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A Probabilistic Inventory Problem for Imperfect Quality with Partial Shortage Backordering, Carbon Emission, and Its Computation Using Genetic Algorithm in Python

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ABSTRACT

In this article, a multiplayer inventory model is discussed that explains the relationship between a single manufacturer and multiple retailers in a supply chain system. There are products with imperfect quality in the quantity of lots produced, resulting from the production and transportation processes. A partial shortage backorder policy is implemented to handle shortage conditions. The cost of greenhouse gas emissions from loading and transportation equipment is also considered in the total cost. The quantity of imperfect products can affect the quantity of products to be shipped. Classical optimization techniques with an integrated approach are used for analytical inventory model analysis. Due to the complexity of the model, the optimal solution is performed using a numerical computational approach. The computational and optimization procedures are implemented in a new algorithm based on the genetic algorithms, which are executed by Python programming.

1. INTRODUCTION

In the manufacturing process, items produced in factories under the supervision of manufacturers are not always in perfect condition in terms of quality, and this assumption serves as a fundamental premise in contemporary inventory modeling. The presence of imperfect items can be described either with deterministic or probabilistic approaches. Many research studies have been conducted focusing on inventory models that consider the presence of imperfect items. Inventory models dealing with probabilistically imperfect items and incorporating controlled lead times have been explored [1], which followed a lead-time demand distribution approach. Building on this research, inventory models were further investigated, while also considering the concept of shortage backordering [2]. Additionally, an inventory model for products with imperfect quality has been examined, taking into account the assumption of quality improvement over lead time [3]. Other relevant research related to the aspect of imperfect quality also has been conducted by some researchers [4-8]. The development of an inventory model relies on predefined assumptions. Nevertheless, there are still numerous conditions and scenarios within supply chain management that have not yet been incorporated into inventory models dealing with imperfect-quality products.

Recent inventory management research has increasingly focused on environmental concerns, specifically the carbon emissions (greenhouse gases) generated by production machinery, equipment, loading/unloading operations, and vehicles. Several studies have included carbon emission factors in their inventory models [9-19]. The shortage condition is a situation that is frequently encountered in practice. Interestingly, this condition is not actively sought to be circumvented by participants within the supply chain system. Various strategies are employed to minimize the occurrence of product shortages, with one such approach being the adoption of a partial backordering policy. This policy, in turn, influences the quantity of product shipments from manufacturers to fulfill retailer demands. Therefore, the partial backordering policy can also be integrated into the analysis of carbon emission management stemming from loading and unloading equipment and vehicles. The relationship between carbon emission policies and partial backordering also has been explored [20].

Furthermore, the presence of imperfect quality products can accelerate the depletion of product inventory beyond the planned timeframe, as imperfect quality items cannot be sold. This aligns with the partial backordering policy, which seeks to address such shortages. Therefore, in this context, there is a hypothesis that the existence of imperfect quality products and the partial backordering policy can be analyzed concurrently within the framework of carbon emission management policies. To date, no prior research has explored this particular combination of factors. From the existing literature, it is evident that no prior research has addressed the construction of a mathematical model for product with imperfect quality while simultaneously considering greenhouse gas emissions, reorder procedures, and policies for managing shortages through backordering as part of the objective function formulation.

The optimization analysis of inventory models involving multiple variables and parameters is generally quite complex. It is not uncommon for the optimum analytical solution to become infeasible to express explicitly. Numerical approaches often provide a realistic choice, where solutions can be obtained through computation based on algorithms designed to determine the optimal solution. One of the programming languages suitable for developing algorithms to handle complex computational cases is Python.

Based on the above description, it can be explained that this research involves 2 research questions (RQ):

RQ 1. How does a multi-player inventory model consider aspects such as imperfect quality, shortage backordering, and carbon emission management?

RQ 2. How does a genetic algorithm using the Python programming language determine the optimal solution of the inventory model computationally?

The research results, which are the answers to the research questions, are innovative in that they introduce a new multi-player inventory model that has not existed before. Previous research has not included a green inventory model that integrates aspects of imperfect quality, shortage backordering, and carbon emission in one mathematical model. Furthermore, innovation lies in the genetic algorithm developed to determine the optimal solution for the green inventory model computationally. Genetic algorithms are rarely used in inventory model literature, and the use of the Python programming language to develop this genetic algorithm in the context of inventory models has not been done before this research. In this research, a new green inventory model will be formulated, considering the existence of imperfect quality products and incorporating a partial backordering policy. The optimization analysis will be constructed based on integration and synchronization schemes. Subsequently, the numerical solutions will be determined using a genetic algorithm developed using Python.

The rest of this explanation is structured as follows. In Section 2 general assumptions, formulation of mathematical model, and analytical analysis for the optimum value are formulated. The algorithm and numerical computation of the inventory model are discussed in Section 3. Finally, some conclusion from our works and provide Several recommendations for future research are presented in the final section.

