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Effect of Double Porous Layer on Rough Step Slider Bearing Lubricated with Couple Stress Fluid

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ABSTRACT

An investigation into how a double porous layer at the base impacts the performance of a Rayleigh step slider bearing lubricated with couple stress fluid is presented. The bearing is considered to be rough in nature. The modified Darcy's law is used to express the fluid pressure in the porous regions. To estimate the effect of surface roughness of the bearing the random variables pertaining to roughness parameters are analysed applying the Christen'S stochastic model. Applying Stokes' micro-continuum theory for couple stress yields an estimation of pressure via a modified Reynolds equation. Integrating over the bearing length the estimated film pressure, the load carrying capacity or lifting force, frictional force and coefficient of friction are incorporated into the study. A comparison to a single porous layer underscores the superior efficiency offered by the double porous layer. The study further elucidates the most favourable inlet film height and the optimal bearing length that facilitate maximum load carrying capacity. It is evident that the inlet film thickness can be reached the maximum up to 1.8 to 2 to obtain the maximum load carrying capacity of the bearing. The present study is compared with the existent results and that is more efficient for the current bearing model.

1. INTRODUCTION

The scientific community has shown increasing interest in Newtonian lubricants mixed with various additive combinations, primarily due to the enhancements they provide to lubricating properties. These efficient fluids demonstrate non-Newtonian flow characteristics, rendering the conventional continuum theory, applicable to fluids obeying Newton's law of viscosity, unsuitable. In response, Stokes [1] proposed a micro-continuum theory for couple stress fluids to describe the flow characteristics of these non-Newtonian fluids. This theory couples certain essential motions of the fluid's material with the conventional Newtonian Continuum theory, resulting in Stokes' non-Newtonian micro-continuum theory. This comprehensive theory considers effects often neglected in conventional fluid theories, including the impact of couple stresses, the influence of body couples, the effect of a non-symmetric stress tensor, and the effect of additive particle size.

In 1918, Lord Rayleigh introduced the concept of the step bearing, known as the Rayleigh step bearing [2]. When compared to other slider bearings, Rayleigh step bearings exhibit superior load-carrying capabilities. Consequently, these bearings have been widely adopted in the industrial sector, particularly in automotive equipment, to enhance efficiency. Numerous researchers have further studied this bearing configuration under various lubrication conditions since its inception.

Naduvinamani and Siddangouda [3] investigated the impact of surface roughness on a porous step slider bearing, illustrating that a couple stress fluid can increase the bearing's load capacity and lower the coefficient of friction compared to a Newtonian fluid. Ramanaiah and Sarkar [4] further examined the influence of couple stress, noting that its effect is more pronounced in thinner areas of the lubricant film, as indicated by increased pressure profiles toward the leading edge. The steady and dynamic properties of Rayleigh step slider bearings under the influence of non-Newtonian couple stress were also explored by Lin et al. [5], who observed significant improvements in dynamic stiffness and damping coefficients but a reduction in the volume flow rate.

Further studies by Naduvinamani and Siddangouda [6], Wang and Jin [7], Naduvinamani et al. [8], Sekar et al. [9], and Maiti [10] have explored various aspects of these bearings, including the effect of micropolar fluid, non-Newtonian index, non-linear factors of lubricants, magnetic parameters, and the lubricating properties of micropolar fluids. Collectively, these studies highlight the improved performance and load capacity of bearings when using non-Newtonian fluids, with some noting a decrease in friction.

The introduction of porosity into these systems can create a significant drop in load-carrying capacity, which can be mitigated by reducing the permeability of the porous matrix. As permeability increases, more fluid is trapped between bearing surfaces. Introducing an additional porous layer can create a double-porous system, which further reduces the amount of lubricant entering the porous zone, extending the accumulation time of lubricant between the wall surfaces, and ultimately enhancing the load-carrying capacity of the bearing. Studies by Verma [11], Anthony and Elamparithi [12], Kataria and Patel [13], and Shah and Kataria [14] have delved into various aspects of double porous systems, including the effects of limited permeability, roughness, MHD, slip and squeezing velocity, and the application of two porous layers to the lower plate of a slider bearing.

In this context, ferrofluid was employed as the lubricant, with slip and squeeze velocities taken into account for the analysis. Notably, a significant increase of approximately 6.35% was observed in the dimensionless load-carrying capacity when the permeability of the upper porous layer exceeded that of the lower layer. Rao et al. [15] conducted a study on a double porous layered long journal bearing lubricated with a couple stress fluid. The findings revealed an increase in the load-sustaining capacity due to the presence of a double porous layer and couple stress parameter. However, these factors were inversely proportional to the coefficient of friction.

Additional research by Singh and Sharma [16] analysed the hybrid double layer porous journal bearing, uncovering an increase in stiffness and damping coefficient when micropolar fluid was employed in comparison to Newtonian fluid. Similarly, Srinivasan's [17] investigation revealed that the double layer had the effect of augmenting load capacity and frictional drag while simultaneously decreasing the coefficient of friction. Comparing single and double layer porous step slider bearing with couple stress fluid, Naduvinamani and Ganachari [18] found the double porous layer to possess a superior load carrying capacity.

Furthermore, Naduvinamani and Siddangouda [3] examined the impact of surface roughness on the thin lubricant film between the surfaces of a porous step-slider bearing. It was discovered that negative mean and skewed roughness contributed to an increase in load carrying capability.

