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HVAC/R Systems Modelling: Assessing Mathematical Model for Gas Compressor

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ABSTRACT

Ventilation, cooling, heating, and air conditioning (HVAC) systems represent a substantial portion, approximately 40%, of energy consumption in buildings. Among the key components of HVAC/R systems, the compressor, fans, and electrical resistances are major consumers of electricity. In this context, the compressor stands out due to its significant dynamic behaviour, serving as the pivotal element in heat pump operations. However, many manufacturers solely provide empirical data regarding compressor performance, lacking comprehensive models for assessing performance across various operational conditions. This paper addresses the methodology for evaluating mathematical models utilizing experimental compressor data. Through this approach, we aim to bridge the gap between empirical observations and predictive modelling, enabling a more nuanced understanding and optimization of HVAC/R system performance. The proposed model uses interpolation and it takes as input the evaporation and condensation temperatures. It shows an R – square and adjusted R – square value of 0.99. By using the experimental data and an open – source library for interpolation this paper paves the way for a flexible and practical compressor modelling.

1. INTRODUCTION

Heat pumps are one the major energy consumers in buildings [1]. In a self-sustaining air treatment unit, energy consumption mainly revolves around ventilators, electrical heaters, and the heat pump, with the latter accounting for over 65% of electricity usage, specifically through its compressor motor.

Traditionally, manufacturers design heat pumps using software for iterative calculations to reach an equilibrium point or rely on empirical evaluations of operational points. However, this often necessitates building and testing a prototype in a climatic chamber during the design phase to verify real-world performance against theoretical predictions, incurring both time and financial costs.

The limited availability of mathematical models for evaluating compressor performance under varying operating conditions presents a significant challenge in optimizing HVAC systems. Moreover, when considering the intricacies of 2-dimensional approximation polynomial methods, the complexity deepens. Compressors, crucial components in these systems, exhibit dynamic behaviour that demands precise mathematical representation to ensure accurate performance assessment across diverse operational scenarios.

Addressing this challenge requires the development and refinement of mathematical models tailored specifically for compressors operating within HVAC systems. These models must effectively capture the nuances of compressor behaviour under different conditions, considering factors such as varying loads, ambient temperatures, and refrigerant properties. Furthermore, in the context of 2-dimensional interpolation polynomial methods, the models need to account for spatial variations in compressor performance, adding another layer of complexity to the analysis.

Underwood [2] presents an analytical approach to model compressors as an integral component of a heat pump. This model is based on physical parameters and performance parameters that not always are available, but what is more important is that these parameters are not easy to measure. Thus, the analytical models find application only when the requirements are strict.

Similarly, in concept, Yao et al. [3] have followed the same approach as Underwood [2]. The compressor performance is calculated without considering its dynamics, based on mass flow and discharge temperature.

Efforts to enhance compressor performance evaluation through mathematical modelling must also explore advancements in 2-dimensional interpolation polynomial techniques. These techniques offer a powerful framework for representing complex systems with spatial dependencies, providing a more accurate portrayal of compressor behaviour across different operating conditions. As shown in the documentation [4] the manufacturer has provided a performance table and polynomial model calculation on the points which are not presented in respective tables.

Vugrin et al. [5] have worked on confidence region estimation for nonlinear regression. The linear approximation method showed the best performance for confidence region estimation. Similar is the case of the compressor static performance modelling.

Aprea and Renno [6] have presented an experimental study for evaluation of the dynamic model of a variable speed compressor. The results of this study are very satisfactory because it is able to describe the COP (Coefficient of performance) with an error less than 0.5%. However, in order to implement this model theoretical data that not always is available is needed.

Zhao et al. [7] have proposed a novel method for the evaluation of the compressor performance. This method is based on neural networks which would make the calculation more complex. The neural network that gave the best performance had one hidden layer with three neurons. The standard deviation of the model was less than 1% and the biggest error less than 3%.

Aprea et al. [8, 9] have worked on the optimized control of a variable speed compressor. Their approach is to find the local optimum of the compressor's performance. This is an experimental study that uses the third order polynomial model, but does not include the identification of the polynomial coefficients.

Zhao et al. [10] have developed a model for the evaluation of steady – state chiller performance by using polynomial neural network compressor model. The presented model when compared to the test data has shown an accuracy of around 95%. At these conditions we can state that this model would describe at a satisfactory level the compressor performance in steady – state.

