INTERNATIONAL JOURNAL OF HEAT AND TECHNOLOGY

ISSN: 0392-8764
Vol. 35, No. 1, March 2017, pp. 25-31
DOI: 10.18280/ijht.350104

A publication of IIETA

http://www.iieta.org/Journals/IJHT

Hydromagnetic bioconvection flow in the region of stagnation-point flow and heat transfer in non-Newtonian nanofluid past a moving surface with suction: similarity analysis

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ABSTRACT

Steady 2-D flow of stagnation point and heat transfer in a direction past a moving surface in the presence of nanoparticles and gyrotactic microorganisms in a nanofluid, with the existence of a magnetic field applied perpendicular to the surface is studied. Similarity analysis is applied to transform the model of non-linear PDEs to non-linear ODEs. Results obtained are then discussed numerically with the help of the shooting method. The result are shown graphically including velocity profiles, temperature profiles and density profiles (of motile microorganisms) for different values of physical parameters like suction parameter, thermophoresis parameter, Brownian motion parameter, Magnetic parameter and the stretching parameter, Lewis, Schmidt and bioconvection Peclet number.

Keywords: Nanofluid, Stagnation Point, Thermophoresis, Brownian Motion, Stretching Sheet, Gyrotactic Microorganism.

1. INTRODUCTION

A point in a flow field where the local velocity of the fluid is zero is known as stagnation point. It exists on the surface of items immersed in the flow field, where the fluid is brought to rest by the item. The flow of stagnation describes the motion of the fluid which is close to the region of stagnation which occurs at all solid bodies flowing in a fluid which encounter high level pressure, heat transfer and maximum rates of mass deposition. Stagnation point flow has now become an interesting area amongst the scientists and the researchers because it plays a very important role in industrial processes and has scientific significance as well. E.g. cooling of electronic devices and the nuclear reactors, reduction in drag, thermal oil recovery and most of the hydrodynamic actions in engineering applications. Some of the work is given in references [1-6].

Nanofluid holds nanometer-sized particles in which fluids are emerged in the base fluid with poor thermal conductivity such as water, ethylene glycol mixture and oils. Recently, many researchers have been attracted to explore the problem of heat transfer features in nanofluid and they claim that, in the existence of nanoparticles within a fluid, the effective thermal conductivity of the fluid rises up appreciably. Example of such processes are fuel cells, microelectronics, chiller, hybrid powered industries and more in pharmaceutical applications. Nanofluids was studied first by Choi [7].Thermal conductivity of the fluid is increased by adding a very little quantity of nanoparticles to conventional heat transfer fluids which presented by Choi et al [8]. Later on, a detailed survey of convective transport was given by Buongiorno [9]. In recent years, many authors [10-16] have contributed to the study of convective flows of nanofluids. Bioconvection has remarkable importance in biological systems, in bio-microsystems and biotechnology. The nanofluid bioconvection deals with the study which gives the density stratification and formation of impulse pattern which is through the behavior of condensed self-propelled microorganisms, buoyancy forces, and nanoparticles.

The presence of motile microorganisms in the system increases the rate of mass transfer, heat transfer and improves nanofluid stability. In the past decade, there was lot of work done on the convective heat transport in nanofluids but nanofluids containing nanoparticles and gyrotactic microorganisms have not been extensively investigated. Kuznetsov and Avramenko [17] studied the different characteristics of bioconvention issues in suspensions containing solid particles. This phenomenon of bioconvection is the formation of convective motion of fluid due to skyward swimming microorganisms having mean density higher than water (Pedley and Kessler [18]). Geng and Kuznetsov [19] studied the impact of tiny suspended particles on the development of bioconvection plumes and found that particles affect the system and are the origin of transition of bioconvection plume to a different steady state. Gyrotactic movement is the typical behavior for algal suspensions. Whenever these microorganisms are in a moving flow, their line of swimming is controlled through the stability between the gravity acting on the microorganisms and torques with viscous drag appearing from shear flow (Pedley and Kessler [20]). In the case of motile microorganisms, the nanoparticles are not self-propelled, also they move because of thermophoresis and Brownian motion occurring within nanofluid as shown by Aziz et al. [21]. Different facet of bioconvection problems are given by Kuznetsov [22-26]. Mutuku et al [27] studied the bioconvection effect past a verticle plate in nanofluid in presence of gyrotactic microorganisms. Because of the applications of MHD effects in engineering, science and technology, the object of this paper is to study the stagnation point flow past a moving surface in a nanofluid containing gyrotactic microorganisms in influence of magnetic field with suction by using similarity transformation.

