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# Parametric Cost Prediction Models for Light General Aviation Aircraft in the Early Design Phase

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**ABSTRACT**

Among many methods used to predict the cost, parametric cost analysis is widely used. This technique employs parameters not directly correlated to the product cost, like quantity/quality characteristics, to predict the aircraft cost. The data used in this paper for prediction relates to aircraft still "in production" and "in service" and presents cost-estimation models for light general aviation aircraft whose maximum take-off weight (MTOW) is less than 2000 kg. The Aircraft are classified into two categories based on the landing gear configuration. Important design parameters that are mostly known or easily calculated at the beginning of the preliminary design phase and affect the aircraft design are considered. Multi-linear regression analysis based on the p-value (also known as p-value analysis) is applied to develop the cost-estimation models. These empirical models presented in the paper can predict the cost to an error accuracy of less than ± 5% for all categories, and in the majority of cases, the cost prediction error accuracy of less than 3%. In addition, the models offer the possibility of performing parametric studies to obtain the cost sensitivity to the key design parameters.

## 1. INTRODUCTION

The global forecast for the aircraft industry is set to rise due to the growing adoption of light and ultralight aircraft in the military, commercial, and civil arenas. According to a report from MarketsAndMarkets [1], the CAGR (compound annual growth rate) is currently 5.9%. Thus, the light aircraft market is planned to rise from $7.5 to $11.9 billion between 2022 and 2030. This report classified the light and ultralight aircraft market according to Region (North America, Europe, Asia Pacific, and rest of the world), System (airframes, avionics, cabin interior, and aircraft systems), Flight operation (CTOL and VTOL), Material (aluminium, composites, titanium, and others), Propulsion (conventional fuel and electric-hybrid), Technology (manned and unmanned), End Use (civil & commercial, and military), and Aircraft Type (light and ultralight). However, new opportunities will arise that require a competitive advantage with this growth. The utilization of new materials and technologies would inevitably mean new designs that address technical, legal, ecological, and operational aspects. The growth in the light aviation area inevitably means many new designs that are fit for specific purposes. The greatest challenge that befalls the designer, is to determine the sensitivity of key design parameters that affect not only the performance but the overall cost of the aircraft. The overall cost affects the direct operating cost (DOC) and the seat mile cost (SMC) which in turn are key financial metrics that determine the viability of the design for commercial exploitation. Additionally, the maximum take off weight (MTOW) which is a function of wing size, fuselage size, geometric layout, aerodynamic and propulsive aspects, determines the cost. Each design aspect ultimately results in the utilization of resources such as Material, Manpower, Equipment, Services, Facilities, and Time, resulting in costs that can be direct or indirect.

In many business and engineering decisions, including the aircraft industry, cost estimation is challenging since it needs to factor in various expenses such as materials, labor quantities, sales, floor space, utilities, and overheads. These expenses are utilized as inputs to analysis like decision tree analysis or Monte Carlo simulation [2, 3]. Therefore, the cost estimation must be accurate to avoid making wrong decisions or overrunning the total developed cost [4]. Reliable cost estimation may also improve the chances of getting external funding - a vital point for start-ups. To tackle this issue, the authors developed a research method to investigate potential cost models for GA aircraft that can serve as a guideline for, e.g., start-ups and research works. More specifically, Shahriar et al. [5] presented a comprehensive summary of all the cost-estimating techniques in design, development, manufacturing, and production cycles. Although parametric models exist, usually not of high fidelity. DAPCA-IV was developed by RAND Corporation in the late '60s and is based on empirical relations where information on the airframe, engine, avionics, labor, and material is processed. Furthermore, the DAPCA-IV program requires information that is generally not available during early design phases.

The analogous cost estimation method is the most common approach for estimating a new product cost. "Similar products have similar costs" is the basis for this technique [6, 7]. The benefit of using the available past cost data is obtaining a valuable estimate with minimal time [8, 9]. References [10-12] offer additional research on the analogous cost estimation method. Parametric cost estimation is another widely used method to estimate a new product cost. Its main principle is to generate what is known as "cost estimation relationships". Furthermore, a single model may have several relationships [13-15]. The engineering build-up method is another approach for cost-estimating a new product. In this approach, the part design details, the process interactions, and a deep process understanding should be available [16, 17]. More information is available in the references [18-20]. The commonly used method's strengths and weaknesses and related applications are presented in the reference [21], point to note is that software development, data-driven risk analysis, sensitivity analysis, long-range planning, architectural studies, cross-checking, design-to-cost trade studies are cited as the main advantages in parametric methods. The models developed later on in this work are particularly suited to performing sensitivity analysis, and design-to-cost trade studies. These methods are significantly important in the Preliminary and detailed phases of development [22].

Various parameters, such as time constraints, availability of data, level of detail required, and adequacy of project/program definition, are used to decide the type of cost estimation method. However, in selecting a methodology, the analyst should note that the cost-estimating task predicts upcoming costs by extrapolating the existing schedule data and historical costs. Figure 1 shows the relative significance of each method throughout the product development phases [22], Phase A (Conceptual), Phases B and C (Preliminary), and Phase D (Detailed).