In the work proposed by some researchers [11-13], a model that can evaluates the working conditions of the compressor based on the design parameters. The parameters that model calculates are volume displacement, input power and refrigerant mass flow. The mean error of parametric evaluation is around 3%. The accuracy of the model is above the accepted threshold for the HVAC/r systems, but the disadvantage that this model shows is the complexity. The required data to build the model is not always available and it can vary with time.

Ndiaye and Bernier [14] have proposed a model that considers the dynamic and steady state of a compressor during an on – off operation. The model is based on the state dynamics and take into consideration the constructive parameters of the machine. For parameters such as suction capacity, discharge capacity and refrigerant mass flow the dynamic equations are written meanwhile for the input power the third order polynomial model is used. The identification of the third order polynomial is not treated in this paper, while the rest of the equations depends on the design parameters that are expected to be given by the manufacturer.

Two-dimensional Lagrange interpolation is a mathematical technique used to approximate a function of two variables within a given region using a polynomial. Just like its one-dimensional counterpart, two-dimensional Lagrange interpolation aims to find a polynomial that passes through a set of known data points in a plane. This method is particularly useful in various fields such as computer graphics, image processing, geographic information systems, and numerical analysis.

In this method, the polynomial is constructed by combining basis polynomials, each of which is associated with a specific data point. These basis polynomials are chosen such that they have a value of one at their respective data points and zero at all other data points. By summing these basis polynomials with appropriate weights, the interpolated polynomial is generated, which can then be used to estimate the function values at any point within the given region.

Two-dimensional Lagrange interpolation offers flexibility in handling irregularly spaced data points and provides a continuous approximation of the underlying function. However, as the number of data points increases, the computational complexity of constructing the interpolating polynomial also grows. Therefore, careful consideration of the data distribution and the choice of interpolation method is crucial to achieve accurate results efficiently.

In such cases, experimental data becomes invaluable for creating precise calculation models for compressors across various operational points. This experimental data typically includes tables detailing refrigeration power, electric power consumption, gas mass flow, and electric current corresponding to different combinations of evaporation and condensation temperatures. By leveraging this data, along with knowledge of the gas type used, it becomes feasible to establish relationships between evaporation/condensation pressures and key performance parameters.

The development of advanced mathematical models for compressors requires a multifaceted approach that integrates both theoretical and empirical methodologies. While traditional methods provide a foundation, the complexity of modern HVAC systems necessitates more sophisticated techniques to enhance accuracy and reliability. One of the critical advancements in this area is the use of multivariable polynomial approximations. Unlike simple linear models, these techniques allow for the representation of complex relationships between multiple operating variables. For instance, a polynomial model can simultaneously consider factors such as compressor speed, refrigerant type, ambient temperature, and load conditions. This holistic approach enables a more comprehensive understanding of compressor behaviour and improves the precision of performance predictions.

Recent advancements in machine learning have opened new avenues for compressor modelling. By training algorithms on large datasets of compressor performance under various conditions, it is possible to develop predictive models that can adapt to new scenarios with high accuracy. These data-driven models can identify patterns and relationships that might be difficult to capture using traditional analytical methods. Moreover, machine learning models can continuously improve as more data becomes available, making them highly adaptable to evolving system requirements.

To ensure the practical applicability of these advanced models, it is essential to integrate experimental data. This data provides a real-world benchmark that can validate and refine theoretical predictions. For instance, performance tables that include refrigeration power, electric power consumption, gas mass flow, and electric current across different evaporation and condensation temperatures serve as a crucial resource. By aligning mathematical models with this empirical data, it is possible to achieve a high degree of accuracy in performance assessments.

Combining different modelling techniques can also enhance the robustness of compressor performance evaluations. Hybrid models that integrate polynomial approximations, machine learning, and empirical data offer a balanced approach. These models leverage the strengths of each method, providing both the precision of mathematical models and the adaptability of machine learning algorithms. Such an approach ensures that the models are both theoretically sound and practically relevant.

The continuous evolution of HVAC systems and compressor technologies presents ongoing research opportunities. Future work could explore the development of real-time adaptive models that adjust compressor operation dynamically based on current system conditions. Additionally, further investigation into the effects of new refrigerants and environmental regulations on compressor performance is essential. As HVAC systems become more integrated with smart building technologies, the role of advanced modelling in optimizing energy consumption and system efficiency will only grow in importance.

In conclusion, addressing the challenge of optimizing compressor performance in HVAC systems requires a comprehensive approach that combines advanced mathematical modelling, empirical data integration, and innovative techniques such as machine learning. By developing robust and adaptable models, it is possible to achieve significant improvements in system efficiency and reliability, ultimately contributing to more sustainable and cost-effective air treatment solutions.