2. PROBLEM FORMULATION

Consider the steady 2-D stagnation point flow of a nanofluid in direction to the stretching surface coinciding with plane $y=0$ at near stagnation point at $x=0$ in the nanofluid see fig 1. $u_w(x)$ the stretching velocity and $u_e(x)$ which is the ambient fluid velocity is supposed to extend to $u_w(x)=cx^m$ and $u_e(x)=ax^m$ where a,c,m are constants with $a>0$ and $m>0$ whereas $c>0$ corresponds to stretching sheet.

Also assumed that, at the surface of sheet, the nanoparticle fraction C , the temperature T and the uniform concentration of microorganisms N takes constant values C_w, T_w and N_w respectively however, the values of C,T and N when $y \to \infty$ are indicated by C_∞, T_∞ and N_∞ respectively.

Consider the following model of equations in vector

Equation of Continuity:

$$\nabla.\mathrm{v}=0 \tag{1}$$

Equation of Momentum:

$$\rho_f(\mathrm{v}.\nabla\mathrm{v})=-\nabla p+\mu\nabla^2\mathrm{v}+\sigma B^2(u_e-u) \tag{2}$$

Energy Equation:

$$\mathrm{v}.\nabla\mathrm{T}=\alpha\nabla^2 T+\tau\left[D_B\nabla C.\nabla T+\left(\frac{D_T}{T_\infty}\right)\nabla T.\nabla T\right] \tag{3}$$

Nanoparticle Volume Fraction

$$\mathrm{v}.\nabla C=D_B\nabla^2 C+\left(\frac{D_T}{T_\infty}\right)\nabla^2 T \tag{4}$$

Equation of Conservation for Microorganisms

$$\nabla.\,\mathrm{j}=0 \tag{5}$$

where j is the flux of microorganisms, given by

$$\mathrm{j}=N\mathrm{v}+N\tilde{\mathrm{v}}-D_n\nabla N \tag{6}$$

and $\tilde{\mathrm{v}}$ is given by as $\tilde{\mathrm{v}}=\left(bW_c/\Delta C\right)\nabla C$ (7)

where ρ_f density the base fluid, μ is dynamic viscosity, σ the electrical conductivity, ρ is the density, C is the nanoparticles volume fraction, ρ_p is density of the particle, $(\rho c)_f$ is the heat capacity of the fluid and $(\rho c)_p$ is effective heat capacity of the nanoparticle material, $\alpha=k/(\rho c)_f$ is diffusivity of thermal, D_B is Brownian diffusion coefficient, D_T is coefficient of thermophoresis diffusion, uniform magnetic field the base fluid is given by $B(x)=B_0 x^{\frac{m-1}{2}}$, $B_0>0$ and is assumed to be applied normally to the surface, $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ the ratio of nanoparticles capacity of heat and the base fluid, b is the chemotaxis constant and bW_c is the maximum cell swimming speed.

Associated with boundary conditions are:

$$\begin{aligned} &y=0: u=u_W(x)=cx^m, v=v_w(x), T=T_w, C=C_w, N=N_w \\ &y\to\infty: u=u_e(x)=ax^m,\ C=C_\infty,\ T=T_\infty,\ N=N_\infty \end{aligned} \tag{8}$$

where $v_W(x)$ is the mass flux velocity with $v_W(x)<0$ for suction and $v_W(x)>0$ for injection.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0^2}{\rho_f}(u_e-u) \tag{10}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{11}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{12}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\frac{\partial}{\partial y}\left(N\tilde{v}\right)=D_n\frac{\partial^2 N}{\partial y^2} \tag{13}$$

where $\tilde{v} = \left(bW_c \big/ \Delta C \right) \partial C \big/ \partial y$. Also ν is the kinematic viscosity.