2. LITERATURE REVIEW

The process of modern inventory model formation has involved the assumption of the existence of products with imperfect quality. This is because in inventory management, there will always be products with imperfect quality. In the literature, several inventory models have been developed with the assumption of products having imperfect quality. Some of these studies include Lin [1], who examined a probabilistic multi-player inventory model with a lead-free distribution demand policy. Additionally, Jha and Shanker [2] studied the impact of imperfect quality on an integrated inventory model, which also incorporated controllable lead time policies and shortage backordering. Research on vendor-buyer inventory models for imperfect quality items with controllable lead time assumptions was also conducted by Setiawan and Endrayanto [7]. Focusing on controllable lead time assumptions, Mandal and Giri [3] explored the relationship between quality improvement and reducing products with imperfect quality. Furthermore, Konstantaras et al. [4] analyzed inventory models for products with imperfect quality, allowing for shortage policies and learning in inspection. Huang et al. [9] also analyzed vendor-buyer inventory models for imperfect quality products and shortage backordering strategies. Inventory model analysis for imperfect quality under stochastic demand and partial backlogging conditions was studied by Bhowmick and Samanta [5]. Finally, control and efficiency in inventory management for imperfect quality products have been addressed by Alamri et al. [6]. Based on this literature analysis, shortage backordering is considered an appropriate strategy in inventory management for imperfect quality products. This is because products with imperfect quality reduce the quantity of good products, leading to a rapid decline in inventory levels and potential shortages. On the other hand, control processes and their efficiency are necessary to reduce the potential for shortages due to imperfect quality. One strategy that can be used is partial backordering, although this strategy has not been extensively discussed in the literature as previously mentioned. Furthermore, in those literature sources, the numerical computations used still do not employ heuristic computations (evolutionary algorithms).

The modeling of green inventory has been developed by many researchers, focusing on various aspects of the relationship between supply chain management and the environment. When focusing on carbon management aspects, there are several known results in the literature. Firstly, Huang et al. [9] discussed inventory models with green investment and the effects of various carbon emission policies. Rahimi et al. [10] explained stochastic routing problem inventory models considering profit, service level, and green criteria. Bozorgi [11] examined carbon emission aspects in inventory models for cold items. Marchi and Zanoni [12] discussed inventory management for a closed-loop supply chain model considering logistics. Beccera et al. [14] studied sustainable green inventory models. Mahato and Mahata [20] explored a sustainable partial backordering inventory model linked to order credit policy and all-unit discount with capacity constraints and carbon emissions. Based on the literature review, there has not been a discussion on green inventory focusing on imperfect quality aspects combined with partial backordering strategies. Therefore, this research becomes critically important because carbon emissions and imperfect quality are two significant aspects of modern inventory discussions.

The green inventory model developed in this paper has fundamental differences compared to existing green inventory models in the literature. Firstly, the inventory model is constructed by combining assumptions of carbon emission management with the presence of imperfect products and partial backordering policies. Furthermore, genetic algorithms have not been previously applied in previous studies to optimize the green inventory model. Several comparisons between our inventory model and existing inventory models are presented in the Table 1.

There are two limitations of existing models in some literatures that will be addressed and also serve as the objectives of the inventory model formation process in this paper. First is the limitation in developing assumptions for the green inventory model regarding carbon emission management, which has not integrated several important assumptions in inventory management, namely the presence of imperfect quality products and anticipation of shortage conditions with partial backordering policies. The second limitation to be addressed in this research is that the computation process for optimal solutions in inventory model analysis is still primarily based on classical optimization theory and has not been directed towards a metaheuristic approach. However, given the increasing complexity of models and data, heuristic optimization approaches become more realistic. One such heuristic optimization approach is genetic algorithms, which have high precision in determining optimal solutions.

Based on the analysis and comparison with previous research in the literature, the contributions and novelties of this research are as follows:

(1) A new green inventory model for imperfect quality products under the assumption of using partial backordering strategy. This model's form has not been researched before.

(2) Computation of optimal solution determination using genetic algorithm based on Python programming, which has not been done before in inventory model analysis.

Table 1. Literature review

No.	Aspect Authors	1	2	3	4	5
1	Lin [1]	v	-	-	-	-
2	Jha and Shanker [2]	v	-	-	-	-
3	Mandal and Giri [3]	v	-	-	-	-
4	Konstantaras et al. [4]	v	-	-	-	-
5	Bhowmick and Samanta [5]	v	v	-	-	-
6	Alamri et al. [6]	-	-	-	-	-
7	Setiawan and Endrayanto [7]	-	-	-	-	-
8	Hsu and Hsu [8]	v	-	-	-	-
9	Huang et al. [9]	-	-	-	v	-
10	Rahimi et al. [10]	v	-	-	v	-
11	Bozorgi [11]	-	-	-	v	-
12	Marchi and Zanoni [12]	-	-	-	v	-
13	Beccera et al. [14]	-	-	-	v	-
14	Mahato and Mahata [20]	-	-	-	v	-
15	**This Paper**	v	v	v	v	v

Aspect:
1. Imperfect quality related to shortages/service level policy.
2. Imperfect quality related to partially backorder policy.
3. Numerical computation using Evolutionary Algorithms.
4. Carbon Emission Aspect.
5. Carbon Emission for imperfect quality items.

3. MATHEMATICAL MODEL FORMULATION AND OPTIMUM ANALYSIS

3.1 Mathematical inventory model

A mathematical inventory model will be developed for an inventory system comprising a single manufacturer with multiple retailers. The existence of imperfect quality will be taken into account, and a partial backordering policy will be adopted by all retailers. Before delving into further details, let's first introduce the notations and assumptions being utilized.

Assumptions

1. The inventory management system pertains to a single item product. Planning horizon is assumed to be infinite (without any time constraints).

2. The order level is known, constant, and continuous.

3. There is no lead time and the replenishment rate of product is infinite.

4. The expense (cost) related to carbon emissions is taken into account, and the demand rate remains consistent and well-known.

5. Shortage conditions are permissible and are addressed through a partial backordering policy. We adhere to Mahato and Mahata [20]' approach, where a fixed rate β is used to backorder a portion of the shortages.

6. Imperfect items a present in a lot of sizes q. The fraction of these imperfect items, denoted as γ_i, follows a probability density function represented by $f(\gamma_i)$. To guarantee that the manufacturer possesses adequate production capacity to meet retailer demand, it is postulated that the expected value of $E[\gamma_i] < 1 - \frac{D}{P}$.