2. METHODOLOGY

We suggest consulting references [15-18] for a comprehensive understanding of multivariate interpolation. Here, we briefly outline two-dimensional Lagrange polynomial interpolation.

Throughout the article, for vectors $x, y \in R^2$, we denote by (x, y) the standard Euclidean inner product, and by $||\cdot||$ the corresponding norm. We denote with:

$$\left||x|\right|_p = (\sum_{i=1}^{2}|x_i|^p)^{\frac{1}{p}}, 1 \leq p < \infty \tag{1}$$

the l_p-norm, $1 \leq p < \infty$ and $\left||x|\right|_\infty = |x_i|$.

Let $n \in N$ and $G_{2,n}$ be the tensorial equidistant points, indexed by a multi – index set $A_{2.n} = \left\{\alpha \in N^2 : \left||\alpha|\right|_\infty \leq n\right\}$. For each $\alpha \in A_2$ the tensorial multivariate Lagrange polynomials are:

$$L_\alpha(x) = \prod_{i=1}^{2} l_{\alpha_j,i}(x) \tag{2}$$

$$l_{j,i}(x) = \prod_{k=0,k\neq j}^{n} \frac{x_i - p_{k,i}}{p_{j,i} - p_{k,i}} \tag{3}$$

The Lagrange polynomials are a basis of the polynomial space $\Pi_{2,n} = span\{x^\alpha = x_1^{\alpha 1} x_2^{\alpha 2}\}$ induced by $A_{2,n}$. Since L_α satisfies $L_\alpha(p_\beta)=\delta_{\alpha,\beta}$ for all $\alpha, \beta \in A_{2,n}, p_\alpha \in G_{2,n}$ we deduce that given a function f: $\Omega \rightarrow R$ the interpolant $Q_{G_{2,n}} f \in \Pi_{2,n}$ of f in $G_{2,n}$ is given by the formula:

$$Q_{G_{2,n}} f = \sum_{\alpha \in A_{2,n}} f(p_\alpha) L_\alpha \tag{4}$$

We are going to evaluate the model quality based on the following indexes:

·SSE – Sum of squared errors

·R-squared

·Adjusted R-squared

·RMSE – Root Mean Square Error

Throughout this article, vectors $(x, y) \in R^2$ are considered, with (x, y) denoting the standard Euclidian inner product and $\|\cdot\|$ representing the corresponding norm. We define the l_p-norm for $1 \leq p < \propto$ as:

$$\|x\|_p = (\sum_{i=1}^{2}|x_i|^p)^{\frac{1}{p}}$$
$$l_\propto\text{-norm as: } \|x\|_\infty = max_i|x_i|. \tag{5}$$

To evaluate the quality of our interpolation model, we utilize several statistical metrics:

1. Sum of Squared Errors (SSE):

$$SSE = \sum_{i=1}^{n}(y_1 - \hat{y}_i)^2 \tag{6}$$

where, y_i are the observed values and $\hat{y}_i$ are the predicted values. SSE measures the total deviation of the predicted values from the actual values.

2. R-squared (R^2):

$$R^2 = 1 - \frac{SSE}{SST} \tag{7}$$

where, SST is the total sum of squares:

$$SST = \sum_{i=1}^{n}(y_1 - \bar{y}_i)^2 \tag{8}$$

with $\bar{y}$ being the mean of the observed values. R^2 indicates the proportion of the invariance in the dependent variable that is predictable from the independent variables.

3. Adjusted R-squared:

$$\text{Adjusted } R^2 = 1 - \left(\frac{1-R^2}{n-p-1}\right) \tag{9}$$

where, n is the number of observations and p is the number of predictors. This metric adjusts R^2 for the number of the predictors in the model, providing a more accurate measure of model quality when multiple predictors are involved.

4. Root Mean Square Error (RMSE):

$$RMSE = \sqrt{\frac{SSE}{n}} \tag{10}$$

which represents the square root of the average of the squared differences between predicted and observed values. RMSE is a standard way to measure the error of a model in predicting quotative data.

Our methodology follows a systematic approach, beginning with data collection and pre-processing. We gather a comprehensive set of experimental data points, including refrigeration power, electric power consumption, gas mass flow, and electric current for various combinations of evaporation and condensation temperatures. This data set forms the basis for our interpolation model.