From the review and analysis of the aforementioned literature, it is clear that no research has yet been conducted concerning the effect of surface roughness on a double porous layer step slider bearing lubricated with a couple stress fluid. This paper aims to elucidate the efficiency of double porous layers, drawing comparisons with a similar scenario previously analysed for a single layer by Naduvinamani and Siddangouda [3]. The comparison between the single and double porous layer is consequently tabulated. Graphical representations detailing the effects produced by parameters that influence the flow of the lubricant are plotted and discussed, highlighting their respective impacts.

2. MATHEMATICAL MODELING

As the lubricant flows through the passage of step slider bearing, which is cushioned by a double porous material, the flow of the lubricant through these two channeled porous beds can be modeled as follows. From Figure 1 the model consider's a rough Rayleigh step slider bearing with a step in its upper surface and lower surface which is backed with a double porous layer. Consider h_1 and h_2 to represent the film thickness at the entry level and the exit level of the bearing respectively. In this case the variation in film thickness makes it possible to calculate the step height, which is denoted by h_1-h_2. Consider L_1 as the length up to the step, and L_2 as the remaining length, getting together the condition for the bearing length $L=L_1+L_2$. A solid base to the bottom of the slider serves as the support for the first porous layer which is of thickness δ_1. This is supported below by a second porous layer of thickness δ_2 there by allowing the lubricant to flow through both the pores. It is assumed that the upper porous bed has a lower percolation compared to the lower porous bed due to the smaller pore size of the first layer. The lower surface of the step slider is oriented along the x direction. The sliding velocity of the lower surface is taken to be U.

The layer between the upper and the lower face of the slider is filled with a very thin layer of couple stress fluid. As couple stress fluid has most of its application in the automobile industry it becomes a model of considerable importance to analyse its effect on a Rayleigh step slider.

The following assumptions are made while considering the above situation. The lubricant is taken as an incompressible couple stress fluid. The derivation takes into account that there is absence of body force and body couple. The equation of continuity and the equation of momentum are derived by Stokes [1] for the couple stress fluid assuming the lubricant to be a very thin film takes the following form.

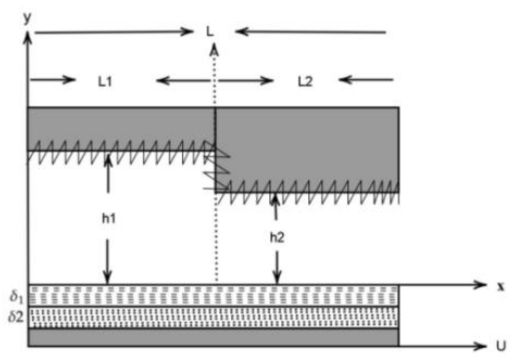

Figure 1. Double porous Rayleighstep slider bearing

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\mu\frac{\partial^2 u}{\partial y^2}-\eta\frac{\partial^4 u}{\partial y^4}=\frac{\partial p}{\partial x} \tag{2}$$

$$\frac{\partial p}{\partial y}=0 \tag{3}$$

The components of velocity in the direction of (x, y) is taken to be (u, v). In the above equation, the pressure is denoted by p, viscosity of the lubricant, that is, the couple stress fluid is taken as μ and η the material constant that characterizes the couple stress fluid.

The boundary condition at the upper and lower surface is given by:

(i) At the upper surface ($y=H$)

$$u=v=0,\frac{\partial^2 u}{\partial y^2}=0 \tag{4}$$

(ii) At the lower surface ($y=0$)

$$u=U,\frac{\partial^2 u}{\partial y^2}=0,v=-\hat{v} \tag{5}$$

where, $\hat{v}$ is the velocity of the fluid in the porous region in the direction of y. The negative sign shows the drag experienced due to the pressure that builds inside the pores which is brought out by the modified Darcy's law expressed as:

$$\hat{v} = \frac{-1}{\mu k}\left[\frac{k^2}{(1-\beta)}\frac{\partial \hat{p}}{\partial y}\right]$$

where, $\hat{p}$ is the pressure in the porous medium, $\beta = \frac{\eta}{\mu}\left(\frac{1}{k}\right)$ which estimates the ratio between size of microstructure to the pore size and k represents the percolation of the fluid in the porous region. The pressure $\hat{p}_1$ and $\hat{p}_2$ satisfies the Laplace equation in the porous region δ_1 and δ_2. The Laplace equation is defined as follows:

$$\frac{\partial^2 \hat{p}_1}{\partial x^2} + \frac{\partial^2 \hat{p}_1}{\partial y^2} = 0 \tag{6}$$

$$\frac{\partial^2 \hat{p}_2}{\partial x^2} + \frac{\partial^2 \hat{p}_2}{\partial y^2} = 0 \tag{7}$$

The related pressure boundary conditions in the porous region are

$$\hat{p}_1 = 0 \; at \; x = 0 \text{ and } x = L \tag{8}$$

$$p(x,0) = \hat{p}_1(x,0) \tag{9}$$

$$\hat{p}_1(x,-\delta_1) = \hat{p}_2(x,-\delta_1) \tag{10}$$

$$\left(\frac{\partial \hat{p}_2}{\partial y}\right)_{y=-(\delta_1+\delta_2)} = 0 \tag{11}$$

$$\frac{k_1}{\mu(1-\beta_1)}\left(\frac{\partial \hat{p}_1}{\partial y}\right)_{y=-\delta_1} = \frac{k_2}{\mu(1-\beta_2)}\left(\frac{\partial \hat{p}_2}{\partial y}\right)_{y=-\delta_1} \tag{12}$$

