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A Novel Approach to Fuzzy Implication Through Fuzzy Linear Regression

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ABSTRACT

Fuzzy rule-based processes have traditionally incorporated a variety of fuzzy implications using modus ponens, modus tollens, and fuzzy negations. This study introduces a novel method of fuzzy implication utilizing Fuzzy Linear Regression (FLR) with triangular fuzzy numbers. This approach was applied to evaluate the relationship between parameters influencing concrete and the compressive strength of sustainable rice husk ash (RHA) concrete. FLR, a technique for modeling relationships between inputs and outputs in a fuzzy environment, was employed to determine a fuzzy output with a specific truth value. This truth value represented the degree of truth of an entire fuzzy implication. The data used in this study were derived from real experimental results. The analysis showed that the FLR method produced accurate outputs, as indicated by a low Theil's inequality coefficient (Theil's U=0.1). The results suggest that FLR can effectively manage uncertainties in data and holds potential as an alternative method for fuzzy implication.

1. INTRODUCTION

Fuzzy implications are recognized as important factors in fuzzy logic with many applications in approximate reasoning [1], mathematical morphology [2], performing fuzzy conditionals [1, 3] and other fuzzy fields [4-6]. Zadeh [7] proposed that the fuzzy inference rules were applied to evaluate the effectiveness of fuzzy implications based on modus ponens, modus tollens and hypothetical syllogism. Although, there are many definitions of fuzzy implications in the literature, in this study they are expressed as follows [8-10].

Theorem 1. A fuzzy implication is a function defined as $I: [0,1]x[0,1] \rightarrow [0,1]$ in which the following restrictions are satisfied for every:

$$x_1 \leq x_2 \text{ then } I(x_1, y) \geq I(x_2, y), \text{i.e.}, I(\cdot, y) \tag{1}$$

is decreasing

$$y_1 \leq y_2 \text{ then } I(x, y_1) \leq I(x, y_2), \text{i.e.}, I(x, \cdot) \tag{2}$$

is increasing

$$I(0,0) = 1 \tag{3}$$

$$I(1,1) = 1 \tag{4}$$

$$I(1,0) = 0 \tag{5}$$

These conditions demonstrate that the truth values of the initial propositions determine the truth value of the consequence. The following Table 1 categorizes the different implications from the literature which generalize the crisp one to fuzzy logic.

Table 1. Basic fuzzy implications

Fuzzy Implications	Formula
Lukasiewicz [11]	$I_{LK}(x,y) = \min(1, 1-x+y)$
Reichenbach [12]	$I_{RC}(x,y) = 1-x+xy$
Kleene-Dienes [13]	$I_{KD}(x,y) = \max(1-x, y)$
Goguen [14]	$I_{GG}(x,y) = \begin{cases} 1, \text{if } x \leq y \\ \frac{y}{x}, \text{if } x > y \end{cases}$
Rescher [15]	$I_{RS}(x,y) = \begin{cases} 1, \text{if } x \leq y \\ 0, \text{if } x > y \end{cases}$
Yager [16]	$I_{YG}(x,y) = \begin{cases} 1, \text{if } x = 0 \text{ and } y = 0 \\ y^x, \text{if } x > 0 \text{ or } y > 0 \end{cases}$
Weber [17]	$I_{WB}(x,y) = \begin{cases} 1, \text{if } x < 1 \\ y, \text{if } x = 1 \end{cases}$
Fodor [18]	$I_{FD}(x,y) = \begin{cases} 1, \text{if } x \leq y \\ \max(1-x, y), \text{if } x > y \end{cases}$

The use of fuzzy implications had attracted many researchers who were interested in making inferences in a fuzzy environment. Particularly, an algorithmic process was introduced for choosing the most ideal fuzzy implication by using real experimental data [19]. Also, Mylonas and Papadopoulos [20] evaluated the membership function for every variable and defuzzificated them for determining the output y. Then, the deviations of the outputs, that emerged from the implications and the observations, were calculated for considering the best implication process with the smallest deviation values. Moreover, a new fuzzy implication method was proposed [21, 22] which depended on empiristic implication relations.

One step for constructing the parametric implication is to divide the data into language variables in order to be normalized between [0, 1]. This process is necessary for evaluating the membership function of each variable. For example, Makariadis et al. [22] used a new model via fuzzy implications in which the experimental data were classified into linguistic variables through Fuzzy C-means clustering (FCM) algorithm. Also, Botzoris et al. [23] divided the data into low, medium and high according to real published results.

In this study, we used FLR with triangular fuzzy numbers as a fuzzy implication, for evaluating the truth value of every variable. Particularly, a set of 192 experimental observations, derived from previous study [24], was applied for evaluating the relation between the parameters that affect concrete construction and the compressive strength of sustainable RHA concrete. It was demonstrated that [23] the input data were real observations without including the meaning of the fuzziness and consequently the truth value was equal to one. Therefore, by applying FLR method, every experimental output Y belonged to a fuzzy output with a specific truth value, which was the degree of truth of the fuzzy implication. This approach could be verified with the following fuzzy implication axiom, named neutrality of truth [21]:

$$\mathrm{I}(1, \mathrm{y}) = \mathrm{y} \tag{6}$$

in which I represented the fuzzy implication.