Next, we implement the two-dimensional Lagrange polynomial interpolation using the aforementioned mathematical formulation. The interpolation process involves conducting the Lagrange polynomials for the given data points and using these polynomials to estimate the function values at other points within the definition region.

When we evaluate the accuracy of our interpolated model using the described statistical metrics, by comparing the predicted values against the actual data, we can assess the model's performance and make necessary adjustments to improve its accuracy.

Finally, we validate our model through cross-validation techniques, ensuring that it generalizes well to unseen data. This validation step is crucial to confirm that our interpolation model is reliable and robust for practical application in HVAC system optimization. So, our methodology leverages two-dimensional Lagrange polynomial interpolation to develop a precise mathematical model for compressor performance evaluation. By integrating advanced statistical metrics, we ensure a comprehensive assessment of model quality, paving the way for enhanced optimization of HVAC systems.

3. EXPERIMENTAL SETUP

To realize the identification of the computational form of the compressor in this case we will use a compressor whose computational form we know, which will serve us as a comparative standard of the final form. The compressor that we will take into consideration is "GMCC KSG280V1VMU" with the following data [4]:

Table 1. GMCC KSG280V1VMU corresponding data

Parameter	Value	Unit
Capacity	7105	W
Input power	2501	W
Flow rate	106.6	Kg/h
Current	11.58	A
Displacement	28	cm^3/rev
Electrical supply	1 – 50 – 230	-
Refrigerant	R32	-
Technology	Rotary	-
Height	360	mm
Sound level	73	dB
Diameter	140	mm
Oil charge	850	ml
Discharge pressure	3.28	MPa
Suction pressure	0.89	MPa
Evaporation temperature	10	°C
Condensation temperature	46	°C
Suction line	16.2	mm
Discharge line	9.8	mm
Superheating	8	°C
Subcooling	5	°C
Maximum discharge temperature	115	°C

The data shown in Table 1 cannot be used to calculate the performance of the compressor at different working points knowing the pressure or temperature in each of the exchangers. For this reason, the manufacturer has provided a graphic representation of the parameters of interest depending on the temperature of evaporation and condensation. Below we will see the curves related to the performance of the compressor.

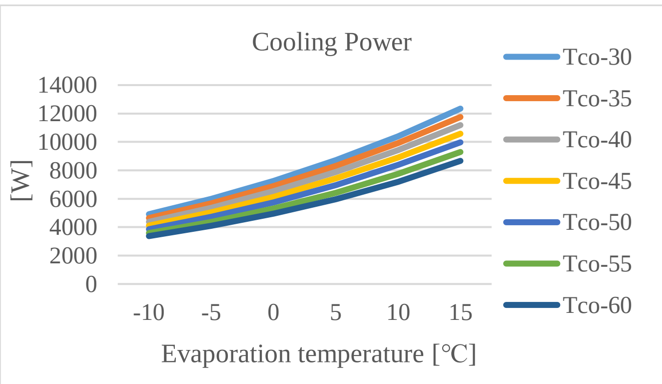

Figure 1. Refrigerant power curve

In Figure 1, we can see the relative curves of thermal capacity in relation to evaporating temperature. Each curve has been generated for a specific condensing temperature, in order to generate a 2D plot.

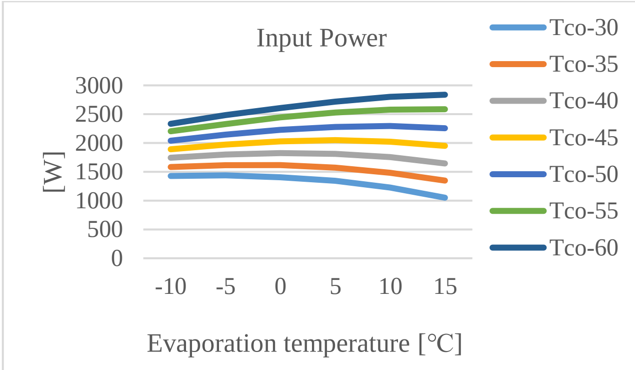

Figure 2. Electrical power curve

Figure 2 shows the curves of electrical power input in relation to evaporating temperature. Even in this case, the same result that we noted for the cooling power is true. The relation of input power is similar in form with the evaporation and condensation temperature.