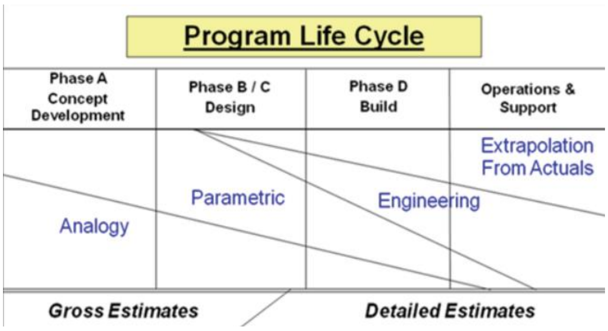


**Figure 1.** Relative significance of cost-estimating methods during the aircraft design phases [22]

For the preliminary design phase (phases B and C), it is clear that the parametric method has the highest relative significance. Therefore, it is recommended to use it where data and samples are lacking. Moreover, the aircraft design process is complicated, more often than not iterative, and often has a long development cycle due to many uncertainties and often due to the incorporation of new technologies, which directly impact the overall costs. Consequently, the prediction of aircraft cost is not only tricky but also a complex task. Accurate cost predictions at an early design stage allow the design drivers' impact to be evaluated and avoid the risk of bad decision-making or overrunning the total developed cost [4]. It is true for estimating the cost of both commercial and light aircraft [23].

However, parametric cost estimation is a widely used method to estimate a new product's cost in the early stages [24]. Its main principle is to generate what is known as "cost estimation relationships". These relationships correlate the product cost with a number of its parameters, usually identified as "cost drivers". For instance, the product size can be a parameter; the manufacturing costs will increase as the size increases. Thus, cost and size can be mathematically correlated through statistical analysis. The mathematical relationship (the model) can have a number of parameters or design variables like weight, size, density, thickness, etc. Note that these parameters (cost drivers) should greatly impact the cost changes. Further, a single model may have several relationships [14, 23, 25].

Conversely, most textbooks and design references use a simple linear regression model, defining the cost as a function of the MTOW. This approach is commonly adopted, with a prediction error of around 10%. Thus, the main contribution of this paper is to develop more accurate cost estimation models (with prediction error less than ±5%) for light general aviation aircraft (MTOW less than 2000 kg). In addition, the linearity of the models facilitates the process of investigating the impact of each design variable on the aircraft cost. Thus, these models help, simplify, and accelerate the decision-making process.

## 2. PARAMETRIC COST-ESTIMATION ANALYSIS

It is a method of generating and applying equations to explain the relationships between measurable system attributes and cost. For $n$ parameters $p=(p_1, \ldots, p_j)$, the typical statistical process is to obtain a value in which it can predict the cost reasonably well using Eq. (1). It represents the general form of the response surface approach [26]:

$$c = f(m, p) + e \tag{1}$$

where, $m=(m_1, \ldots, m_n)$ is a set of system characteristics values, which change over $x$ cases ($c_i\ m_{1i} \ldots m_{ni}$), and it is different for each $i$=1, $x$. In addition, $e$ is the prediction error. Therefore, a parametric cost analysis can be considered a practical use of the response surface approach from a statistical perspective.

Since $f$ is a linear function in $p$, Eq. (1) can be written as:

$$c = \sum_{k=1}^{j} p_k\, f_k(m) \tag{2}$$

Note that the least squares, which reduce the Euclidian distance between the case value ($c_1, \ldots, c_x$) and the predicted value ($t_1, \ldots, t_x$), represent a criterion to define "predicted reasonably well" [27]. These parameters can be calculated using the generalized least squares equation:

$$p = (M'\, M)^{-1}\, M'\, c \tag{3}$$

where, $M'$ is the transpose of $M$, $M$ is a matrix of $x$ rows ($f_{1i}(m) \ldots f_{ji}(m)$), $(M'\,M)^{-1}$ is the inverse of the matrix product $M'\,M$, and $c$ is a vector of $x$ rows $c_i$. The following estimator is used to predict the cost $c$ reasonably well using an arbitrary $m$:

$$t = f(m, p) \tag{4}$$

Consequently, the standard formula for parametric cost equations [28] is:

$$t = e^{a_0}\, e^{a_i w_i} \ldots e^{a_r w_i} m_1{}^{b_1} \ldots m_l{}^{b_s} \qquad (5)$$

where, $i$ and $l$ are indices associated with the type of variables $w$ and $m$, respectively, the linear least squares method is applied to find the coefficients $a_i$ and $b_l$, $w_i$ and $m_l$ are statistically important measures associated with the product, and $m_l$ are standard size measures that normally contain the number of code lines for software or weight for hardware. The $a_0$ indicates the base class complexity, and $a_l$ is created with the unit binary variables $w_i$, which refer to the membership of classes, and the other $a_i$ are the relative associated subclass complexities. Thus, $w_1$=0 and $e^{a_i w_i}$=1 if the particular product is not in subclass $i$. In contrast, $w_k$=1 and $e^{a_i w_i} = e^{a_i}$ will modify the base complexity $a_0$ to offer the subclass complexity $a_0$+$a_i$. For a given part, this technique offers, for example, a method of differentiating between various manufacturing processes [29].

### 2.1 Use of parametric cost analysis (PCA)

Al-Shamma and Ali [30] used PCA in estimating the cost for large civilian aircraft, where the aircraft were classified into 4 groups. They Presented 4 Models for predicting aircraft Costs and classified the aircraft based on their classification using MTOW.