7. The entirely of the lot quantity sorting procedure is finished at the retailer's site before the commencement of each cycle duration T. In this scenario, the time taken for sorting is included in the delivery lead time. Any products displaying imperfect quality will be sent back to the manufacturer during the subsequent lot shipment via a return process.

8. There is an assumption that no extra shipping expenses are incurred for this return process. The manufacturer offers compensation of ω for each imperfect-quality product identified. Additionally, the manufacturer plans to resell these imperfect-quality products through a secondary market.

9. The principles of synchronization and integration are agreed upon by the manufacturer and all retailers for the determination of optimal solution.

Next, the inventory mathematical modeling process will be carried out with the following steps:

1. Formulation of Assumptions: Based on the assumptions used, a mathematical inventory model will be developed. This inventory model includes the total cost formulation needed by each player in the supply chain system, namely the retailers and the manufacturer.

2. Determination of the Total Cost Function: The total cost functions for the retailers and the manufacturer are determined based on the identification of the cost components required according to the previously defined assumptions and inventory levels. The inventory levels are presented in a graph that depicts the inventory condition of each retailer and manufacturer within one cycle.

3. Formation of the Combined Retailer Total Cost Function: This function is obtained by summing up the total cost functions of all individual retailers.

4. Formation of the Joint Total Cost Function: This total cost function is derived based on the integration scheme assumption by summing the total cost functions of all retailers and manufacturer.

Once these four stages are completed, the inventory modeling process is finished. The next step is optimization to obtain the optimal values of the decision variables, *q* and *n*, using optimization principles and theories applied to the supply chain system total cost function obtained in step 4.

The construction of an inventory model for imperfect-quality products, involving a single manufacturer and multiple retailers, will now be explained, based on the previously outlined assumptions. The inventory model consists of a total cost function that encompasses both the manufacturer and retailers, which is subsequently combined into a total cost function for the entire inventory system through an integration process. The objective function for retailers in one cycle is composed of several components, encompassing ordering expenses, product transportation charges, product sorting expenditures, holding costs, backordering costs, and carbon emission handling cost.

The ordering cost C_i^b is the standard cost that each retailer must pay to the manufacturer to enable the production of products needed by each retailer. The products produced by the manufacturer will be shipped to each retailer, and the transportation cost F_i is borne by the retailer. Since it is assumed that there are always imperfect quality items, the retailer will undergo a sorting process when a lot arrives at the retailer's location during each shipment. This sorting process incurs a sorting cost per unit product s_i. The products at the retailer's location will be stored before they can be sold during the sales period. The storage process incurs a holding cost h_i^b. Then, to avoid shortage conditions, each retailer agrees to use a partial backorder strategy with a reorder cost per unit of products per unit of time b_i and with a proportion of shortage that will be ordered set at β. It is then agreed that all carbon management costs will be charged as part of the product fulfillment cost by the manufacturer to the retailer. First is the carbon emission cost from loading equipment EG_2 in the process of loading products into the transport vehicle. The transport equipment uses fuel that generates carbon emissions. Next is the carbon emission cost EG_2 from the type of transport vehicle used to deliver products from the manufacturer's location to each retailer's location. It is assumed that the same type of vehicle is used for each retailer, so EG_3 has the same value for each retailer *i*. Finally, the carbon emission cost component from unloading equipment used to unload products from the transport vehicle that arrives at the retailer's location is considered, assuming that all retailers use the same type of unloading equipment and fuel that can generate carbon emissions.

Firstly, the component of product holding costs will be determined. It is assumed that the sorting process is entirely 100% completed when the products reach the retailer’s location. Imperfect-quality products discovered during this process are promptly separated and returned to the manufacturer during the subsequent product shipment. It is further assumed that no holding expenses are incurred for the identified imperfect-quality products. The assessment of holding costs is derived from the inventory levels are presented in the following diagram.

According to the inventory process illustrated in Figure 1, the cost for holding component in the cost function for each retailer can be formulated as:

$$h_i^b \left(\frac{n}{2} \left(\frac{q_i - \gamma_i - \beta\, q_i}{D} \right)^2 + \frac{q_i^2\, \gamma_i\,(1-\gamma_i)}{D} \right)$$

Meanwhile, the shortage cost component is formulated as $\frac{1}{2} C_b\, \beta n\, q_i\, /\, D$. The emission handling cost is allocated to each retailer *i*. The type of handling equipment and vehicles are determined by the manufacturer. The emission handling cost element is computed by adding up the costs incurred from the loading and unloading equipment as well as transportation equipment emissions, formulated as:

$$J_i EG_1 C_j q_i + C_l EG_2 m_{pp} q_i + C_{ul} EG_3 m_{pp} q_i.$$

Consequently, the function of total cost for each retailer is denoted as $\mathbb{R}_i(.) : \mathbb{R}^2 \to \mathbb{R}$ with:

$$R_{i(q_i,n)} = C_i^b + nF_i + s_i n q_i + h_i^b \frac{1}{2} + \frac{1}{2} C_b \beta n \frac{q_i}{D} + J_i EG_1 C_j q_i + C_1 EG_2 m_{pp} q_i + C_{ul} EG_3 m_{pp} q_i \tag{1}$$

Now, the manufacturing objective function, denoted as the total cost function, will be detailed. Comprising several components, this manufacturing objective function includes setup costs and compensation expenses incurred due to the identification of imperfect-quality products. Drawing from Figure 1 and building upon the research conducted by Lin [1] and also result by Konstantaras et al. [4], the inventory holding cost for the manufacturer under the integration scheme is established as the product of the cost per unit of product held and the cumulative disparity between the manufacturer's inventory level and retailer *i*'s cumulative inventory level. The formulation of holding costs per cycle is accomplished through the following equation:

$$h_p \left[n(qP^{-1} + (n-1)T)q - \frac{n\left(\frac{n}{P}\right)}{2} \right] - h_p[q + 2q + \cdots + (n-1)qT] = h_p \left(\frac{1}{P} nq^2 - \frac{n^2 q^2}{2P} + \frac{1}{2} D^{-1} n(n-1)(1-\gamma) q^2 \right)$$

Hence, the total cost for the manufacturer is formulated by:

$$M(q,n) = C_p + \omega n q \gamma + h_P \frac{nq^2}{P} - h_P \frac{1}{2P} n^2 q^2 + h_p \left(\frac{1}{2} D^{-1} n(n-1)(1-\gamma) q^2 \right) \tag{2}$$

with $q = \sum_{i=1}^{n} q_i$ and $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_n)$.