Integrating of Eq. (6) with respect to y over the porous wall thickness:

$$\left(\frac{\partial \hat{p}_1}{\partial y}\right)_{y=0} = -\int_{-\delta_1}^{0}\frac{\partial^2 \hat{p}_1}{\partial x^2}dy + \left(\frac{\partial \hat{p}_1}{\partial y}\right)_{y=-\delta_1} \tag{13}$$

Integrating Eq. (7) with respect to y over the porous wall thickness with use of Eq. (13) gives:

$$\left(\frac{\partial \hat{p}_1}{\partial y}\right)_{y=0} = -\int_{-\delta_1}^{0}\frac{\partial^2 \hat{p}_1}{\partial x^2}dy - \frac{k_2(1-\beta_1)}{k_1(1-\beta_2)}\int_{-(\delta_1+\delta_2)}^{-\delta_1}\left(\frac{\partial^2 \hat{p}_2}{\partial x^2}\right)dy \tag{14}$$

Since the porous region thickness δ_1 and δ_2 is assumed be small, Eq. (14) reduces to:

$$\frac{\partial \hat{p}_1}{\partial y}\Big|_{y=0} = -\left(\delta_1 + \frac{k_2(1-\beta_1)}{k_1(1-\beta_2)}\delta_2\right)\frac{\partial^2 \hat{p}_1}{\partial x^2} \tag{15}$$

The continuity and momentum Eqs. (1)-(2) are solved applying the boundary conditions stated in Eqs. (4)-(5) holding Eq. (3) to be true to obtain the velocity of flow of the couple stress fluid present between the slider surface in the direction of x as follows:

$$u = U\left(1-\frac{y}{H}\right) + \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[y^2 - yH + 2l^2\left\{1 - \frac{cosh(\frac{2y-H}{2l})}{cosh(\frac{H}{2l})}\right\}\right] \tag{16}$$

where, $l = \sqrt{\frac{\eta}{\mu}}$ represents the couple stress parameter.

The continuity Eq. (1) is integrated over the film thickness to obtain the modified Reynolds equation as:

$$\frac{\partial}{\partial x}\left\{f(H,l)\frac{\partial p}{\partial x}\right\} = 6\mu U\frac{\partial H}{\partial x} - \frac{12k}{(\mu-\mu\beta)}\frac{\partial \hat{p}}{\partial y}\Big|_{y=0} \tag{17}$$

where, $f(H,l) = H^3 - 12l^2H + 24l^3tanh(0.5H/l)$.

Assuming the porous region thickness δ_1 and δ_2 to be very thin, Morgan-Comeron gave an approximation given by Eq. (15) which on application to Eq. (17) gives:

$$\frac{\partial}{\partial x}\left\{\left[f(H,l) - \frac{12\psi}{(1-\beta_1)}\left(1 + \frac{k_2(1-\beta_1)}{k_1(1-\beta_2)}\frac{\delta_2}{\delta_1}\right)\right]\frac{\partial p}{\partial x}\right\} = 6\mu U\frac{\partial H}{\partial x} \tag{18}$$

The lubricant fills the space between the two surfaces which occupies a height H which can be considered to be made up of two parts a smooth and a rough part which can be expressed as:

$$H = h(x) + h_s$$

where, the first part of the sum represents the smoothness and the second part is a random variable measuring the roughness. The value $h(x)$ varies as h_1 for $x \in [0, L_1]$ and it varies as h_2 for $x \in [L_1, L]$. The value of h_s ranges between $-c$ to c, where c is the maximum height deviated at any point when measured from the mean film roughness.

Operating both sides of Eq. (18) by expectancy operator E defined by:

$$E(*) = \int_{-\infty}^{\infty}(*)f(h_s)dh_s$$

where, $f(h_s)$ is the probability density function given by:

$$f(h_s) = \begin{cases} \frac{35}{32}\frac{(c^2-h_s)^3}{c^7} & if \quad h_s \in [-c,c] \\ 0 & elsewhere \end{cases}$$

which gives Eq. (19) as:

$$\frac{\partial}{\partial x}\left\{\left(E[f(\mathrm{H},l)] - \frac{12k_1\delta_1}{(1-\beta_1)}\left(1 + \frac{k_2(1-\beta_1)}{k_1(1-\beta_2)}\frac{\delta_2}{\delta_1}\right)\right)\frac{\partial E(p)}{\partial x}\right\} = 6\mu U\frac{\partial E(H)}{\partial x} \tag{19}$$

where,

$$\begin{aligned} E[f(H,l)] &= f(h,l,\alpha,\sigma,\epsilon) \\ &= h^3 + 3h^2\alpha + 3h(\alpha^2+\sigma^2) + \epsilon + 3\alpha\sigma^2 \\ &+ \alpha^3 - 12l^2\alpha - 12l^2h + 24l^3\tanh\left(\frac{h}{2l}\right) \\ &+ \left(1 - \tanh^2\left(\frac{h}{2l}\right)\right)(12l^2\alpha - \alpha^3 - \epsilon \\ &- 3\sigma^2\alpha) \end{aligned}$$

The mean α, the standard deviation σ and the parameter ϵ, which is the measure of symmetry of the random variable h_s are defined as:

$$\alpha = E(h_s)$$
$$\sigma^2 = E[(h_s-\alpha)^2]$$
$$\epsilon = E[(h_s-\alpha)^3]$$

Introducing the below mentioned dimensionless quantities:

$$\bar{x} = \frac{x}{L}, P = \frac{ph_2^2}{\mu UL}, \bar{l} = \frac{2l}{h_2^2}, \psi = \frac{k_1\delta_1}{h_2^3}, \bar{h} = \frac{h}{h_2}, \overline{h_m} = \frac{h_m}{h_2}$$
$$\overline{h_1} = \frac{h_1}{h_2}, \overline{L_1} = \frac{L_1}{L}, \overline{L_2} = \frac{L_2}{L}, \bar{\alpha} = \frac{\alpha}{h_2}, \bar{\sigma} = \frac{\sigma}{h_2^2}, \bar{\epsilon} = \frac{\epsilon}{h_2^3}$$

The above dimensionless parameters are considered in deriving the following equations.