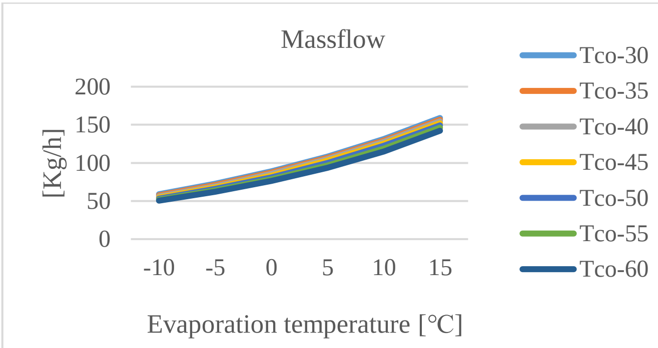

Figure 3. Refrigerant mass flow curve

In Figure 3, we can see the relation of flow rate in units of Kg/h with the evaporating temperature. As in the other cases the condensing temperature is showed using different colors for constant temperatures and we see that relation of inputs shows the same relation in principle with the output.

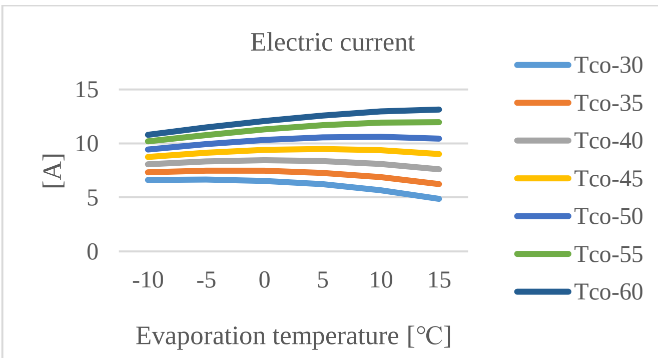

Figure 4. Absorbed current curve

Figure 4 shows the relation of electrical current with the evaporating temperature for the same set of condensing temperatures. On the other hand, the manufacturer has also given the tabular data of the graphs presented above shown on the Table 2.

Table 2. Experimental data of cooling power

T_{ev} °C	T_{co} °C	Cooling Capacity W
-10	60	3371
-10	55	3585
-10	50	3855
-10	45	4110
-10	40	4378
-10	35	4642
-10	30	4911
-5	60	4091
-5	55	4367
-5	50	4725
-5	45	5036
-5	40	5360
-5	35	5683
-5	30	5992
0	60	4954
0	55	5311
0	50	5753
0	45	6130
0	40	6527
0	35	6897
0	30	7257
5	60	5975
5	55	6419
5	50	6967
5	45	7441
5	40	7890
5	35	8307
5	30	8714
10	60	7195
10	55	7744
10	50	8375
10	45	8904
10	40	9430
10	35	9906
10	30	10387
15	60	8666
15	55	9290
15	50	9972
15	45	10578
15	40	11183
15	35	11753
15	30	12343

Table 3. Polynomial coefficients of manufacturer datasheet

Coefficient	Value
P1	8574.61292
P2	311.120343
P3	-11.2296901
P4	4.38595312
P5	-0.554774978
P6	-1.35373316
P7	0.024523568
P8	-0.0158036242
P9	-0.0245053902
P10	0.00885813341

In this case, the manufacturer of this compressor has also provided a third – order polynomial calculation model. This model will serve us as the second verification criterion on the method that we will implement in this case, while the first evaluation criterion will be the direct comparison between the real value measured and the one estimated by the model.

The model presented by the manufacturer is as follows:

$$z = p_1 + p_2 \cdot x + p_3 \cdot y + p_4 \cdot x^2 + p_5 \cdot x \cdot y + p_6 \cdot y^2 + p_7 \cdot x^3 + p_8 \cdot x^2 \cdot y + p_9 \cdot x \cdot y^2 + p_{10} \cdot y^3 \quad (11)$$

In this form, the z parameter symbolizes the calculated parameter, x symbolizes the evaporation temperature, while the y parameter represents the condensation temperature. The proportional coefficients of the polynomial for the refrigeration capacity given by the manufacturer are presented on the Table 3.

4. RESULTS AND DISCUSSION

Based on the experimental data of the Table 2, we are presenting the graphical representation of the samples.

From the distribution of the samples in Figure 5, we can notice that their descriptive function is not linear, but has elements of curvature. In this case there is not a present challenge about the model identification because it is already provided. However, in practice the manufacturers not always provide the model and there are cases in which only the tabular data are given. On the other hand, we have to consider the case of variable speed compressor when the tabular data of only certain frequencies are given. In this situation, we can evaluate the tabular data of the frequencies of interest by interpolating between frequencies. At this point the problem of compressor model identification raises. Staying in the form given by the manufacturer in the Eq. (5), which is also the most used form of compressor calculation, we will write a program in the Python programming language to approximate the polynomial coefficients of the compressor model. In this case we will use the function curve_fit of the optimization library of the SciPy module [19].