3.2 Optimum analysis

The manufacturer and all retailers are assumed to agree to employ an integration scheme as the foundation for determining the optimal decision variables. The integration scheme in the inventory system is translated into the establishment of the overall cost function for the inventory system which is the combination of the manufacturer's objective function and the retailers' objective function, $J(q,n,B) = M_i(q_i,n) + \sum_{i=1}^{k} R_i(q_i,n)$ as follows:

$$\begin{aligned} J(q,n) = {} & h_p \left(\frac{nq^2}{P} - \frac{n^2 q^2}{2P} + \frac{n(n-1)q^2(1-\gamma)}{2D} \right) \\ & + \textstyle\sum_{i=1}^{k} \left(C_i^b + nF_i + s_i n q_i \right) \\ & + \textstyle\sum_{i=1}^{k} \frac{1}{2} h_i^b n \left(\frac{(q_i - \gamma_i - \beta q_i)^2}{D} \frac{q_i^2 \gamma_i (1-\gamma_i)}{D} \right) \\ & + \textstyle\sum_{i=1}^{k} \left(\frac{1}{2} \frac{c_b \beta n q_i}{D} + J_i EG_1 C_j q_i + C_l EG_2 m_{pp} q_i \right) \\ & + \textstyle\sum_{i=1}^{k} \left(C_{ul} EG_3 m_{pp} q_i \right) + C_p + \omega n q \gamma. \end{aligned} \tag{3}$$

where, $q = \sum_{i=1}^{n} q_i$, and $\gamma = (\gamma_1, \dots, \gamma_n)^T$. Since every party in the inventory system also agree to use the synchronization principle, the relationship $q_i = D_i \frac{q}{D}$ hold. Thus, Eq. (3) can be expressed as the following equation:

$$\begin{aligned} J(q,n) = {} & \frac{q^2 n}{D^3} \left(\frac{h_p D^3}{P} - \frac{(1-\gamma) D^2 h_p}{2} \right) + \\ & \frac{q^2 n}{D^3} \sum_{i=1}^{k} h_i^b D_i^2 \gamma_i (1-\gamma_i) + \frac{q^2 n}{D^3} \left(-\beta \sum_{i=1}^{k} h_i^b D_i^2 + \frac{1}{2} \sum_{i=1}^{k} \beta^2 D_i^2 \right) \\ & + \frac{q^2 n}{D^3} \left(\frac{1}{2} \sum_{i=1}^{k} h_i^b D_i^2 \right) + \frac{q^2 n^2}{-\frac{1}{2P} + \frac{(1-\gamma)}{2D}} + \\ & qn \frac{1}{D} \left(\omega \gamma + \frac{1}{D} \sum_{i=1}^{k} s_i D_i - D^{-2} \sum_{i=1}^{k} h_i^b \gamma_i D_i \right) + \\ & qn \frac{1}{D} \left(\frac{\beta}{D^2} \sum_{i=1}^{k} h_i^b \gamma_i D_i + \frac{c_b \beta}{2D} \right) + \\ & \frac{q}{D} \sum_{i=1}^{k} D_i \left(J_1 EG_1 C_j + C_l EG_2 m_{pp} \right) + \sum_{i=1}^{k} D_l\, C_{ul} EG_3 m_{pp} + \\ & \frac{n}{2D} \left(\textstyle\sum_{i=1}^{k} h_i^b \gamma_i^2 + \sum_{i=1}^{k} F_i \right) + \textstyle\sum_{i=1}^{k} C_i^b + C_p. \end{aligned} \tag{4}$$

Due to the length of the product replenishment cycle $\left(T_{tot} = \frac{nq(1-\gamma)}{D} \right)$, then we have:

$$E[T_{tot}] = \frac{nq(1-E[\gamma])}{D} \quad (5)$$

By employing the renewal-reward theorem, the formulated expected average total annual cost per unit of time becomes $EJ(q,n,B) = \frac{E[J(q,n,B)]}{E[T_{tot}]}$. Then, we have:

$$\begin{aligned} EJ\,(q,n) = {} & \frac{q\left(E[\gamma](1-E[\gamma])\sum_{i=1}^{k} h_i^b D_i^2\right)}{D^2(1-E[\gamma])} - \\ & \frac{q(1-E[\gamma])D^2 h_p}{2D^2(1-E[\gamma])} + \frac{q}{D^2(1-E[\gamma])}\left(\frac{h_p D^3}{p} + \frac{1}{2}\sum_{i=1}^{k} h_i^b D_i^2\right) + \\ & \frac{q}{D^2(1-E[\gamma])}\left(\frac{1}{2}\sum_{i=1}^{k}\beta^2 D_i^2 - \beta\sum_{i=1}^{k} h_i^b D_i^2\right) + \\ & q\left(E[\gamma](1-E[\gamma])\sum_{i=1}^{k} h_i^b D_i^2\right) + \frac{qn}{\frac{1}{2}\left(1-\frac{D}{P}(1-E[\gamma])\right)} + \\ & \frac{1}{q(1-E[\gamma])}\frac{c_b\beta}{2D} + \frac{1}{q(1-E[\gamma])}\frac{(c_b\beta)}{2D} \\ & + \frac{1}{q(1-E[\gamma])}\left(\omega E[\gamma] + \frac{1}{D}\sum_{i=1}^{k} s_i D_i\right) \\ & - \frac{\left(\frac{1}{D^2}\sum_{i=1}^{k} h_i^b E[\gamma] D_i + \frac{\beta}{D^2}\sum_{i=1}^{k} h_i^b E[\gamma] D_i\right)}{q(1-E[\gamma])} \\ & + \frac{1}{n(1-E[\gamma])}\sum_{i=1}^{k} D_i\left(J_i EG_1 C_j + C_l EG_2 m_{pp}\right) \\ & + \sum_{i=1}^{k} C_{ul}\, EG_3 m_{pp} + \frac{n}{2}\left(\sum_{i=1}^{k} h_i^b \gamma_i^2 + \sum_{i=1}^{k} F_i\right) \\ & + D\frac{1}{nq(1-E[\gamma])}\sum_{i=1}^{k}\left(C_i^b + C_p\right) \end{aligned} \quad (6)$$

where,

$K_1 = \frac{h_p D^3}{P} - \frac{(1-E[\gamma])D^2 h_p}{2} + E[\gamma](1-E[\gamma])\sum_{i=1}^{k} h_i^b D_i^2 + \frac{1}{2}\sum_{i=1}^{k} h_i^b D_i^2 - \beta\sum_{i=1}^{k} h_i^b D_i^2 + \frac{1}{2}\sum_{i=}^{k}\beta^2 D_i^2$,

$K_2 = \frac{1}{2}\left(1 - \frac{D}{P(1-E[\gamma])}\right)$,

$K_3 = \omega E[\gamma] + \frac{1}{D}\sum_{i=1}^{k} s_i D_i - \frac{1}{D^2}\sum_{i=1}^{k} h_i^b E[\gamma] D_i + \frac{\beta}{D^2}\sum_{i=1}^{k} h_i^b E[\gamma] D_i + \frac{c_b\beta}{2D}$,

$K_4 = \sum_{i=1}^{k} D_i\left(J_i EG_1 C_j + C_l EG_2 m_{pp} + C_{ul} EG_3 m_{pp}\right) + \frac{n}{2}\left(\sum_{i=1}^{k} h_i^b \gamma_i^2 + \sum_{i=1}^{k} F_i\right)$,

$K_5 = \left(\sum_{i=1}^{k} C_i^b + C_p\right)$.

Next, the optimization process for the formulated expected average total annual cost per unit of time (function (6)) will be carried out using classical optimization principles, specifically partial derivatives. The first step is to determine the extreme values of function (6) by taking the partial derivative with respect to each decision variable, *q* and *n*. Thus, we obtain the following equation:

$$\frac{\partial EJ(q,n)}{\partial q} = \frac{K_1}{D^2(1-E[\gamma])} + \frac{nK_2}{2} - \frac{K_3}{q^2(1-E[\gamma])} - \frac{DK_5}{nq^2(1-E[\gamma])} = 0 \quad (7)$$

$$\frac{\partial EJ(q,n)}{\partial n} = \frac{qK_2}{2} - \frac{K_4}{n^2(1-E[\gamma])} - \frac{DK_5}{n^2 q(1-E[\gamma])} = 0 \quad (8)$$

To obtain the optimal values of (q and n) (i.e., q^* and n^*), we perform algebraic manipulation of Eqs. (7) and (8) and derive the analytical solution as follows:

$$q^* = \frac{DK_5}{\sqrt{\frac{(n^*)^2 K_2(1-E[\gamma])}{2}\sqrt{\frac{K_3 + \frac{DK_5}{n}}{\left(\frac{K_1}{D^2} + \frac{(n^*)K_2(1-E[\gamma])}{2}\right)}} - K_4}} \quad (9)$$

Since function (6) is a convex function, the value of q^* obtained in Eq. (9) represents a minimum extreme point. Furthermore, determining an explicit analytical solution for the optimal solution based on Eq. (9) is not feasible, so the determination of the optimal solution can be achieved using a numerical computational approach, which will be explained in the next section.

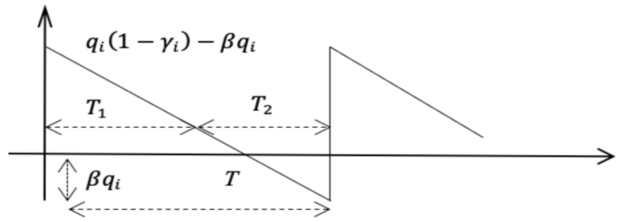

Figure 1. Retailer's inventory level

4. NUMERICAL COMPUTATION

Based on the joint total cost function in Eq. (3) and the optimal value q^* in Eq. (9), the analytical solution obtained is still in implicit form and cannot be directly interpreted in real conditions. Therefore, numerical computation is required to obtain a representative solution from the analytical solution. Due to the complex form of function (3) and the presence of many parameter values, classical optimization techniques using partial derivative criteria become irrelevant. The genetic algorithm used is a general genetic algorithm applied to function (3) with variables q and n, processed using the genetic procedures in the genetic algorithm to obtain the best fitness. These procedures include selection, crossover, and mutation. The value of γ is assumed to follow a uniform distribution in the interval [0,1].