The expression for pressure is obtained from Eq. (19) by integrating it twice and applying the boundary conditions for pressure, that pressure vanishes at the boundary and $P = P_c$ at $\bar{x} = \overline{L_1}$ gives:

$$P = \frac{6(\bar{h} - \overline{\mathrm{h_m}})}{\mathrm{G}(\bar{\mathrm{h}}, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon})}\,\bar{x} + a_1 \tag{20}$$

where, P_c is the pressure which is common to both the entry and the exit region which prevails at the point of the step of the slider bearing.

where,

$$\mathrm{G}(\bar{h}, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon}) = F(\bar{h}, \bar{l}, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon}) - \frac{12\psi}{(1-\beta_1)}(1 + (KR)B_1)$$

$$KR = \frac{k_2}{k_1}, B_1 = \left(\frac{1-\beta_1}{1-\beta_2}\right)\left(\frac{\delta_2}{\delta_1}\right)$$

$$F(\bar{h}, \bar{l}, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon}) = \bar{h}^3 + 3\bar{h}^2\bar{\alpha} + 3\bar{h}(\bar{\alpha}^2 + \bar{\sigma}^2) + 3\bar{\sigma}^2\bar{\alpha} + \bar{\epsilon} + \bar{\alpha}^3 - 12\bar{l}^2\bar{h} - 12\bar{l}^2\bar{\alpha} + 24\bar{l}^3 tanh(\frac{\bar{h}}{2\bar{l}}) + (1 - tanh^2(\frac{\bar{h}}{2\bar{l}})) \times (12\bar{l}^2\bar{\alpha} - \bar{\epsilon} - \bar{\alpha}^3 - 3\bar{\sigma}^2\bar{\alpha})$$

when $k_2 = 0$ the double layer porous bearing will become a single layered porous bearing.

Using the boundary condition for pressure gives:

$$a_1 = 0, P_c = \left\{\frac{(\overline{\mathrm{h_1}} - \overline{\mathrm{h_m}})}{\mathrm{G}(\overline{\mathrm{h_1}}, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon})}\right\}\overline{L_1} \tag{21}$$

for the entry level and:

$$a_1 = 0, P_c = \left\{\frac{(\overline{\mathrm{h_1}} - 1)}{\mathrm{G}(1, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon})}\right\}\overline{L_2} \tag{22}$$

for the exit level.

Let $\overline{h_m}$ be the film thickness at step where the pressure P_c defined using Eqs. (21) and (22) coincide:

$$\overline{h_m} = \frac{\overline{\mathrm{h_1}}\,\overline{\mathrm{L_1}}(\mathrm{G}(1, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon})) + \overline{\mathrm{L_2}}(\mathrm{G}(\overline{\mathrm{h_1}}, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon}))}{\overline{\mathrm{L_2}}(\mathrm{G}(\overline{\mathrm{h_1}}, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon})) + \overline{\mathrm{L_1}}(\mathrm{G}(1, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon}))} \tag{23}$$

The dimensionless pressure for the entry region $\overline{\mathrm{L_1}}$ through which lubricant enter the bearing is obtained as:

$$P_1 = 6\left[\frac{\overline{\mathrm{L_2}}\,\overline{\mathrm{h_1}} - \overline{\mathrm{L_1}}}{\overline{\mathrm{L_2}}\left(\mathrm{G}(\overline{\mathrm{h_1}}, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon})\right) + \overline{\mathrm{L_1}}(\mathrm{G}(1, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon}))}\right]\bar{x} \tag{24}$$

The dimensionless pressure for the entry region $\overline{\mathrm{L_2}}$ through which lubricant exits the bearing is obtained as:

$$P_2 = 6\left[\frac{(\overline{\mathrm{L_1}}\,\overline{\mathrm{h_1}} - \overline{\mathrm{L_1}})(1 - \bar{\mathrm{x}})}{\overline{\mathrm{L_2}}\left(\mathrm{G}(\overline{\mathrm{h_1}}, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon})\right) + \overline{\mathrm{L_1}}(\mathrm{G}(1, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon}))}\right] \tag{25}$$

Integrating Eq. (24) between 0 to $\overline{L_1}$ and Eq. (25) between $\overline{L_1}$ to L gives the load carrying capacity of the bearing which can be expressed in dimensionless form as:

$$W = \frac{wh_2^2}{\mu UL^2} = 3\left[\frac{\overline{\mathrm{L_1}}\left(\overline{\mathrm{L_1}}\,\overline{\mathrm{L_2}} - \overline{\mathrm{L_1}}^2 + 1\right)(\overline{\mathrm{h_1}} - 1)}{\overline{\mathrm{L_1}}\left(\mathrm{G}(\overline{\mathrm{h_1}}, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon})\right) + \overline{\mathrm{L_1}}(\mathrm{G}(1, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon}))}\right] \tag{26}$$

The force of friction experienced per unit width on the surface y=0 is defined by:

$$f = \int_0^L (\tau_{yx})_{y=0} \tag{27}$$

where,

$$\tau_{yx} = \mu\frac{\partial u}{\partial y} - \eta\frac{\partial^3 u}{\partial y^3} \tag{28}$$