However, the very light aircraft in GA are considerably less than 2000 kg, the presented models are unlikely to predict the cost accurately as the bounds of the data would be invalidated. Therefore, predicting the cost for lighter general aviation aircraft still poses a problem. Conventionally MTOW has been used to predict costs, and that does not allow parametric changes to be investigated. On the other hand, there is a substantial market for Light General Aviation aircraft, and cost estimation tools for new designs based on current production aircraft that take into account new materials and technologies are all the more important. In this paper, we present cost estimation models for very light General aviation aircraft.

## 3. AIRCRAFT DESIGN PARAMETERS

Lappas and Bozoudis [31] used fuselage length, empty weight, MTOW, SFC, max speed, and ceiling parameters in their paper to predict cost per flying hour. The choice of the variables is logical, as the variables chosen affect the capital costs and recurrent costs. For example, wing area affects many aspects, principally the weight, therefore more material has to be used, as well as increasing lift and drag, to obtain the impact of many performance aspects. Noticeably, the choice of the variables and their relative significance and impact on the desired objective function is not obvious and dependent on the experience and wider knowledge base of the aircraft designers. The TLAR (top level aircraft requirements) allows few design choices to be made and performance and operational requirements impact the other choices, which are often honed by trade-off analysis, for which parametric is essential.

Having accurate and up-to-date data for obtaining cost estimation models is the first and foremost requirement. The knowledge of the Cost in design passes B and C is critically important. Any model of any significant fidelity must only use variables that are known in these phases. Later in the section we outline such variables. Data from retired or even old aircraft is discarded as it is based on old technologies, manufacturing methods, and materials. Therefore, only aircraft that are in production and also in service are included in this paper.

Moreover, only piston-engine aircraft with the same features and design characteristics are considered. Although there is a growing trend toward all-electric aircraft, internal combustion engines will continue to be used and designed for the foreseeable future. The sampled data (aircraft) used in modeling and testing are classified into two categories, namely the tricycle (category A) and taildragger (category B). The aircraft used to build the cost-estimation models are listed in Table 1 for category A and Table 2 for category B.

**Table 1.** The prices and maximum takeoff weights of category A

| Aircraft Type | Price (M$) | MTOW (kg) |
|---|---|---|
| Beechcraft Bonanza | 0.815 | 1656 |
| Cessna 172 Skyhawk | 0.438 | 1160 |
| Cessna 182 Skylane | 0.480 | 1406 |
| Cessna 206 Stationair | 0.665 | 1633 |
| Cirrus SR20 | 0.440 | 1383 |
| Cirrus SR22 | 0.710 | 1633 |
| Diamond DA40 | 0.455 | 1198 |
| Mahindra GA8 Airvan | 0.827 | 1814 |
| Mooney M20 | 0.779 | 1528 |
| Piper PA-28 | 0.467 | 975 |
| Tecnam P2010 | 0.4 | 1160 |

**Table 2.** The prices and maximum takeoff weights of category B

| Aircraft Type | Price (M$) | MTOW (kg) |
|---|---|---|
| American Champion Citabria | 0.190 | 748 |
| American Champion Explorer | 0.216 | 820 |
| American Champion High-country Explorer | 0.245 | 820 |
| American Champion Scout | 0.247 | 975 |
| American Champion Denali Scout | 0.272 | 975 |
| CubCrafters Top Cub | 0.270 | 1043 |
| CubCrafters XCub | 0.317 | 1043 |
| Maule M-7 | 0.255 | 1134 |
| Aviat Husky | 0.314 | 1021 |
| Maule M-9 | 0.327 | 1270 |
| American Champion Citabria | 0.190 | 748 |

From the list above, it can be noted that both conventional and composite aircraft have been included. All electric aircraft are relatively new, with just a few certified; hence, they have been excluded. So, the design variables need to be generic, specifically, those that are not a function of technology.

In order to establish the cost models, a typical set of variables has been chosen [30]. In this work, the selected parameters critically affect almost all design and cost-estimation aspects and are evaluated, estimated, or known at the conceptual design phase. The selected variables have by far the most impact on the overall cost and are described in the following:

1. $W_{to}$ – The maximum take-off weight – is the primary parameter and must be considered during all design phases. Its importance impacts endurance, range, wing loading, thrust loading, and all aspects of aircraft performance (including take-off, climb, cruise, turning, etc.). Various techniques are available for estimating this parameter in the conceptual design phase.

2. *PAX* – Number of passengers (Number of Seats) – is one of the crucial requirements and has a bearing on fuselage

length and/or the diameter of the fuselage. This requirement is generally available at the conceptual phase. As the number of seats increases, the aircraft's structural weight goes up, and hence, the aircraft cost increases. The increased fuselage length increases the tail arm, which affects static and dynamic stability, requiring larger volume coefficients, leading to larger empennage areas, and, eventually, the cost indirectly affects the *PAX* parameter. Since the occupants' weight is significant compared to the empty weight, it is likely to be significant.

3. $R$ – Aircraft range – is largely dependent on the amount of fuel that can be carried and computed by the Breguet Range Equation, which requires the knowledge of including initial and final weights ($W_o$ and $W_f$), drag and lift ($D$ and $L$) characteristics, specific fuel consumption (SFC), and cruise speed ($V_c$). For example, increasing the range requires a robust, lightweight structure, a higher L/D ratio, and efficient engines. All of these improvements will result in increased costs.

4. $F$ – Fuel capacity – is directly proportional to aircraft range, limited by the maximum take-off weight and performance. It also affects the wing geometry and fuselage design due to volume storage requirements.