Table 2. Value of parameters

Symbol	Value	Symbol	Value
D_1	150	D_2	160
h_1^b	3	h_2^b	4
C_p	4	s_1	3
J_1	100	J_2	120
EG_1	0.004	EG_2	0.003
C_l	4	C_{ul}	2
F_2	55	F_3	60
D_3	170	β	0.4
h_3^b	4	ω	25
s_2	3	s_3	3
J_3	125	m_{pp}	10
EG_3	0.03	C_j	3
C_b	2	F_1	55
P	500	h_p	5

We will explain the numerical computational result using genetic algorithm to find approximate values of q^* and n^* in a complex equation. We will input the previously determined parameters into our Python algorithm and initialize initial estimated values. Furthermore, we will describe the iterative process following the principles of the genetic algorithm, which is an effective method for solving nonlinear equations. This algorithm will be executed using Python programming, allowing us to approximate solutions for an equation that cannot be solved explicitly. The results will provide us with approximate values for practical use in our inventory system.

The first step is to provide the required parameter values as shown in Table 2.

The standard value of γ for each retailer is set to 0.1. The units of these values are adjusted according to the nature of the corresponding parameters as listed in Table 2. The units for quantities of products are in 1000 IDR, for unit costs are in IDR, and for weight is in kilograms. It is hypothesized that the defect rate, denoted as γ_i is consistent across all retailers and adheres to a uniform distribution on the interval [0,1]. The optimal values for q and n are calculated using a numerical approach applied to Eqs. (7) and (8). The numerical approach method employed is the genetic algorithm. The genetic algorithm was chosen due to its higher precision. Subsequently, the algorithm is formulated and executed in Python using the "random" and "pandas" packages and also "matplotlib.pyplot" packages to visualize the relationship between the changes in several parameters and the optimal values of q and n. The subsequent pseudocode outlines the genetic algorithm employed to identify the optimal decision for q and n.

The values of the other important parameters are explained as follows. The population size is set to 50. This size is considered sufficient to achieve variation in the best individual solutions while not being too large to ensure the computation process does not become slow. The crossover probability is set to 0.8. This relatively high probability is intended to ensure that the crossover process occurs frequently, allowing for good gene combinations from the two parents. The mutation process is set with a probability of 0.2 to provide a balance between exploring new solutions.

The convergence of the genetic algorithm is based on two criteria:

(1) Maximum number of generations: Implemented through the 'num_generations' parameter, which controls how many iterations the algorithm will perform before automatically stopping. This is a very common criterion used to ensure the algorithm does not run indefinitely.

(2) Threshold for fitness improvement: Compares the current best fitness value obtained with the previous one to determine whether there is a significant improvement. If there is no significant improvement:

('abs(best_fitness_current - best_fitness_previous) < threshold).

for several consecutive generations, the algorithm stops early. This helps to prevent unnecessary computations and accelerate convergence.

These criteria are designed to ensure that the genetic algorithm operates efficiently and effectively in determining the optimal solution without performing unnecessary calculations.

There are two complexities of the algorithm formed in this paper (Algorithm 1) namely time complexity and space complexity. Time complexity is affected by the initial population which takes O(polulation_size) (in Big-O notations) time.

(1) Fitness Evaluation. Each individual in the population is evaluated in O(1) (constant) time, so the whole population is evaluated in O(population_size) for each generation.

(2) Tournament Selection. This process is performed twice for each pair of parents, with a complexity of O(tournament_size) per selection such that it results in O(2x tournament size).

(3) Crossover and Mutation. This process is performed with O(1) complexity for each pair of children.

(4) Population Sorting and Trimming: The population is sorted and truncated every generation.

Algorithm 1. Algorithm for find optimal solution

```
1: Import: random, pandas, matplotlib.pyplot
2: procedure INITIALIZE_POPULATION(population-size)
3:     return [(random.uniform(500, 1500),
random.uniform(1,2))
        for in range(population-size)]
4: end procedure
5: procedure FITNESS (q, n, D, γ)
6:     Calculate fitness based on given equations
7:     return fitness value
8: end procedure
9. procedure TOURNAMENT_SELECTION (population,
   tournament-size)
10: return min (random.sample(population, tournament-
   size), key=λ x: fitness (x[0], x[1], D, γ))
11: endprocedure
12: procedureCROSSOVER(parent-1,parent-2)
13: crossover-point = random.randint(1, length(parent1) - 1)
14: child-1 = parent-1[1..crossover-point]+parent- 2[crossover-
   point+1..length(parent-2)]
15: child-2 = parent-2[1..crossover-point]+parent- 1[crossover-
   point+1..length(parent-1)]
16: return child-1, child-2
17: endprocedure
18: procedureMUTATION(individual,mutation-probability)
19: if random() < mutation-probability then
20: mutation-index = random.randint(1,length(individual))
21: if mutation_index == 1 then
22: individual[mutation-index] = random.uniform(500, 1500)
23:         else
24: individual[mutation-index] = random.uniform(1, 2)
25: end if
26: endif
27: return individual
28: endprocedure
29: procedure population-size, GENETIC-ALGORITHM
(num-
   generations, tournament-size, crossover-probability,
mutation-
   probability)
30: population = initialize-population(population-size)
31: for generation from 1 to num-generations do
32: parent-1 = tournament-selection (population, tournament-
size)
33: parent-2 = tournament-selection (population, tournament-
size)
34: child-1, child-2 = crossover(parent-1, parent-2)
35: child-1 = mutation(child-1, mutation-probability)
36: child-2 = mutation(child-2, mutation-probability)
37: population.extend ([child-1, child-2])
38: population.sort (key=λ x: fitness(x[0], x[1], D, γ))
39: population = population[:population-size]
40: end for
41: return population[0]
42: endprocedure
43: optimal-solution=genetic-algorithm(num-
generations=100,
   population-size=50, tournament-size=5, crossover-
   probability=0.8, mutation-probability=0.2)
44: print ("Optimal Value of q: " + optimal-solution [0])
45: print ("Optimal Value of n: " + optimal-solution [1])
```