Using Eq. (16) in (28) and replacing it in Eq. (27) leads to the estimation of the dimensionless form of friction as:

$$F = \frac{-fh_2}{\mu UL} = \int_0^1 \left[\frac{1}{h} + \frac{h}{2}\frac{\partial p}{\partial x}\right]dx, \tag{29}$$

$$F = \frac{\xi + (-1 + 1/\overline{\mathrm{h_1}})\overline{\mathrm{L_1}}(\xi) + 3\overline{\mathrm{L_1}}(\overline{\mathrm{h_1}} - 1)(\overline{\mathrm{L_2}}\,\overline{\mathrm{h_1}} - 1 + \overline{\mathrm{L_1}})}{\xi} \tag{30}$$

where, $\xi = \overline{\mathrm{L_2}}\left(\mathrm{G}(\overline{\mathrm{h_1}}, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon})\right) + \overline{\mathrm{L_1}}\left(\mathrm{G}(1, \bar{l}, \psi, \bar{\alpha}, \bar{\sigma}, \bar{\epsilon})\right)$

The coefficient of friction is given by:

$$C = \frac{F}{W} \tag{31}$$

3. RESULTS AND DISCUSSIONS

The effect produced by porosity arranged in a two layered surface which is rough and has a thin film waiting of couple stress lubricating the surface of Rayleigh step slider bearing is studied for its performance. The effects are compared with that of a single layered porous surface of a similar geometry derived by Naduvinamani and Siddangouda [3]. The differences are best brought forward by analyzing the following parameter: couple stress parameter $\bar{l}$, mean roughness $\bar{\alpha}$, standard deviation $\bar{\sigma}$, skewed surface roughness $\bar{\epsilon}$ and bearing length $\overline{L_1}$. The investigations on the porous bearings have determined that the load carrying capacity of the bearing is reduced on account of permeability. As the lubricant starts seeping into the pores, the lubricant remains between the bearing surfaces for a very small amount of time which eventually leads to the reduced load carrying capacity. When compared to the single porous layer, the fluid takes a greater amount of time to percolate in a double porous layer. When both these layers are of different permeability, the penetration of the fluid in the two layers differ from one and another. This difference gives sufficient time for the saturation of the fluid between the bearing surfaces. This leads to the enhanced load carrying capacity of a double porous layer when compared to a single porous layer geometry.

3.1 Dimensionless work load

The observation on the load sustaining capacity W with the film thickness $(\overline{\mathrm{h_1}})$ for distinct values of $\bar{l}$, $\bar{\alpha}$, $\bar{\epsilon}$, $\bar{\sigma}$ and $\overline{L_1}$ are plotted in Figures 2, 3, 4, 5 and 6 respectively. The observation reveals that the couple stress fluid parameter $\bar{l}$ significantly contributes towards the enhancement of load sustaining capacity W as shown in Figure 2. It is noted that increasing the value of mean roughness $\bar{\alpha}$, skewed surface roughness $\bar{\epsilon}$, standard deviation $\bar{\sigma}$, decreases the load carrying capacity. From Figure 6 it is noted that the load sustaining capacity reaches the maximum point at $\overline{L_1} = 0.8$ and $\overline{L_2} = 0.2$. From the graphs it is evident that the inlet thickness can be maximum up to 1.8 to 2 to obtain a maximum load carrying capacity. Values beyond this do not help sustain maximum load.

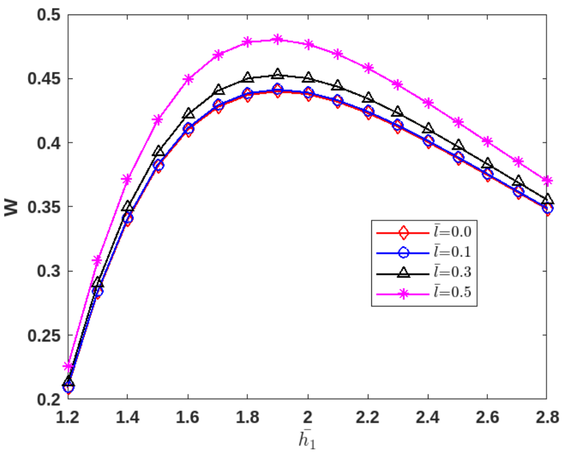

Figure 2. Plot of W with $\overline{h_1}$ for distinct values of $\bar{l}$

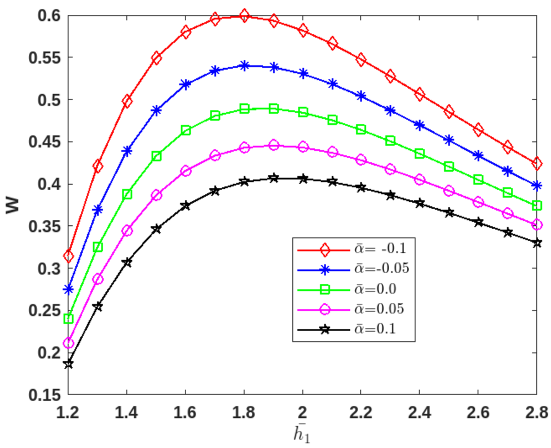

Figure 3. Plot of W with $\overline{h_1}$ for distinct values of $\bar{\alpha}$

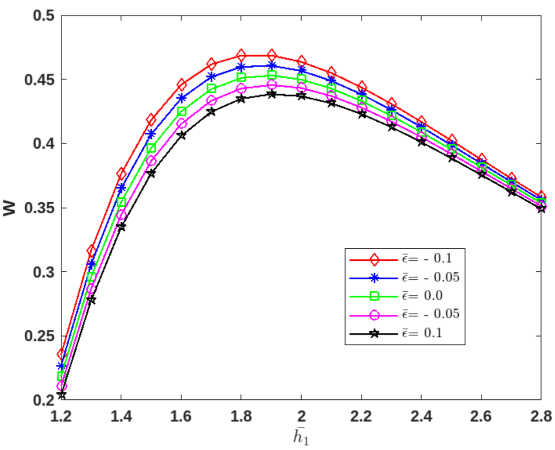

Figure 4. Plot of W with $\overline{h_1}$ for distinct values of $\bar{\epsilon}$