5. $V_c$ – Cruise speed – directly impacts the stage or mission time. *Increasing* the cruise speed involves obtaining increased power. This situation translates into larger engines, which implies heavier engines, and in turn, the wing loading (structure) and aircraft cost go up.

6. $P$ – Engine power – A certain excess power is needed to obtain good performance. This case limits the permitted total drag, affecting the aerodynamic design and, to a large extent the structural design, as well.

7. $L_f$ – Aircraft length – mainly depends on the seating arrangement, the number of passengers, and fore and after body requirements.

8. $b$ – Wing span affects wing loading and aspect ratio selection.

9. $S$ – Aircraft wing area – depends on wing loading obtained from the constraint analysis that requires several simultaneous constraints to be met.

## 4. THE PROPOSED METHOD

There is a dearth of information on aircraft prices, and very little is available in the public domain in this respect. Of the few sources, one is Wikipedia [32], where the prices presented are based on the year 2018. This source was referenced in the Flyer magazine entitled “2018 buyers guide” by Pope [33]. Although Wikipedia is not regarded as a reliable source for academic writing or research [34], the costs presented are used to illustrate the cost prediction methods. The presented method uses verifiable design variables for the class of aircraft under consideration to predict not only the cost but also to highlight the cost sensitivity to the design variables. The variability in the aircraft cost comes from many options that the manufacturers offer, and all the prices presented in reference [32] seem plausible. The presented models can easily be modified on the availability of more accurate costs.

This work uses multi-linear regression analysis based on p-value to establish the aircraft cost estimation models. It is employed to minimize a collection of design parameters into a smaller set with high prediction accuracy. Thus, it is a dimensionality reduction technique. In the previously published literature, cost prediction was based only on the maximum take-off weight. Additional design parameters will enhance the estimation accuracy, and additional variables will make parametric trade-offs possible. In the preliminary design phase, simple linear regression is often used based on price per kilogram ($/kg); the same technique is also presented here as a fast prediction method for comparative purposes only and to show the case that a multi-variable cost prediction method produces more refined cost estimates.

### 4.1 Linear regression (LR) method

The direct operating costs (DOC) are a key measure of the financial viability of the aircraft. A good estimate of the aircraft cost is needed to estimate this measure in the preliminary design stage. In the early design stages, a simple cost-estimation technique called linear regression (LR) uses only the maximum take-off weight as the independent variable. The LR method is fast, easy, and widely used in nearly all modern aircraft design textbooks [35, 36]. However, it has poor accuracy, which is its main disadvantage. Low accuracy is acceptable at early stages, but many trade-offs are required at advanced stages, rendering this method unsuitable. Due to the main design parameters changes, better cost-estimation models are required, since all design trade-offs have a cost implication. As mentioned in the previous section, the list of key design variables directly or indirectly affects the total cost. Note that not all parameters are needed to estimate the cost. The final models will retain only the parameters that significantly affect the cost, as the p-value analysis identifies. The aircraft prices for each category used to establish cost estimation models are listed in Table 2. The maximum take-off weights of the relevant aircraft are also included in the table, whereas Figure 2 shows the graphical representation of the data in categories A and B.

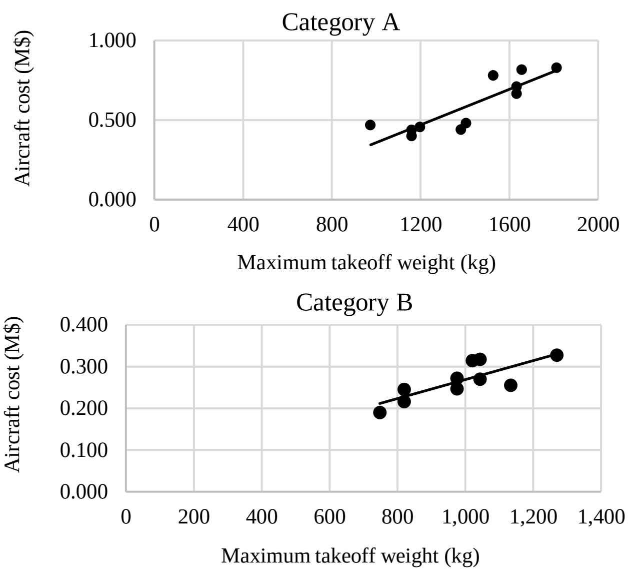


**Figure 2.** The cost-weight relationships of categories A and B

The best empirical model is obtained by applying the LR method is represented by Eqs. (6) and (7). It may be noted that the extrapolation of the empirical relationships below 750 kg may produce an invalid prediction:

$$C_{est.A[L]} = 0.0006 \times W_{to} - 0.2002 \quad (6)$$

$$C_{est.B[L]} = 0.0002 \times W_{to} + 0.0678 \quad (7)$$

### 4.2 Multi-linear regression (MLR)

MLR is applied to obtain the correlations between more than one variable, which have cause-impact relationships. It is also performed to get estimates for the topic by the relationship. In the MLR technique, an effort is considered to describe synchronically the disparity of the independent variables in the dependent variable. The mathematical formula of the MLR model is:

$$m = p_0 + p_1 a_1 + \cdots + p_n a_n + \varepsilon \tag{8}$$

where, $p_i$ is a parameter, $a_i$ is the independent variable, $m$ is the dependent variable, $\varepsilon$ is the error.