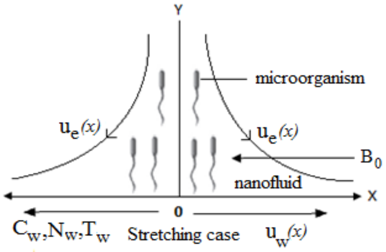

Figure 1. A Sketch of the physical problem.

Applying the stream function $u = \dfrac{\partial \psi}{\partial y}$, $v = -\dfrac{\partial \psi}{\partial x}$, $\theta = \dfrac{T - T_\infty}{T_w - T_\infty}$, $\phi = \dfrac{C - C_\infty}{C_w - C_\infty}$ and $\chi = \dfrac{N - N_\infty}{N_w - N_\infty}$ in equation (9-13) then we get following equations.

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = u_e \frac{du_e}{dx} - \nu \frac{\partial^3 \psi}{\partial y^3} + \frac{\sigma B_0^2}{\rho_f}\left(u_e - \frac{\partial \psi}{\partial y} \right) \tag{14}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2} + \tau \left[D_B \frac{\partial \phi}{\partial y}\frac{\partial \theta}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial \theta}{\partial y} \right)^2 \right] \tag{15}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 \theta}{\partial y^2} \tag{16}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \chi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \chi}{\partial y} + \frac{\partial}{\partial y}\left(\chi \tilde{v} \right) = D_n \frac{\partial^2 \chi}{\partial y^2} \tag{17}$$

along with the following conditions

$$\begin{aligned} &\frac{\partial \psi}{\partial y} = cx^m, \ \frac{\partial \psi}{\partial x} = v_w(x), \quad \theta = \phi = \chi = 1 \ at \ y = 0 \\ &\frac{\partial \psi}{\partial y} = u_e(x) = ax^m, \theta \to 0, \phi \to 0, \chi \to 0 \ as \ y \to \infty \end{aligned} \tag{18}$$

Following [28] we introduced the similarity variables as

$$\begin{aligned} &\psi = \sqrt{u_e(x)\nu x}\, f(\eta), \eta = \sqrt{\frac{u_e(x)}{\nu x}}\, y, \ \theta(\eta) = \frac{T - T_\infty}{\Delta T}, \\ &\phi(\eta) = \frac{C - C_\infty}{\Delta C}, \ \chi(\eta) = \frac{N - N_\infty}{\Delta N} \end{aligned} \tag{19}$$

Using equation (19) we get

$$\begin{aligned} &u = u_e(x) f'(\eta), \\ &v = -\frac{m+1}{2}\sqrt{\frac{u_e(x)\nu}{x}}\left[f(\eta) + \frac{m-1}{m+1}\eta f'(\eta) \right] \end{aligned} \tag{20}$$

Here we assume $v_w(x) = -\dfrac{m+1}{2}\sqrt{\dfrac{u_e(x)\nu}{x}}\, S$, the mass flux velocity to obtain the similarity solution where S is the parameter of mass flux velocity with $S < 0$ for injection and $S > 0$ for suction.

Putting equation (19) to equations (14)-(17) we get

$$f''' + \frac{m+1}{2} ff'' + m - mf'^2 + M(1 - f') = 0 \tag{21}$$

$$\theta'' + P_r\left(\frac{m+1}{2} f\theta' + N_b \theta'\phi' + N_t \theta'^2 \right) = 0 \tag{22}$$

$$\phi'' + \frac{m+1}{2} Le\, f\, \phi' + \frac{N_t}{N_b}\theta'' = 0 \tag{23}$$

$$\chi'' + \frac{m+1}{2} \mathrm{Sc}\, f \chi' - \mathrm{P_e}\left[\phi'\chi' + (\sigma + \chi)\phi'' \right] = 0 \tag{24}$$

with boundary value conditions as

$$\begin{aligned} &f'(0) = \lambda, \ f(0) = S, \ \theta(0) = \phi(0) = \chi(0) = 1 \\ &f'(\infty) = 1, \ \theta(\infty) = 0, \ \phi(\infty) = 0, \ \chi(\infty) = 0 \end{aligned} \tag{25}$$

Here $\lambda = \dfrac{c}{a} > 0$ is stretching parameter, $L_e = \dfrac{\nu}{D_B}$ the Lewis number, $P_r = \dfrac{\nu}{\alpha}$ the Prandtl number, $P_e = \dfrac{bW_c}{D_B}$ is bioconvention Peclet number, $S_c = \dfrac{\nu}{D_n}$ is Schmidt number, $N_b = \dfrac{\tau D_B \Delta\phi}{\nu}$ is the parameter of Brownian motion, $N_t = \dfrac{\tau D_T \Delta\theta}{\nu T_\infty}$ is parameter of thermophoresis, $\sigma = \dfrac{N_\infty}{\Delta N}$ is the dimensionless parameter and $M = \dfrac{\sigma B_0^2}{\rho_f\, a}$ is the magnetic parameter.