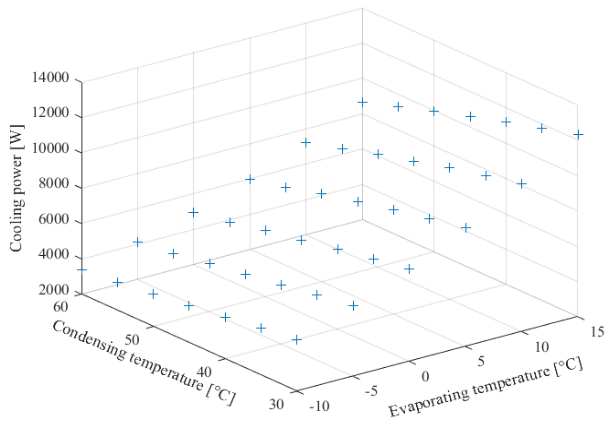

Figure 5. Distribution of samples of experimental data

This is a function that fits the parameters of an unknown function through the non-linear least squares method. The function given to the curve_fit function input is:

```
def comp_model(X, a, b, c, d, e, f, g, h, i, l):
    x1, x2 = X
    return a + b*x1 + c*x2 + d*x1**2 + e*x1*x2 + f*x2**2 +
g*x1**3 + h*x2*x1**2 + i*x1*x2**2 + l*x2**3
```

In the above function, x1 and x2 stand respectively for the evaporation and condensation temperature, while the parameters a, b, c, d, e, f, g, h, i and l represent the proportional coefficients of the polynomial model. After the above function is defined, using the following function we can get the result related to the polynomial coefficients. By executing the above code, the result we get regarding the polynomial coefficients is presented at the Table 4.

Table 4. Polynomial coefficients generated from SciPy module

Coefficient	Value
P1	8568.53
P2	311.078
P3	-10.832
P4	4.391
P5	-0.5534
P6	-1.363
P7	0.02455
P8	-0.0159
P9	-0.02459
P10	0.00892

In Figure 6, we are presenting the chart built with the coefficients shown on the Table 4 and also the samples used for the interpolation.

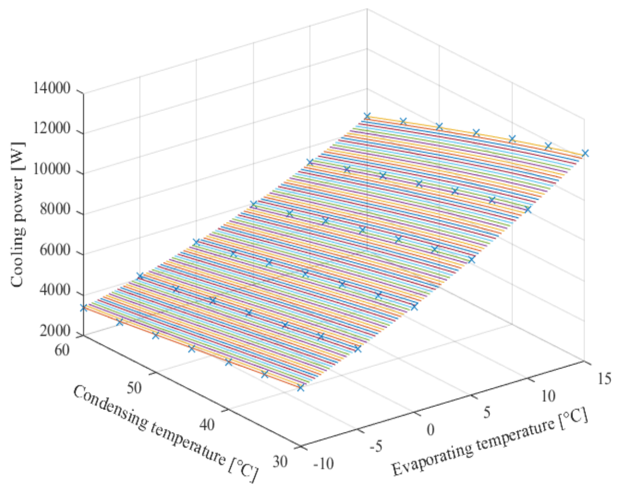

Figure 6. Distribution of samples and model surface with Python

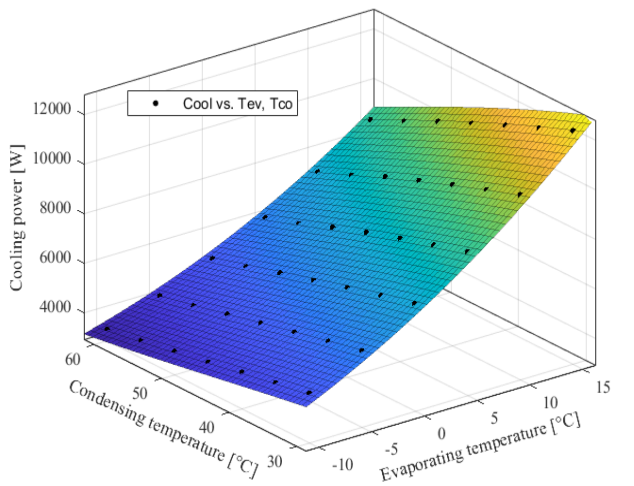

Figure 7. Distribution of samples and model surface with MATLAB

From Figure 6, it is clear that the approximation with this method has given satisfactory results, but on the other hand, the library used does not give us statistical indicators regarding the quality of the approximation. In the following, we will try to use the Curve Fit application in MATLAB, with a form of polynomial approximation with two variables of the third order.