Consequently, the MLR assumptions are linear, with freedom from risky values, normal distribution, and the absence of multiple links between the independent variables. The MLR analysis is available in most statistical platforms like MS Excel. Section 5 explains how to apply the MLR and the source data (the values of the dependent and independent variables) [37].

### 4.3 p-value analysis

This analysis aims to reduce the number of parameters used in the model, built using correlation analysis with acceptable error. This reduction of parameters in the resultant model can simplify the aircraft cost prediction, identify the significance in terms of parametric analysis, and accelerate decision-making. The importance of each design parameter is obtained by examining the p-value of the regression model coefficients. It is inversely proportional to the p-value (i.e., if the p-value is low, the parameter importance is high, and vice versa).

However, this analysis is an iterative process. The parameter of the highest p-value is removed from the parameter regression list at the end of each iteration. When the p-value of all parameters becomes lower than 0.05, the iteration process is stopped. The parameters with the lowest p-value are used to formulate the final model.

## 5. ESTABLISHING COST-ESTIMATION MODELS

Table 3 lists the input values of the design parameters for each aircraft, which are used to establish the cost estimation models using multi-linear regression analysis. For comparison purposes, Eqs. (6) and (7) are also used for estimating the aircraft cost.

**Table 3.** Input design-parameter values of all categories

| Aircraft Category | Aircraft Type | Design Parameters | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $W_{to}$ (kg) | P (kW) | $L_f$ (m) | S ($m^2$) | b (m) | $V_c$ (km/h) | R (km) | F (L) | PAX |
| A | Beechcraft Bonanza | 1656 | 220 | 8.38 | 16.8 | 10.21 | 325 | 1700 | 280 | 6 |
| | Cessna 172 Skyhawk | 1160 | 134 | 8.3 | 16.2 | 11 | 230 | 1300 | 201 | 4 |
| | Cessna 182 Skylane | 1406 | 170 | 8.84 | 16.2 | 10.97 | 260 | 1700 | 329 | 4 |
| | Cessna 206 Stationair | 1633 | 220 | 8.61 | 16.2 | 10.97 | 285 | 1300 | 329 | 6 |
| | Cirrus SR20 | 1383 | 150 | 7.92 | 13.5 | 11.68 | 265 | 1100 | 229 | 5 |
| | Cirrus SR22 | 1633 | 230 | 7.92 | 13.5 | 11.68 | 340 | 1960 | 348 | 5 |
| | Diamond DA40 | 1198 | 134 | 8.1 | 13.5 | 11.9 | 255 | 1500 | 155 | 4 |
| | Mahindra GA8 Airvan | 1814 | 220 | 8.95 | 19.32 | 12.28 | 222 | 1500 | 340 | 8 |
| | Mooney M20 | 1528 | 209 | 8.13 | 16.3 | 11.13 | 430 | 2000 | 250 | 4 |
| | Piper PA-28 | 975 | 120 | 7.1 | 15 | 9.14 | 210 | 900 | 189 | 4 |
| | Beechcraft Bonanza | 1160 | 130 | 7.97 | 13.9 | 10.3 | 240 | 1150 | 240 | 4 |
| B | American Champion Citabria | 748 | 110 | 6.91 | 15.33 | 10.2 | 210 | 864 | 150 | 2 |
| | American Champion Explorer | 820 | 120 | 6.74 | 15.98 | 10.25 | 204 | 900 | 136 | 2 |
| | American Champion High-country Explorer | 820 | 130 | 6.74 | 15.98 | 10.25 | 212 | 860 | 136 | 2 |
| | American Champion Scout | 975 | 130 | 7 | 16.72 | 11 | 209 | 1737 | 260 | 2 |
| | American Champion Denali Scout | 975 | 156 | 7 | 16.72 | 11 | 237 | 1737 | 260 | 2 |
| | CubCrafters Top Cub | 1043 | 130 | 7.15 | 16.5 | 10.7 | 204 | 920 | 189 | 2 |
| | CubCrafters XCub | 1043 | 130 | 7.25 | 16.24 | 10.5 | 246 | 1300 | 185 | 2 |
| | Maule M-7 | 1134 | 130 | 7.2 | 16.14 | 10 | 222 | 1287 | 151 | 4 |
| | Aviat Husky | 1021 | 142 | 6.9 | 17 | 10.8 | 226 | 1330 | 189 | 2 |
| | Maule M-9 | 1270 | 175 | 7.2 | 16.14 | 10 | 260 | 1985 | 310 | 5 |

**Table 4.** The correlation matrix of Category B

| | *Price* | *P* | *PAX* | *S* | $W_{to}$ | *Vc* | *R* | *F* | $L_f$ | *b* |
|---|---|---|---|---|---|---|---|---|---|---|
| ***Price*** | 1 | | | | | | | | | |
| ***P*** | 0.792 | 1 | | | | | | | | |
| ***PAX*** | 0.394 | 0.738 | 1 | | | | | | | |
| ***S*** | 0.609 | 0.372 | -0.098 | 1 | | | | | | |
| $W_{to}$ | 0.803 | 0.848 | 0.781 | 0.449 | 1 | | | | | |
| ***Vc*** | 0.621 | 0.729 | 0.690 | 0.002 | 0.585 | 1 | | | | |
| ***R*** | 0.618 | 0.779 | 0.633 | 0.431 | 0.734 | 0.712 | 1 | | | |
| ***F*** | 0.578 | 0.788 | 0.529 | 0.388 | 0.658 | 0.541 | 0.913 | 1 | | |
| $L_f$ | 0.585 | 0.416 | 0.527 | 0.184 | 0.735 | 0.367 | 0.520 | 0.492 | 1 | |
| ***b*** | 0.023 | -0.241 | -0.745 | 0.666 | -0.293 | -0.497 | -0.064 | 0.036 | -0.308 | 1 |