3. NUMERICAL APPROCH

The equations (21)-(24) along with the boundary condition (25) are a coupled nonlinear boundary value problem. Fourth order Runge–Kutta scheme with shooting method is applied to examine the flow geometry for the system of above equations. This method transforms equations into a group of initial value problems with unknown initial values, which can be obtained by guessing. Then, the fourth order Runge–Kutta scheme is applied to integrate the group of initial value problems till the given boundary conditions are fulfilled. These coupled ODEs third order in f , second order in θ,ϕ and χ are reduced to a system of nine simultaneous equations for nine unknowns. This system can be solved numerically with Runge–Kutta scheme, we need nine initial conditions but two in f one in each of θ,ϕ and χ are known and at $\eta\to\infty$ values of f', θ,ϕ and χ are known to us. By using these four conditions we can construct both not known initial conditions at $\eta=0$ with the help of shooting method technique. The way that section titles and other headings are displayed in these instructions, is meant to be followed in your paper.

The important step to select the proper finite value of η_∞. Hence to obtain η_∞, we began with initial guesses and solve the BVP having the set of equations (21)-(24) to get $f''(0)$, $\theta(0),\phi(0)$ and $\chi(0)$ .We repeated same process by considering other larger value of η_∞ till two of the consecutive values of $f''(0)$, $\theta(0),\phi(0)$ and $\chi(0)$ vary only after required significant digit. For numerical computations, the thickness of boundary layer i.e. η_∞ which is to be obtained by applying to boundary conditions (25). We obtained the value $\eta_\infty=8$ is adequate for all profiles to fulfil the infinite boundary conditions (25) asymptotically. Using step size $h=0.001$ we found the numerical solution with $\eta_{\max}$ and convergence criteria of 10^{-7} is used.

Now we define following variables as

$$f_1=f,\ f_2=f',\ \ f_3=f'',\ \ f_4=\theta,\ \ f_5=\theta'$$
$$f_6=\phi,\ \ f_7=\phi',\ \ f_8=\chi,\ \ f_9=\chi' \tag{26}$$

The equations (21)-(24) which are coupled order differential equations along with boundary conditions (25) which are transformed by equation (26) as

$$f_3'=-\left(\frac{m+1}{2}\right)f_1f_3-m+mf_2^2+M\left(f_2-1\right) \tag{27}$$

$$f_5'=-\Pr\left(\frac{m+1}{2}f_1f_5+N_bf_5f_7+N_tf_7^2\right) \tag{28}$$

$$f_7'=-\left(\frac{m+1}{2}Lef_1f_7+\frac{N_t}{N_b}f_5'\right) \tag{29}$$

$$f_9'=-\left(\frac{m+1}{2}Sc\,f_1f_9\right)+Pe\left[f_7f_9+\left(\sigma+f_8\right)f_7'\right] \tag{30}$$

where prime represents derivative with respect to η and initial boundary conditions:

$$f_1(0)=S,\ f_2(0)=\lambda,\ f_3(0)=a_1,\ f_4(0)=1,\ f_5(0)=a_2$$
$$f_6(0)=1,\ \ f_7(0)=a_3,\ \ f_8(0)=1,\ \ f_9(0)=a_4 \tag{31}$$

where a_1,a_2,a_3,a_4 are the assumed initial conditions for shooting method.