Figure 7 shows the approximated surface generated from MATLAB. We notice that 2 plot shown respectively in Figure 6 and Figure 7 show almost identical results. From MATLAB we get the model data as follows:

$$f\,(x, y) = p00 + p10*x + p01*y + p20*x^2 + p11*x*y + p02*y^2 + p30*x^3 + p21*x^2*y + p12*x*y^2 + p03*y^3$$

Coefficients (with 95% confidence bounds)

Table 5. Polynomial coefficients generated from curve fit module

Coefficient	Value
P00	8569 (7814, 9323)
P10	311.1 (294.5, 327.7)
P01	-10.83 (-63.65, 41.99)
P20	4.391 (3.921, 4.86)
P11	-0.5534 (-1.309, 0.2021)
P02	-1.363 (-2.564, -0.16190
P30	0.02455 (0.009542, 0.03956)
P21	-0.0159 (-0.02578, -0.006011)
P12	-0.02452 (-0.03286, -0.01618)
P03	0.008926 (4.679e-05, 0.01781)

where, the data regarding the "goodness of fit" are:
- ·SSE=1.231e+04
- ·R – square=0.9999
- ·Adjusted R – square=0.9999
- ·RMSE=19.62

In this case we notice that the coefficients found by using the open-source library in Python presented in Table 4, are almost identical to the coefficients obtained using the MATLAB program shown in Table 5. R – square parameter shows practically that it the ideal value which is 1 and the same stands also for the Adjusted R – square parameter. The last quality index, RMSE, indicates that the models described at an acceptable level and model performance. The value 19.62 is insignificant considering the values of dependent variable. The polynomial model of the third order as a whole gives satisfactory performance when used for the calculation and simulation of air handling machines, referring to the applications carried out through the method presented by Daci and Bundo [20].

5. CONCLUSIONS

The problem of evaluating the computational model of compressors, in its form, is generally a solved problem in the HVAC/R industry. Although through the method described in this paper, we manage to build a model that describes the performance of the compressor with high accuracy, this model does not consider the dynamics of the compressor. Underwood [2] has shown that the dynamics of the heat exchangers is much slower compared to the dynamics of the compressor, so the latter can be neglected. However, in order to implement advanced control techniques with a focus on optimization, it would be necessary to know the dynamics of the compressor. To perform the model identification of the compressor or of the heat pump in the dynamic aspect, the collected data would have to be in the form of time series, therefore the data available in this case cannot be used.

The main problem with the calculation of this component is related to the case when the manufacturer does not provide the coefficients of the polynomial model, but only experimental data, as often happens in this industry or when the compressor is equipped with an inverter and the manufacturer provides the model only at certain frequencies. In this case, considering the linear model between two consecutive frequencies, synthetic data can be generated approximately at the new frequencies.

Another very important element would be related to the instantaneous testing of air handling machines and their control in real time. Based on the data of the manufacturer of the compressor component, we would not have a guarantee that the theoretical data would coincide with the practical ones. Meanwhile, using a supervision and control system, the data collected in real time can be used to evaluate the compressor model and then apply control based on the model built on the actual data.

Using this data and the Python code shown above, a third-order polynomial model can be obtained with the same accuracy as the MATLAB program. The quality indicators of this model, such as R-square, Adjusted R-square and RMSE, are almost at the ideal approximation values. The added value that this method brings is related to two factors. The first factor is related to the cost, where since Python is an open-source programming language, the technological costs for the implementation of this method are zero. On the other hand, by generating this model through an open-source programming language, the possibility of further automation of the process increases by integrating it with other simulation or data management programs.

REFERENCES

[1] Department of Climate Change, Energy, the Environment and Water, Australian Government. Factsheet HVAC Energy Breakdown. https://www.environment.gov.au/system/files/energy/files/hvac-factsheet-energy-breakdown.pdf, accessed on Mar. 3, 2024.

[2] Underwood, C.P. (2016). 14-Heat Pump Modelling. In Advances in Ground-Source Heat Pump Systems, Woodhead Publishing, pp. 387-421.