In applying multiple linear regression (MLR) properly, it should be noted that the number of independent and dependent parameters must be less than one than the number of aircraft in the specified category under consideration. However, this work uses nine design (independent) parameters listed in section three and one independent (aircraft cost) parameter. Thus, to include all these parameters in constructing the cost models, the number of aircraft in each category should be more than ten. Otherwise, correlation analysis is applied first to reduce the number of independent parameters by removing parameters with the lowest correlation coefficients. Starting with Category A, there are 11 aircraft, and thus there was no need to apply correlation analysis. After performing the first pass of MLR, the fuel parameter with the biggest p-value was eliminated. The aircraft length was removed in the second pass, while the span parameter was removed in the third. The fourth pass showed that all parameters had a value less than 0.05, and the process was stopped. Thus, the mathematical equation of the cost-estimation model (category A) is:

$$\begin{aligned} C_{est.A\,[p-value]} = {} & 16184.74 \times S + 195 \times R \\ & +1514 \times V_c + 2622.74 \times P \\ & +156908 \times PAX \\ & -860.82 \times W_{to} - 384648 \end{aligned} \tag{9}$$

Logically all the retained variables affect aircraft cost, while investigating category B showed that the number of sampled data is ten aircraft. Thus, only eight design parameters should be considered in performing the MLR. Correlation analysis should be performed to decide which parameters should be eliminated. The correlation matrix is obtained in Table 4.

The wing span ($b$) has the lowest correlation coefficient value and is removed in the first pass. In subsequent passes, more variables are dropped, resulting in a reduced mathematical model of category B, which is:

$$\begin{aligned} C_{est.B\,[p-value]} = {} & 1123.798 \times F - 235.5 \times R \\ & +2896.625 \times V_c + 254.8 \times W_{to} \\ & +82245.46 \times S \\ & -1959.83 \times P - 1614.28 \end{aligned} \tag{10}$$

The aircraft in these categories are covered by the CAA requirements for experimental aircraft, known as e-Conditions [38]. The empirical models will benefit small-scale aircraft designers and manufacturers up to a maximum take-off weight of 2000 kg for accurately estimating the aircraft cost quickly.

## 6. RESULTS AND DISCUSSION

After establishing the cost-estimation models, either Eq. (9) (Tri-Cycle) or Eq. (10) (Taildragger) is used to predict the cost of a new aircraft. Table 5 lists the aircraft category, type, and actual price. It also lists the predicted cost, the error (difference between actual and predicted cost), and the error percentage, which are calculated using both methods (MLR and LR). Statistical analysis has several measures for evaluating the performance of the empirical cost-estimation models. One of these measures is the mean value of the variations between the actual and estimated values, denoted as the mean error (ME). In contrast, the mean value of the percentage variations is called the mean percentage error (MPE).

**Table 5.** The results of MLR and LR models for each category

| Category | Aircraft Type | Actual Aircraft Cost (M$) | Multi-Linear Regression (p-value Analysis) | | | Linear Regression (LR) | | |
|---|---|---|---|---|---|---|---|---|
| | | | Estimated Cost (M$) | Error (Diff.) | Error (%) | Estimated Cost (M$) | Error (Diff.) | Error (%) |
| A | Beechcraft Bonanza | 0.815 | 0.804 | - 0.011 | - 1.382 | 0.793 | 0.022 | 2.650 |
| | Cessna 172 Skyhawk | 0.438 | 0.460 | 0.022 | 4.976 | 0.496 | - 0.058 | - 13.196 |
| | Cessna 182 Skylane | 0.480 | 0.466 | - 0.014 | - 2.944 | 0.643 | - 0.163 | - 34.042 |
| | Cessna 206 Stationair | 0.665 | 0.675 | 0.010 | 1.544 | 0.780 | - 0.115 | - 17.233 |
| | Cirrus SR20 | 0.440 | 0.437 | - 0.003 | - 0.683 | 0.630 | - 0.190 | - 43.091 |
| | Cirrus SR22 | 0.710 | 0.713 | 0.003 | 0.402 | 0.780 | - 0.070 | - 9.803 |
| | Diamond DA40 | 0.455 | 0.460 | 0.005 | 1.15 | 0.519 | - 0.064 | - 13.978 |
| | Mahindra GA8 Airvan | 0.827 | 0.827 | 0.000 | 0.047 | 0.888 | - 0.061 | - 7.400 |
| | Mooney M20 | 0.779 | 0.781 | 0.002 | 0.21 | 0.717 | 0.062 | 8.010 |
| | Piper PA-28 | 0.467 | 0.455 | - 0.012 | - 2.65 | 0.385 | 0.082 | 17.602 |
| | Tecnam P2010 | 0.4 | 0.398 | - 0.002 | - 0.508 | 0.496 | 0.096 | 23.950 |
| B | American Champion Citabria | 0.190 | 0.195 | - 0.005 | - 2.601 | 0.222 | 0.032 | 16.727 |
| | American Champion Explorer | 0.216 | 0.219 | 0.003 | 1.568 | 0.237 | 0.021 | 9.537 |
| | American Champion High-country Explorer | 0.245 | 0.235 | - 0.010 | - 4.035 | 0.237 | - 0.008 | - 3.429 |
| | American Champion Scout | 0.247 | 0.243 | - 0.004 | - 1.585 | 0.269 | 0.022 | 8.720 |
| | American Champion Denali Scout | 0.272 | 0.273 | 0.001 | 0.436 | 0.269 | - 0.003 | - 1.27 |
| | CubCrafters Top Cub | 0.270 | 0.271 | 0.001 | 0.354 | 0.283 | 0.013 | 4.650 |
| | CubCrafters XCub | 0.317 | 0.312 | - 0.005 | - 1.532 | 0.283 | - 0.034 | - 10.873 |
| | Maule M-7 | 0.255 | 0.257 | 0.002 | 0.778 | 0.301 | 0.046 | 18.126 |
| | Aviat Husky | 0.314 | 0.319 | 0.005 | 1.741 | 0.278 | - 0.036 | - 11.502 |
| | Maule M-9 | 0.327 | 0.328 | 0.001 | 0.251 | 0.329 | 0.002 | 0.68 |