4. RESULT AND DISCUSSION

The main characteristics of fluid flow which are heat transfer, nanoparticle volume fraction and density of motile microorganisms are obtained and outcomes are shown graphically. The obtained outcomes and numerical values are presented in figs 2-6. A detailed discussion of the resulting parameters like bioconvection Péclet number Pe, Brownian motion parameter Nb, thermophoresis parameter Nt, Magnetic parameter M, Lewis number Le, the parameter of suction S on velocity, temperature, nanoparticles volume fraction and the motile microorganism profiles and the stretching parameter λ is also presented. Here all computations are calculated for the constant value $\Pr=6.2$ and $\lambda=2$ (Stretching sheet).

Figure 2 represents the graph of velocity with different values of suction parameter S, magnetic parameter M and all other parameters remaining fixed. In the presence of the magnetic field, Lorentz force and velocity field are affected. Thus, Magnetic parameter and Suction parameter increase, resulting in an increase of the retarding force and so the velocity and thickness of boundary layer decrease. It is clearly noted that the profiles of velocity, temperature, nanoparticles and motile microorganism density satisfy asymptotically the far field boundary conditions [Equation (25)]. Figure 3 represents the effect of the Nb (Brownian motion parameter), Nt (thermophoresis parameter) and the dimensionless parameter σ on velocity profile.

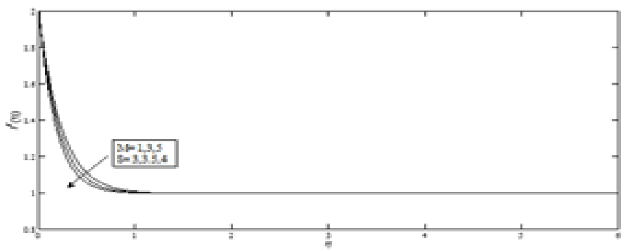

Figure 2. Velocity profile for different values of M and S for $\Pr=6.2$, $Nt=Nb=0.5$, $Pe=Sc=\sigma=1$, $Le=2$ when $m=1$ and $\lambda=2$

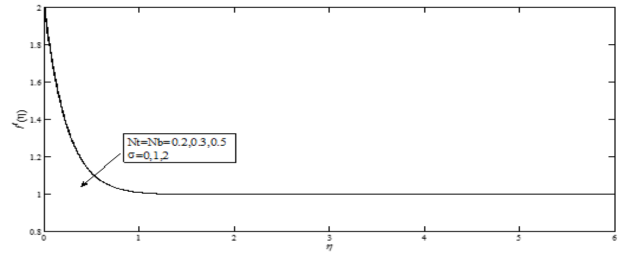

Figure 3. Velocity profile for different values of $Nt=Nb$ and parameter σ for $M=1$, $Pe=Sc=1$, $Le=2$ when $m=1$ and $\lambda=2$

Figures 4-5 shows the change in temperature with the parameters like suction parameter S, magnetic parameter M, Nb (Brownian motion parameter) and Nt (thermophoresis parameter). Increasing the suction parameter S, causes a reduction in the temperature and the thickness of thermal boundary layer as represented in figure 4, which shows that increasing suction parameter S results in more nanofluid sucked out thus reducing the temperature. From application view, enhanced rate of heat transfer on the surface is vital because it has a bearing in metallurgical processes. Figure 5 represents the impact of the change of magnetic parameter M, Nb (Brownian motion parameter) and Nt (thermophoresis parameter). Increasing M, $Nt = Nb$ increases the temperature and thickness of boundary layer. As shown, the presence of nanoparticles in the base fluid increases the thermal conductivity which results in increased temperature and thickness of thermal boundary layer.

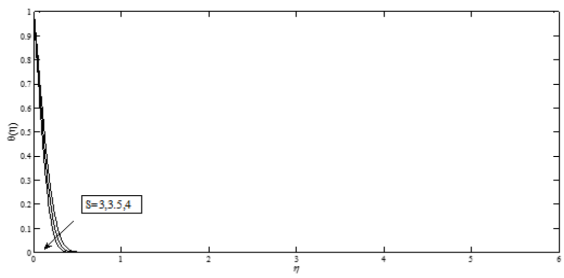

Figure 4. Temperature profile with different values of S for $\Pr = 6.2, Nt = Nb = 0.5, M = 1, Le = 2$ $Pe = Sc = \sigma = 1$ when $m = 1$ a nd $\lambda = 2$