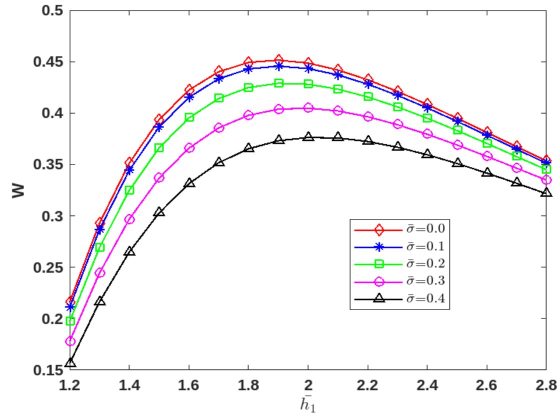

Figure 5. Plot of W with $\overline{h_1}$ for distinct values of $\bar{\sigma}$

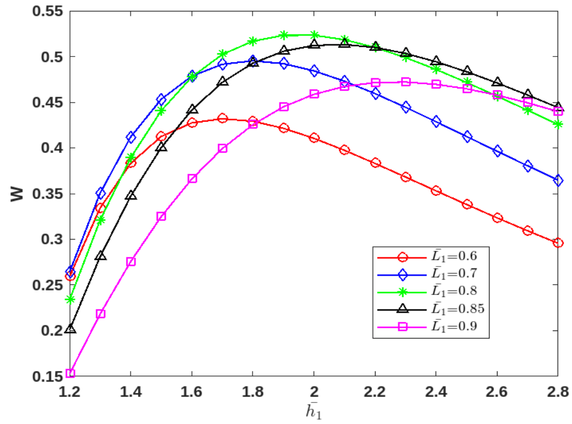

Figure 6. Plot of W with $\overline{h_1}$ for distinct values of $\overline{L_1}$

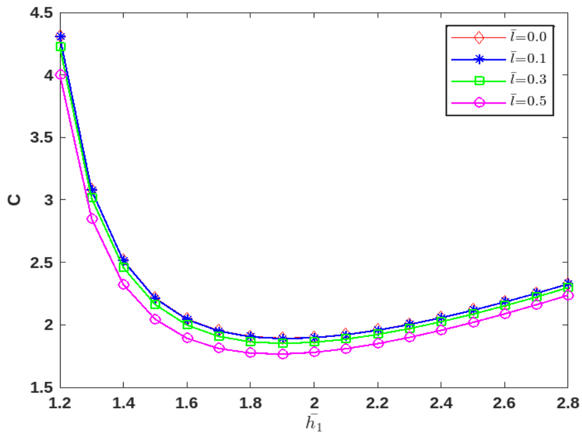

Figure 7. Plot of C with $\overline{h_1}$ for distinct values of $\bar{l}$

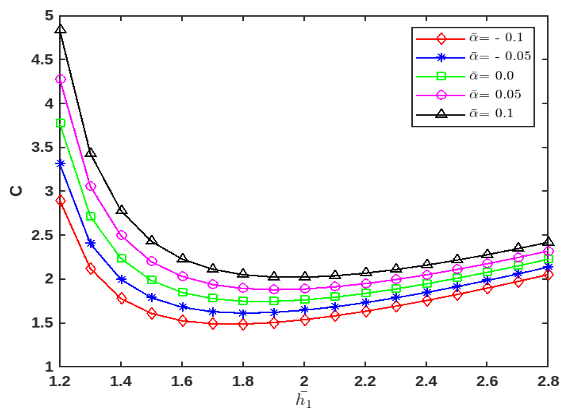

Figure 8. Plot of C with $\overline{h_1}$ for distinct values of $\bar{\alpha}$

3.2 Dimensionless coefficient of friction

Graphical representation of the variation of the coefficient of friction C with the film thickness $\bar{h}_1$ for distinct values of $\bar{l}$, $\bar{\alpha}$, $\bar{\epsilon}$, $\bar{\sigma}$ and $\bar{L}_1$ are shown in Figures 7, 8, 9, 10 and 11 respectively. It is observed that the couple stress parameter $\bar{l}$ significantly decreases the coefficient of friction C as shown in Figure 7 and the coefficient of friction increases the coefficient of friction by increasing the roughness parameters $\bar{l}$, $\bar{\alpha}$, $\bar{\epsilon}$ and $\bar{\sigma}$. It is noted that the coefficient of friction C decreases, when increasing the bearing length $\bar{L}_1$.