**Table 6.** The measures' results of the MLR and LR models

| Estimation Method | Measure | Aircraft Category | |
|---|---|---|---|
| | | A | B |
| MLR (p-value analysis) | ME | -0.00006 | 0.000008 |
| | MAE | 0.00773 | 0.00374 |
| | MPE | 0.01467 | 0.058 |
| | MAPE | 1.49964 | 1.4879 |
| | $R^2$ | 0.99616 | 0.98829 |
| LR | ME | - 0.00006 | 0.00525 |
| | MAE | 0.06444 | 0.02173 |
| | MPE | 1.85 | 3.14 |
| | MAPE | 11.93 | 8.55 |
| | $R^2$ | 0.7497 | 0.3719 |

Note that these differences may be negative or positive values with the possibility of canceling each other out. Thus, estimating costs can be employed as a biased measure. The absolute difference between the estimated and actual values is another measure denoted as the mean absolute error (MAE). A measure widely used in evaluating models, owing to its convenience and simplicity, is denoted as the mean absolute percentage error (MAPE). Lastly, the $R^2$ measure determines how a set of observations (parameters' values) fits with the linear model. Table 6 lists these measures' values applied to all categories.

Examining the measures of the proposed method (MLR) shows that the ME measure has a value of less than ± 0.00006 (i.e., less than \$60) for both categories, while in considering the second measure (MAE), both categories have achieved a value of less than 0.0078 (i.e., less than \$7800). A more indicatable measure is the MPE, where both categories have achieved a value less than 0.06. The MAPE measure performs similarly to the MPE but with higher values (less than 1.5). The Last measure is the $R^2$, the best one to identify the prediction model's accuracy. Both categories have an $R^2$ value of more than 0.98, meaning the model can predict with less error.

The commonly used method (LR) has only been applied for comparison purposes. Aircraft designers have used the simplistic model in the past due to the non-availability of better empirical models. Accuracy measures (i.e., ME, MAE, MPE, MAPE, and $R^2$) were significantly higher. Specifically, the $R^2$ measure is sufficient to reflect the method's performance accuracy. Category A in the LR method has an $R^2$ value of less than 0.75, whereas category B shows a very low value (0.3719). Hence, this method has a very poor cost estimation accuracy.

The minimum and maximum values of the MPE measure are extracted to ease understanding of the proposed method's accuracy. The difference between them (max and min) is calculated, and the percentage of the highest difference value is considered the model error accuracy (error %). It should be noted that the error % in Table 5 is considered to highlight the model's accuracy. These important statistical findings of all aircraft in each category are summarised in Table 7.

It can be observed from Table 7 that the proposed method predicts cost with improved accuracy, with a difference between the predicted and the actual cost of less than ± 5% for the two categories of aircraft considered. In contrast, the traditional method (LR) shows very poor accuracy and is not applicable even in the early design stages.

**Table 7.** The percentage error accuracies of MLR and LR models

| Aircraft Category | MLR Model | | | | LR Model | | | |
|---|---|---|---|---|---|---|---|---|
| | Max Error % | Min Error % | Variation Range % | Error Accuracy % | Max Error % | Min Error % | Variation Range % | Error Accuracy % |
| A | 4.976 | - 2.944 | 7.92 | ± 4.976 | 23.950 | - 43.091 | 67.041 | ± 43.091 |
| B | 1.568 | - 4.035 | 5.603 | ± 4.035 | 18.126 | - 11.502 | 29.628 | ± 18.126 |

## 7. CASE STUDIES

Two aircraft in each group not used in establishing the cost-estimation models presented as Eqs. (9) and (10) are considered to evaluate their suitability. Actual prices for the test aircraft and the associated design parameter values are listed in Table 8.

Table 9 presents the estimated aircraft cost using the developed models (Eqs. (9) and (10)). For comparison purposes, the LR method is also applied (Eqs. (6) and (7)). The error (difference) and the error accuracy between the actual and the estimated costs are also calculated (see Table 9).

Most textbooks state that it is acceptable if the predicted cost is within 10% of the actual cost at early design stages. We have presented new empirical models that will allow designers to predict the cost of new designs with an accuracy of less than 5%. The cost predictions using the test aircraft validate the developed model's suitability, using key design variables with the ability to perform parametric studies.