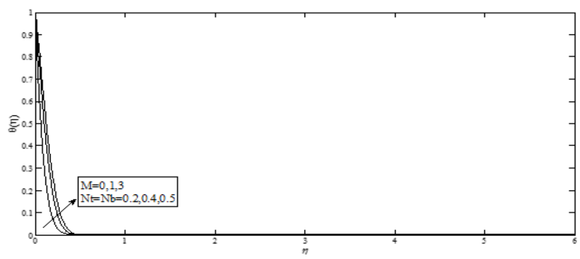

Figure 5. Temperature profile with different values M and $Nt = Nb$ for $\Pr = 6.2, Le = 2$ $Pe = Sc = \sigma = 1$ when $m = 1$ and $\lambda = 2$

Figure 6 represents the effects of the different parameters on the dimensionless density of motile microorganisms. This profile is mostly affected by Sc and Pe. The point to note is that, when we increase Sc, S, Pe and σ , it decreases the dimensionless density of microorganism concentration thickness and also motile microorganism density.

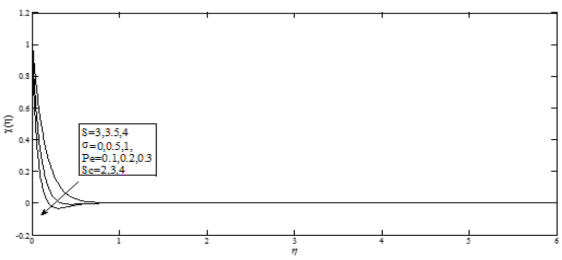

Figure 6. Motile microorganism density profile with different values of S, Pe, Sc, σ for $\Pr = 6.2, Nt = Nb = 0.5, M = 1, Le = 2$ when $m = 1$ and $\lambda = 2$

5. CONCLUSION

Similarity solution of stagnation-point flow and heat transfer past a moving surface which contains nanoparticles and gyrotactic microorganism in the existence of uniform magnetic field with suction is obtained. The governing model of PDEs are transformed into non-linear ODEs by appropriate similarity technique. It is found the convective process is controlled by the parameters Lewis number Le, bioconvection parameters Pe, the Brownian motion parameter Nb and the thermophoresis parameter Nt. Also, as suction increases, it increases the heat transfer rate at the surface.

ACKNOWLEDGMENT

One of the author Govind R Rajput would like to thank NMIMS Deemed-to-be University for the finacial support under Minor Research Project.

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NOMENCLATURE

a	postive constant
A	velocity ratio parameter
B	variable magnetic field
b	chemotaxis constant
M	dimensionless magnetic number
T	temperature of the fluid
u	velocity component along x-axis
v	velocity component along y-axis
T_∞	temperature of the fluid in the free stream
T_w	temperature of the fluid at surface
C	nanoparticle volume fraction
C_w	nanoparticle volume fraction at the surface
C_∞	nanoparticle volume fraction in the free stream
c	constant
Pr	Prandtl number
D_T	Thermophoresis diffusion coefficient
D_B	Brownian diffusion coefficient
Dn	diffusivity of microorganisms
j	flux of microorganism
m	positive exponent
Nb	Brownian motion parameter
Nt	Thermophoresis parameter
p	pressure
Nw	wall concentration of microorganism
Pe	bioconvection Peclet number
S	suction/injection parameter
Sc	Schmidt number
$u_e(x)$	ambient fluid velocity
$u_w(x)$	stretching/shrinking velocity

Wc	maximum cell swimming speed
a_1, a_2, a_3, a_4	constants

Greek symbols

η	dimensionless similarity variable
σ	electrical conductivity
θ	dimensionless temperature
ϕ	dimensionless nanoparticle volume fraction
μ	dynamic viscosity
ν	kinematic viscosity
ρ_f	nanofluid density
$(\rho c)_f$	heat capacity of the fluid
$(\rho c)_p$	heat capacity of the nanoparticle material
λ	stretching/shrinking parameter
ψ	stream function
α	thermal diffusivity of the nanofluid
ΔC	characteristic nanoparticle volume fraction
ΔN	characteristic motile microorganisms density difference
τ	ratio of the effective heat capacity of the nanoparticle to that of the fluid

Subscripts

∞	condition at free steam
w	condition at the surface

Superscripts

'	differentiantion with respect to η