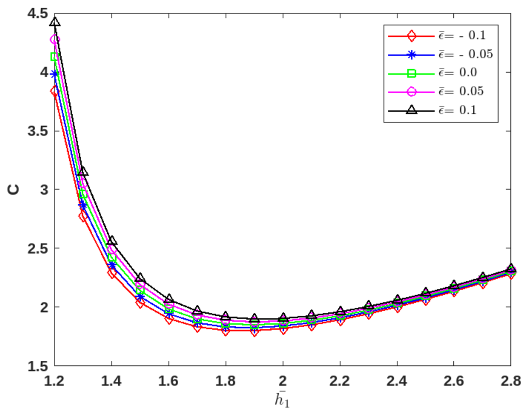

Figure 9. Plot of W with $\bar{h}_1$ for distinct values of $\bar{\epsilon}$

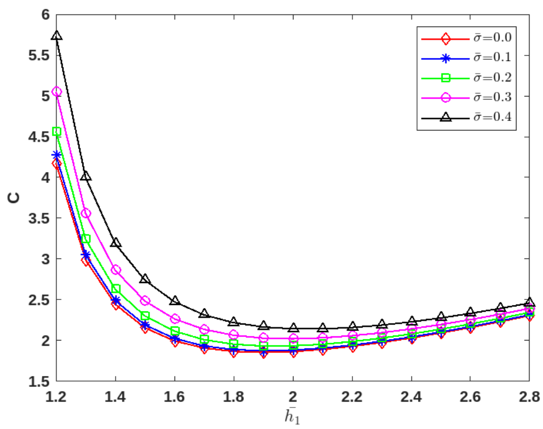

Figure 10. Plot of W with $\bar{h}_1$ for distinct values of $\bar{\sigma}$

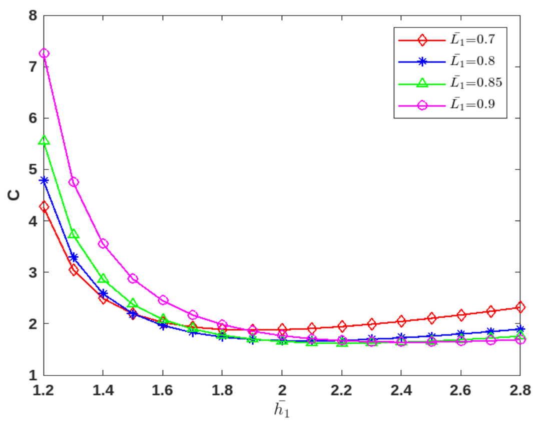

Figure 11. Plot of W with $\bar{h}_1$ for distinct values of $\bar{L}_1$

Tables 1, 2, 3, 4 and 5 bring out the comparison between the single layer porous [3] and double layer porous for different values of couple stress parameter $\bar{l}$, mean roughness $\bar{\alpha}$, standard deviation $\bar{\sigma}$, skewed surface roughness $\bar{\epsilon}$ and bearing length $\bar{L}_1$. A double-layered porous backing to the lower plate improves the bearing's efficiency even when it is not totally saturated with oil by reducing oil leakage into the bearing's wall and increasing the load capacity of the bearing. Due to the double layered porous the lubricant remains in film region for long duration, so it sustains high load and low coefficient of friction. Therefore, the double porous layer has high load carrying capacity than single layered porous but it is inversely proportional to coefficient of friction.

Table 1. Comparison between single and double porous layers for different values of couple-stress parameter

	$\bar{h}_1$	$\bar{l}$=0		$\bar{l}$=0.1		$\bar{l}$=0.3	
		Double	Single [3]	double	Single [3]	double	Single [3]
	1.4	0.3402	0.3373	0.3412	0.3383	0.3493	0.3463
	1.8	0.4372	0.4349	0.4386	0.4363	0.4501	0.4476
W	2.2	0.4232	0.4217	0.4244	0.4229	0.4344	0.4329
	2.6	0.3747	0.3738	0.3756	0.3747	0.3832	0.3823
	1.4	2.5184	2.5382	2.5116	2.5315	2.4568	2.4767
	1.8	1.9088	1.9174	1.9039	1.9124	1.8639	1.8724
C	2.2	1.9609	1.966	1.9567	1.9618	1.9229	1.928
	2.6	2.186	2.1895	2.1822	2.1858	2.1522	2.1557

Table 2. Comparison between single and double porous layers for different values of standard deviation

	$\bar{h}_1$	$\bar{\sigma}$=0		$\bar{\sigma}$=0.1		$\bar{\sigma}$=0.2	
		Double	Single [3]	Double	Single [3]	double	Single [3]
	1.4	0.3511	0.3481	0.3441	0.3412	0.3247	0.3221
	1.8	0.4492	0.4467	0.4428	0.4404	0.4248	0.4225
W	2.2	0.4324	0.4309	0.4281	0.4266	0.4156	0.4142
	2.6	0.3811	0.3802	0.3784	0.3775	0.3704	0.3696
	1.4	2.4449	2.4648	2.4914	2.5112	2.6306	2.6505
	1.8	1.867	1.8755	1.889	1.8976	1.9551	1.9637
C	2.2	1.9297	1.9348	1.9441	1.9492	1.9873	1.9924
	2.6	2.1602	2.1637	2.171	2.1745	2.2034	2.2069

Table 3. Comparison between single and double porous layers for different values of skewed surface roughness