**Table 8.** The design parameter values of the additional aircraft for case studies

| Aircraft Category | Aircraft Type | Actual Aircraft Cost (M\$) | Design Parameters | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $W_{to}$ (kg) | $P$ (kW) | $L_f$ (m) | $S$ (m$^2$) | $b$ (m) | $V_c$ (km/h) | $R$ (km) | $F$ (L) | $PAX$ |
| A | Diamond DA20 | 0.235 | 800 | 93 | 7.16 | 11.6 | 10.87 | 250 | 1010 | 91 | 2 |
| | Piper PA-46 Matrix/M350 | 0.917 | 1969 | 260 | 8.6 | 16.26 | 13.11 | 395 | 2487 | 454 | 6 |
| B | American Champion Xtreme | 0.283 | 885 | 156 | 7 | 15.7 | 9.8 | 239 | 772 | 150 | 2 |
| | American Champion Decathlon | 0.246 | 885 | 130 | 7 | 15.24 | 9.8 | 227 | 622 | 150 | 2 |

**Table 9.** Cost prediction accuracy of the models for the test aircraft

| Category | Aircraft Type | Actual Aircraft Cost (M$) | Multi-Linear Regression (p-value Analysis) | | | Linear Regression (LR) | | |
|---|---|---|---|---|---|---|---|---|
| | | | Estimated Cost (M$) | Error (Diff.) | Error (%) | Estimated Cost | Error (Diff.) | Error (%) |
| A | Diamond DA20 | 0.235 | 0.232 | - 0.003 | - 1.404 | 0.246 | - 0.011 | - 4.83 |
| | Piper PA-46 Matrix/M350 | 0.917 | 0.890 | - 0.027 | - 2.953 | 0.899 | 0.018 | 1.96 |
| B | American Champion Xtreme | 0.283 | 0.274 | - 0.008 | - 3.035 | 0.250 | - 0.033 | - 11.7 |
| | American Champion Decathlon | 0.246 | 0.238 | - 0.007 | - 3.079 | 0.250 | - 0.033 | - 11.7 |

## 8. MODEL SENSITIVITY

This study uses the year 2018 cost data to predict aircraft cost. For new aircraft designs, the estimated cost should be modified up to the year of production, taking into account the new technologies and all market measures like deflation/inflation, CPI, etc. Thus, these costs can be factored into the estimated cost as an appropriate weighting factor.

On the other hand, testing the developed model sensitivity needs independent assessment. The feature of model linearity allows simpler calculations to be performed to estimate the aircraft cost. Moreover, it facilitates parametric studies using the design parameters to establish the impact of the change and sensitivity. Each parameter coefficient results in an impact. The highest impact is obtained with the largest coefficient value and vice versa. Additionally, the sign of the coefficient is significant in determining the proportionality direction. The estimated cost is directly proportional if the sign is positive, whereas a negative sign indicates inverse proportionality. For instance, the model of category A has five positive coefficients (direct proportional, which are $S$, $R$, $\boldsymbol{V_c}$, $P$, and $PAX$) and only one negative coefficient ($W_{to}$). In addition, the $PAX$ parameter has the greatest effect since it has the biggest coefficient value. Reducing one passenger will decrease the aircraft cost by 157000$, whereas increasing one square meter of the wing area will increase the estimated cost by only 16200$. The range parameter has the lowest impact and the lowest coefficient value. Similar deductions can be made to the model predicting the category B aircraft cost.

## 9. CONCLUSIONS

Designing a new aircraft mostly starts with some mission requirements, which impact by many design choices. The design choices not only affect the MTOW of the aircraft, but also the cost. In this paper major design variables available in the preliminary design phase are used to predict the cost. The Power loading (P/W) and wing loading (W/S) are the key entities in satisfying the constraints, which could be takeoff distance, climb rate, turning, gliding or cruise performance. Changing the weight or the wing area or the power can greatly affect the overall cost.

This work introduced the multi-linear regression analysis along with the p-value analysis as a parametric cost estimation approach to estimate the cost of light general aviation aircraft. The aircraft that are still "in production" and "in service" are used to develop the empirical models and are classified into two categories based on their landing gear configuration. Moreover, nine parameters that significantly affect the aircraft design, and its ultimate cost, are considered. Equations (9) and (10) can be used to predict the cost. The findings showed that the developed models could effectively estimate the aircraft cost at early design stages with a prediction error of less than ± 5%., but in most cases the prediction error is less than ± 3%. Thus, giving the designers an accurate method for cost prediction, which will enable either venture capital or bank loans to be secured against an accurate cost forecast. Moreover, among the available cost estimation methods, this accuracy of the presented models is the best and offers the possibility to perform parametric studies to highlight the cost sensitivity to the changes in key design variables.

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## NOMENCLATURE

| | |
|---|---|
| $b$ | wingspan, m |
| $C_{est.x\,[y]}$ | cost estimate where suffix x denotes, and suffix y denotes, $ |
| $F$ | fuel capacity, L |
| $L_f$ | aircraft length, m |
| $V_c$ | Cruise speed, km/h |
| $PAX$ | number of passengers (single class) |
| $R$ | range, km |
| $S$ | wing area, $m^2$ |
| $P$ | engine power, kW |
| $W_{to}$ | maximum take-off weight, kg |

## Subscripts

| | |
|---|---|
| $x$ | Aircraft category A or B |
| $y$ | L – (Linear regression) or<br>p-value – (P-value analysis) |