[3] Yao, Y., Wang, W., Huang, M. (2015). A state-space dynamic model for vapor compression refrigeration system based on moving-boundary formulation. International Journal of Refrigeration, 60: 174-189. https://doi.org/10.1016/j.ijrefrig.2015.07.027

[4] Area Cooling Solutions. Compressor hermetic rotary GMCC KSG280V1VMU, R32 Performance Datasheet. https://www.areacooling.com/p/ksg280v1vmu/, accessed on Mar. 22, 2024.

[5] Vugrin, K.W., Swiler, L.P., Roberts, R.M., Stucky-Mack, N.J., Sullivan, S.P. (2007). Confidence region estimation techniques for nonlinear regression in groundwater flow: Three case studies. Water Resources Research, 43(3): W03423.

[6] Aprea, C., Renno, C. (2004). An experimental analysis of a thermodynamic model of a vapour compression refrigeration plant on varying the compressor speed. International Journal of Energy Research, 28(6): 537-549. https://doi.org/10.1002/er.983

[7] Zhao, L.X., Zhang, C.L., Gu, B. (2009). Neural-network-based polynomial correlation of single-and variable-speed compressor performance. HVAC&R Research, 15(2): 255-268. https://doi.org/10.1080/10789669.2009.10390836

[8] Aprea, C., Renno, C. (2009). Experimental model of a variable capacity compressor. International Journal of Energy Research, 33(1): 29-37. https://doi.org/10.1002/er.1468

[9] Aprea, C., Mastrullo, R., Renno, C. (2000). Refrigeration capacity control in variable-speed compressor applications. In 7th World Congress CLIMA, pp. 15-18.

[10] Zhao, L.X., Shao, L.L., Zhang, C.L. (2010). Steady-state hybrid modeling of economized screw water chillers using polynomial neural network compressor model. International Journal of Refrigeration, 33(4): 729-738. https://doi.org/10.1016/j.ijrefrig.2009.12.028

[11] Liu, J., Li, Q., Wang, F., Zhou, L. (2012). A new model of screw compressor for refrigeration system simulation. International Journal of Refrigeration, 35(4): 861-870. https://doi.org/10.1016/j.ijrefrig.2012.01.016

[12] Ding, G. (2006). Simulation technology for refrigeration and air conditioning appliances. Chinese Science Bulletin, 51: 1913-1928. https://doi.org/10.1007/s11434-006-2072-6

[13] Fu, L., Ding, G., Su, Z., Zhao, G. (2002). Steady-state simulation of screw liquid chillers. Applied Thermal Engineering, 22(15): 1731-1748. https://doi.org/10.1016/S1359-4311(02)00084-4

[14] Ndiaye, D., Bernier, M. (2010). Dynamic model of a hermetic reciprocating compressor in on–off cycling operation (Abbreviation: Compressor dynamic model). Applied Thermal Engineering, 30(8-9): 792-799. https://doi.org/10.1016/j.applthermaleng.2009.12.007

[15] Hecht, M., Gonciarz, K., Michelfeit, J., Sivkin, V., Sbalzarini, I.F. (2020). Multivariate interpolation on unisolvent nodes--lifting the curse of dimensionality. arXiv Preprint. arXiv:2010.10824. https://doi.org/10.48550/arXiv.2010.10824

[16] Acosta, H., Veettil, S.K.T., Wicaksono, D., Hecht, M. (2021). Minterpy–Multivariate interpolation in Python. https://github.com/casus/minterpy.

[17] Zavalani, G., Hecht, M. (2024). High-order numerical integration on regular embedded surfaces. arXiv Preprint. arXiv:2403.09178. https://doi.org/10.48550/arXiv.2403.09178

[18] Tola, S., Daci, A., Zavalani, G. (2022). A simulation of an elastic filament using Kirchhoff model. Mathematics and Statistics, 10(1): 25-34.

[19] SciPy API. scipy.optimize.curve_fit, v1.14.1. https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html, accessed on Mar. 18, 2024.

[20] Daci, A., Bundo, J. (2020). Mathematical model for calculating the equilibrium point of the refrigerant circuit. IOP Conference Series: Materials Science and Engineering, 909(1): 012013. https://doi.org/10.1088/1757-899X/909/1/012013

NOMENCLATURE

W	Watt
Kg/h	Kilogram per hour
A	Amper
cm^3/rev	Centimeter cub per revolution
Mm	Millimeter
dB	deciBel
ml	Milliliter
MPa	Mega Pascal
℃	Celsius degree

Subscripts

ev	evaporating
co	condensing