Then for space complexity includes a population that requires space with a complexity of O(population_size) and additional variables such as fitness value, the best individual that requires O(1) space. Algorithm 1 has advantages in terms of efficiency, namely efficient in finding optimal solutions that are close to the optimal solution in a relatively short time. In this case, the selection, crossover, and mutation processes help to maintain population diversity so as to avoid early convergence on local solutions. Furthermore, the memory required by the algorithm is quite controllable as it only requires space that is linear to the population size. Despite its high complexity, Algorithm 1 is faster and gives better results than traditional optimization methods. Convergence criteria are based on two things:

(1) Maximum number of generations. The algorithm stops after reaching a predetermined number of generations ('num_generations').

(2) Insignificant improvement in fitness. The algorithm will stop early if there is no significant improvement in the best fitness value during consecutive generations ('max_no_improvement'). This is measured by comparing the improvement in fitness value with a threshold value ('threshod').

The convergence process of the algorithm can be explained as follows. Genetic algorithm 1 tends to converge towards the optimal solution as the generations increase. The selection, crossover and mutation mechanisms can help in exploring and exploiting the solution space effectively.

Furthermore, the efficiency of the genetic algorithm created in this research is compared with several other heuristic algorithms.

(1) Simulated Annealing (SA)

a. SA algorithm has a time complexity that is quite similar to Genetic Algorithm 1 in terms of exploring the solution space. However, when the program is run, the SA algorithm is more dependent on the temperature parameters and cooling schedule of the computer processor used to run the algorithm.

b. The convergence process of the SA algorithm is generally slower than the genetic algorithm group because it is more exploratory than exploitative.

c. SA algorithms tend to be slow so it is less efficient to find solutions to large and complex optimization problems.

(2) Tabu Search (TS)

a. The time complexity of the TS algorithm depends on the length of the taboo list and the number of iterations.

b. Convergence of the TS algorithm is more exploratory in the early iterations, but tends to exploit the existing solutions more in the fissile region.

c. The TS algorithm is less efficient than the genetic algorithm

(3) Particle Swarm Optimization (PSO)

a. The time complexity of the PSO algorithm is similar to the genetic algorithm in terms of maintaining and updating the population which in this case are the particles.

b. Convergence. PSO can achieve faster convergence in some cases due to social interaction between particles.

c. In general the genetic algorithm may be more efficient in problems that require exploration of a wider solution space than the PSO algorithm.

Using the Algorithm 1 and starting with individuals within the range of (500, 1500), the best individual values were obtained through the genetic algorithm process over 100 iterations, with the best fitness value around 207.858804659295. This resulted in optimal values for q and n within the ranges q^*=1488.253454433092 and n^*=1.9974241415185063, respectively. Therefore, it can be stated that the optimal value q^* is approximately 1488 and n^* is approximately 2. By using the same algorithm with slight variations in the input values related to imperfect product quality rate (γ), the optimal decision of q and n will change in response to variations in the gamma value. In general, the optimal value of q will increase as the value of γ becomes larger. Meanwhile, the value of n, although changing, remains within the range of 1.99 .... Hence, given that n signifies the quantity of shipments, the optimal value for n remains n=2. The changes in the values of q and n due to variations in the γ value are presented in the following Table 3 and Figure 2.

Table 3. Changes in optimal values of q and n in response to gamma variations

No.	γ	q^*	n^*
1	0.1	1322.153002	1.977192
2	0.2	1470.386571	1.947446
3	0.3	1461.037243	1.991255
4	0.4	1491.971485	1.990678
5	0.5	1413.801326	1.987244
6	0.6	1487.459066	1.992530
7	0.7	1426.698571	1.991902
8	0.8	1487.803543	1.999788

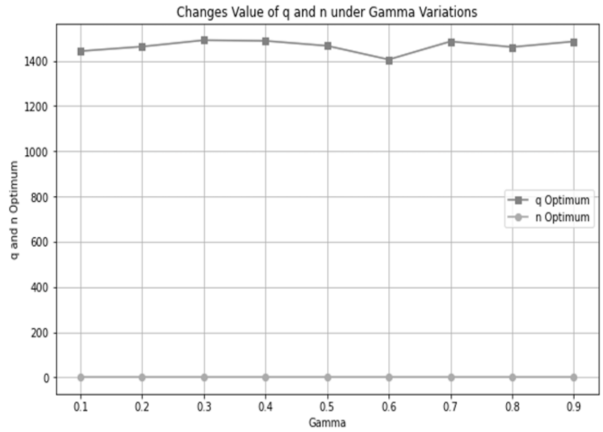

Figure 2. Plots of q and n under gamma variations

According to the numerical simulation in Table 3, it can be noticed that with the rise in imperfect quality rates, the value of q generally becomes larger. However, it can be noted that this change is not significantly large and remains within tolerable limits. Furthermore, the number of shipments (shipment quantity) does not change, allowing control over emission costs resulting from the shipping process. Therefore, this inventory model can be concluded as being resilient to the rate of defective items. Nevertheless, manufacturers should make efforts to reduce the rate of imperfect quality. Increasing q, after all, implies additional costs, underscoring the importance of minimizing imperfect quality. Next, the influence of the parameter β, or the proportion of shortage that will be ordered, on the optimum values of q and n will be examined. By employing Algorithm 1 with additional code to explore different values of β. Next, the influence of the parameter β, or the proportion of shortage that will be ordered, on the optimum values of q and n will be examined. By employing Algorithm 1 with additional code to explore different values of γ for $\gamma=0.1$, the resulting optimal values for q and n are as follows: for γ=0.1, the resulting optimal values for decision variables are shown in Table 4.