	$\bar{h}_1$	$\bar{\epsilon}$=-0.05		$\bar{\epsilon}$=0.0		$\bar{\epsilon}$=0.05	
		Double	Single [3]	Double	Single [3]	Double	Single [3]
	1.4	0.3649	0.3616	0.3542	0.3511	0.3441	0.3412
	1.8	0.4596	0.457	0.4511	0.4486	0.4428	0.4404
W	2.2	0.4384	0.4368	0.4332	0.4316	0.4281	0.4266
	2.6	0.3844	0.3835	0.3814	0.3805	0.3784	0.3775
	1.4	2.3591	2.3789	2.4252	2.4451	2.4914	2.5112
	1.8	1.8321	1.8406	1.8605	1.8691	1.889	1.8976
C	2.2	1.91	1.9151	1.927	1.9322	1.9441	1.9492
	2.6	2.1475	2.151	2.1592	2.1628	2.171	2.1745

Table 4. Comparison between single and double porous layers for different values of the average roughness of the surface

	$\bar{h}_1$	$\bar{\alpha}$=-0.05		$\bar{\alpha}$=0.0		$\bar{\alpha}$=0.05	
		Double	Single [3]	Double	Single [3]	Double	Single [3]
	1.4	0.4383	0.4336	0.3876	0.3839	0.3441	0.3412
	1.8	0.5403	0.5367	0.4886	0.4856	0.4428	0.4404
W	2.2	0.5038	0.5017	0.4641	0.4623	0.4281	0.4266
	2.6	0.4333	0.4321	0.4047	0.4037	0.3784	0.3775
	1.4	1.9918	2.0117	2.2305	2.2503	2.4914	2.5112
	1.8	1.6083	1.6169	1.7434	1.7519	1.889	1.8976
C	2.2	1.7271	1.7322	1.8321	1.8372	1.9441	1.9492
	2.6	1.9805	1.984	2.0731	2.0767	2.171	2.1745

Table 5. Comparison between single and double porous layers for different values of the bearing length $\overline{L_1}$

	$\overline{h_1}$	$\overline{L_1} = 0.7$		$\overline{L_1} = 0.8$		$\overline{L_1} = 0.85$	
		Double	**Single [3]**	**Double**	**Single [3]**	**Double**	**Single [3]**
W	1.4	0.4119	0.3738	0.3892	0.3484	0.3472	0.3083
	1.8	0.4953	0.4666	0.5167	0.4795	0.4927	0.4522
	2.2	0.4594	0.4426	0.51	0.4852	0.5099	0.4803
	2.6	0.3963	0.3868	0.4566	0.4414	0.4711	0.4517
C	1.4	2.4914	2.5112	2.5831	2.6062	2.8593	2.8868
	1.8	1.889	1.8976	1.7415	1.7512	1.7771	1.7884
	2.2	1.9441	1.9492	1.6757	1.6813	1.6215	1.628
	2.6	2.171	2.1745	1.8009	1.8047	1.6885	1.6929

4. CONCLUSIONS

The comparative work between single porous layer which is published by Naduvinamani and Siddangouda [3] and double porous layer on the rough step slider bearing is carried in this article. The characteristics of load carrying capacity and coefficient of friction are analyzed for different values of couple stress parameter $\bar{l}$, mean roughness $\bar{\alpha}$, standard deviation $\bar{\sigma}$, skewed surface roughness $\bar{\epsilon}$ and bearing length $\overline{L_1}$. and the outputs have been both plotted as well as tabulated. These emanate the following result.

• The double layer porous has high load carrying capacity than the single porous layer it is in reverse nature to that of coefficient of friction.

• Mean roughness $\bar{\alpha}$, standard deviation $\bar{\sigma}$ and skewed surface roughness $\bar{\epsilon}$ are in inverse trend to load carrying capacity.

• Mean roughness $\bar{\alpha}$, standard deviation $\bar{\sigma}$ and skewed surface roughness $\bar{\epsilon}$ are directly proportional to coefficient of friction.

• The surface roughness when negatively skewed enhances the load carrying capacity. It also brings down the values of coefficient of friction.

• The surface roughness when positively skewed enhances the coefficient of friction. It also brings down the values of load carrying capacity.

• It is noted that the load sustaining capacity reaches the maximum point at $\overline{L_1} = 0.8$ and $\overline{L_2} = 0.2$.

• It is evident that the inlet thickness can be maximum up to 1.8 to 2 to obtain a maximum load carrying capacity. Values beyond this does not help sustain maximum load.

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NOMENCLATURE

h_1	Thickness of entry film region $(0 \leq x \leq L_1)$
h_2	Thickness of exit film region $(L_1 \leq x \leq L)$
k	Porous matrix permeability
L	Bearing length $(L_1 + L_2)$
L_1, L_2	Bearing lengths in the entry and exit regions respectively
$\bar{l}$	Dimensionless couple-stress parameter $= \frac{2l}{h_2^2}$
$\hat{p}_1, \hat{p}_2$	Fluid pressure in porous region-I and porous region-II
P_1, P_2	Dimensionless film pressure in the entry and exit region respectively
W	Dimensionless load carrying capacity $= \frac{wh_2^2}{\mu U L^2}$
F	Dimensionless frictional force$= \frac{-fh_2}{\mu UL}$
C	Dimensionless coefficient of friction
μ	Lubricant viscosity
η	Material constant for couple stresses
ψ	Permeability parameter $= \frac{k_1 \delta_1}{h_2^2}$
$\bar{\alpha}$	Dimensionless mean roughness $= \frac{\alpha}{h_2}$
$\bar{\sigma}$	Dimensionless standard deviation $= \frac{\sigma}{h_2}$
$\bar{\epsilon}$	Dimensionless skewed surface roughness $= \frac{\epsilon}{h_2^3}$
δ_1, δ_2	Thickness of porous layer-1 and porous layer-2 respectively
k_1, k_2	Porous matrix permeability for the layer-1 and porous layer-2 respectively