Stream banks typically consist of less permeable sediments compared to the more permeable aquifer material. As the width of the stream bank increases, the overall resistance to flow also increases, thereby reducing the water exchange between the stream and the aquifer. Conversely, a narrower stream bank offers a shorter and less resistive flow path, facilitating greater water movement into the subsurface. This highlights the importance of stream bank geometry in the interaction between surface water and groundwater systems.

Figure 4 shows the hydraulic profile for t=5 day for different streambank widths b=0, 1, 2. It is observed that a reduction in stream bank width leads to an accelerated infiltration of stream water into the aquifer, consequently raising the water table level. This process becomes particularly significant when there is a sudden rise in the adjacent stream, which can have a pronounced impact on the phreatic aquifer. Figure 5 illustrates the evolution of the water head profile at different times t =5, 10, 20, and 30 days under the influence of stream stage variation with a dimensionless parameter λ = 2. The plotted profiles demonstrate how the water head responds dynamically to the rapid changes in stream stage. As time progresses, the effect of the variable stream becomes more pronounced, with sharp gradients in the water head near the stream boundary. This clearly reflects how sensitive the aquifer system is to abrupt surface water fluctuations, particularly when the stream responds rapidly to external influences such as heavy rainfall or upstream discharge changes.

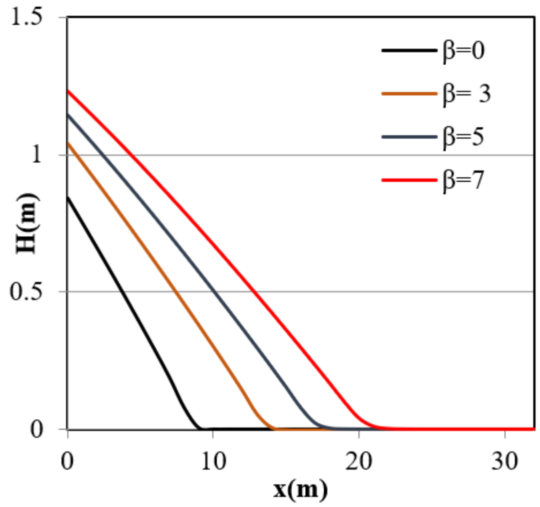

Figure 6. Water head profile for t=10 at increasing sloping angle as β=0, 3, 5, 7

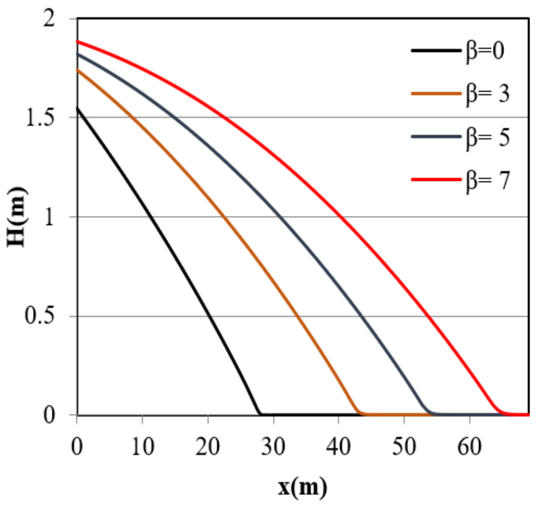

Figure 7. Water head profile for t=30 days for increasing sloping angle as β=0, 3, 5, 7

Figures 6 and 7 illustrate the variation in the water table at two different time instances, t = 10 days and t = 30 days, for varying bed slopes β = 0°, 3°, 5°, and 7°. From these graphs, it is obvious that the change in water table elevation is directly influenced by the bed slope angle. As the bed slope increases, the hydraulic gradient driving flow into the aquifer also increases, resulting in greater infiltration and a more pronounced rise in the water table. This trend clearly indicates that steeper bed slopes enhance the interaction between the stream and the aquifer, promoting more water movement into the subsurface. In the case of a horizontal bed (β=0°), the behaviour of the water table closely aligns with the results reported in previous studies [3, 10], thereby validating the accuracy and consistency of the current model. These observations underscore the importance of channel geometry, particularly bed slope, in influencing groundwater recharge dynamics.

Here the quantitative comparison is given in the form of tables. Table 1 shows the water table variation due to different bed slopes and time.

The slope angle β represents the gradient of the land surface, then

1) β=0°: Flat surface
2) β=3°&5°: Increasingly sloped surfaces

The time indicates the duration over which infiltration, seepage, or other hydrological processes occur. Over time, the water table either rises or falls depending on recharge and discharge.

Table 1. Water table profile for t=10 and 20 days for different sloping angles (i.e., for β=0, 3, 5, 7)

x	T=10 days			T=20 days		
	β=0	**β=3**	**β=5**	**β=0**	**β=3**	**β=5**
5	0.38	0.68	0.84	1.05	1.36	1.50
10	0	0.30	0.50	0.72	1.12	1.32
15	0	0.00	0.15	0.36	0.86	1.11
20	0	0	0.00	0.00	0.59	0.88

Stream bank width is a significant factor influencing groundwater-stream interaction due to its lower permeability compared to an aquifer. The effect of stream bank widths is shown in Table 2. The stream bank width b=1 means a narrow stream bank through, and b=3 a wider stream bank is considered for time T=10 and 20 days.

Table 2. Effect of stream bank width (for b=1 and 3) and time (T=5, 10, 15, and 20 days) on water table fluctuation

x	b=1		b=3	
	T=10	**T=20**	**T=10**	**T=20**
5	0.84	1.51	0.30	0.87
10	0.51	1.38	0.1	0.70
15	0.15	1.12	0.00	0.52
20	0.00	0.88	0	0.32

5. CONCLUSION

In this study, the primary objective is to develop a novel numerical solution to simulate water table fluctuations within a finite aquifer system that is hydraulically connected to a stream via a semi-pervious clogging layer. The model specifically examines the combined influence of bed slope, stream stage, and sedimentary clogging layers on the groundwater flow dynamics. A numerical solution to the nonlinear Boussinesq equation is formulated using the finite difference method, allowing for accurate representation of the transient groundwater behavior under various boundary and hydrological conditions.

The simulation results indicate that groundwater flow within the aquifer is significantly influenced by the bed slope and the hydraulic resistance imposed by the clogging layer.

The sloping angle alters the velocity and direction of flow, while the semi-pervious layer between the aquifer and the stream acts as a regulating barrier, affecting recharge rates and water exchange. Notably, greater water accumulation is observed in finite aquifers compared to infinite aquifer models, highlighting the critical role of aquifer geometry in water storage and movement.

The findings of this study have important implications for water resource management and environmental engineering. The developed model offers a robust framework for predicting water table fluctuations in aquifers that interact with surface water bodies through low-permeability interfaces. Practical applications include enhancing streambank stabilization and erosion control, optimizing groundwater recharge systems, and sustaining floodplain ecosystems. Moreover, the model provides valuable insights for designing pollution mitigation strategies and ensuring safe drinking water by better managing water quality in groundwater-surface water interaction zones.

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