Based on the numerical simulations, The optimal value of q tends to decrease as beta increases, specifically when beta is greater than or equal to 0.1. However, the optimal value of q tends to increase again (returning to the values observed when beta was 0.1) when beta is greater than or equal to 0.5. On the other hand, the optimal value of n remains relatively unchanged. From these observations, it can be concluded that if retailers intend to adopt a partial backordering policy, the value of beta should be chosen to be small. A larger value of beta (approaching 1) would essentially imply a full backordering policy.

Based on the numerical results, it can be concluded that the proposed model has advantages compared to other inventory models mentioned in Section 2, Literature Review. The proposed model effectively illustrates the potential for carbon reduction processes, one of which is by reducing imperfect quality products. When this green inventory model is implemented, carbon emission reduction can be achieved simultaneously. For instance, the relationship between the strategy to reduce imperfect quality and the reduction of carbon emissions in the transportation process is evident. This implies improvements in the transportation process, such as better vehicle selection and route optimization, leading to reduced carbon emissions. Existing inventory models in previous research, or traditional inventory models, do not yet show the connection between imperfect quality and carbon emissions as clearly as the proposed model does.

Table 4. Changes in optimal values of q and n in response to beta variations

No.	β	q^*	n^*
1	0.1	1492.210969	1.984411
2	0.2	1434.313475	1.999060
3	0.3	1476.862559	1.998576
4	0.4	1489.889392	1.974049
5	0.5	1442.761040	1.999444
6	0.6	1491.153190	1.965408
7	0.7	1455.133886	1.997027
8	0.8	1497.088201	1.998467
9	0.9	1484.334895	1.996898

Based on this data in Table 4, Figure 3 can be generated:

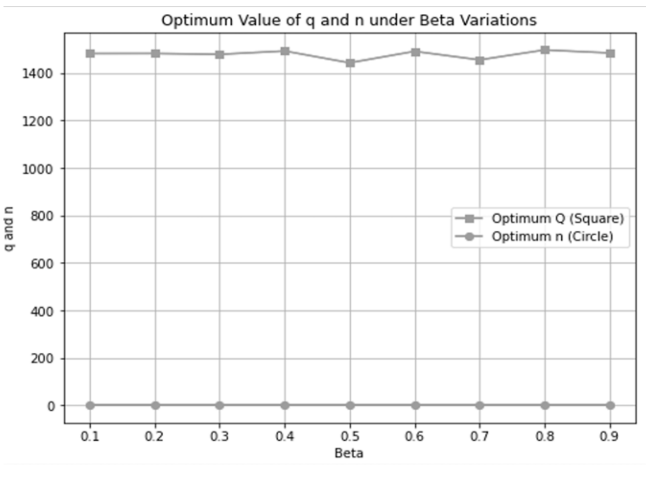

Figure 3. Plots q and n under beta variations

5. CONCLUSIONS

A multi-compartment inventory model was developed for a single manufacturer and multiple retailers. The inventory model was designed considering imperfect quality products and carbon emission costs. Due to the involvement of numerous parameters, obtaining an explicit analytical solution was not feasible, and only implicit solutions could be derived analytically. The focus of this study was on utilizing genetic algorithms implemented in Python to find numerical approximations. From the simulation outcomes, it can be deduced that the inventory model created demonstrates resilience in the face of variations in crucial parameters like the rate of imperfect quality and the proportion of ordered shortages. This implies that although variations in these parameters affect the optimal value of q, the resulting changes are within tolerable limits. Furthermore, alterations in these parameter values also impact the computed value of n, but only at the decimal level. Consequently, the optimal value of n, which should be an integer, remains unchanged at n=2.

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NOMENCLATURE

q_p	Batch production size at manufacturing site
q_i	Lot size or production
B_i	The maximum quantity of reorder per unit
n	Number of shipments in each batch produced by the manufacturer. The decision variable
D_i	Demand from the retailer *i*
D	Cumulative demand
P	Manufacturer's production rate $P>D$, with $D = \sum_{i=1}^{n} D_i$
C_p	Set up cost the production process
C_i^b	Cost for ordering per unit
γ_i	Percentage of products with imperfect quality in lot q
$f_i(\gamma_i)$	The probability density function of γ_i
ω	Cost for compensation per unit of imperfect quality of products
s_i	Sorting cost per unit of products for retailer *i*
b_i	Reorder cost per unit of products per unit time
h_p	Manufacturer's holding cost
h_i^b	The expense associated with holding one unit of the product for each retailer *i*
F_i	Freight cost per shipment from the manufacturer to retailer *i*
T	The time length between one shipment and the next (retailer's replenishment cycle)
T_1	The period during the production process at the manufacturing site
T_2	The period when the manufacturer fulfills the demands of all retailers from the inventory kept the manufacturing site
T_t	Cycle time, $T_t=T_1+T_2=n_T$
J_i	The distance from the manufacturing location to each retailer i (km)
EG_1	Carbon gas emissions from specific vehicle for delivering a unit product ($kgCO_2$) per 1 kg product
EG_2	Carbon gas emissions from loading equipment ($kgCO_2$) per 1 kg product
EG_3	Carbon gas emissions from unloading equipment ($kgCO_2$) per 1 kg product
C_l	Cost of loading per unit product
C_{ul}	Cost of unloading per unit product
C_j	Carbon emission cost per unit distance
C_b	Cost for backodering process per unit per time
m_{pp}	Weight per unit product
$R(.)$	Retailer i' s objective function
$M(.)$	Manufacturer's objective function
β	Proportion of shortage that will be ordered