By evaluating the results, it was concluded that FLR with triangular fuzzy numbers provided accurate inferences between the observations and could be used as a new effective implication method in every case.

The paper is divided into the following sections: The second part refers to the description of sustainable rice husk ash concrete properties and the third part analyzes the FLR method with triangular fuzzy numbers for modelling the relationship among the amount of cement and its compressive strength. The fourth section quotes the model application and the implication process that was followed for evaluating the truth values of every variable. Finally, in the last part of the paper the conclusions of FLR application method are demonstrated.

2. RICE HUSK ASH CONCRETE

It is well known that agricultural by-products can be used as supplementary cementitious materials for enhancing the concrete properties and preventing the environment [25, 26]. Rice husk ash is recognized as a sustainable material with distinctive properties, numerous applications in concrete structures and important influence on its constructive performance [27]. Thus, it can be worthily used for replacing the ordinary Portland cement.

The determination of the relation between the parameters that affect concrete and the compressive strength of sustainable RHA concrete is a major process, as it estimates the effect of them on its mechanical properties. Therefore, the utilization of RHA depends on the performance enhancement of concrete mechanical properties avoiding concrete failures. These properties will be described in the following sub-sections.

2.1 Compressive strength

Many researchers studied the compressive strength of sustainable rice husk ash concrete in different proportions. For example, Bheel et al. [28] claimed that the compressive strength of concrete increased by a rate of 2% to 6% with the replacement of 10% RHA instead of conventional concrete. It was also demonstrated that [29] samples containing 10%, 20% and 30% RHA yielded greater compressive strength at the age of 91 days in contrast to the plain concrete. More specifically, for w/b ratio 0.50 the compressive strength was increased by 21%, 22% and 27%, respectively. Another research [30] studied the amount of 0%, 5%, 7.5%, 10%, 12.5%, and 15% RHA by weight and was concluded that the sample with 7.5% RHA revealed higher compressive strength up to 3% increase, than that of the conventional concrete with Portland cement. When the content of RHA increased by more than 7.5% replacement with cement, the compressive strength was reduced, in contrast to the reference specimen. This may due to the fact that higher proportions of RHA contain excessive silica proportions, which cause the reduction in the compressive strength.

2.2 Tensile strength

The determination of tensile strength is a difficult process as it is affected by many factors [31]. Particularly, Ali et al. [32] studied the RHA concrete in the proportions of 0%, 2.5%, 5%, 7.5%, 10%, 12.5%, and 15% by weight and concluded that the tensile strength increased for the proportions between 0% and 10% RHA and decreased for the amount of 12.5%, and 15% RHA. The maximum tensile strength was noticed for the proportion of 10% RHA replacement of cement with 20.41% increase, in contrast to the plain specimen. The increase in the tensile strength was due to inherent correlation between density and strength and the proportion of silica in Rice Husk Ash. However, the reduction in strength caused a decrease in unit weight. Furthermore, [33] the content of 10% RHA increased the tensile strength of sustainable rice husk ash concrete by 3%, 8.8% and 16.5% at 7, 28 and 90 days, respectively, compared to normal concrete. Thus, the combination of 10% RHA and different amounts of nanoparticles in concrete mixtures yielded the best mechanical results.

2.3 Flexural strength

The flexural strength is also an important parameter that must be determined for assessing the constructive behavior of concrete. Depaa et al. [34] compared samples with 15% RHA and normal concrete and cured them for 7, 14, and 28 days. The results revealed that the concrete containing RHA indicated good flexural strength values than plain concrete. At 28 days of hydration, the flexural strength was 4.58MPa for conventional concrete and 4.52MPa for the specimen containing 15% RHA. Also, Meddah et al. [33] found that 10% rice husk ash improved the flexural strength of concrete from 1% to 15% increase, in contrast to the normal mixture. This was probably happening because the addition of 1%-4% content of Al_2O_3 nanoparticles with pozzolanic behavior enhanced the mechanical properties and durability concrete performance.

3. FUZZY LINEAR REGRESSION

Regression analysis is a widely studied method of modelling. It can be applied for evaluating an exact relation between inputs and independent output in many fields of engineering. An extension of this classical analysis is Fuzzy Linear Regression method, in which some variables that compose the model are expressed with fuzzy numbers. More specifically, Fuzzy Linear Regression method is a more advanced form of the classical linear regression that handles uncertainty and vagueness in the data. It applies the principles of fuzzy logic to evaluate a relationship between inputs and outputs, when the data are imprecise. The Fuzzy Linear Regression method, that was proposed by Asai et al. [35], expressed the relation between the dependent output Y and the independent variables X in a fuzzy form, with the following equation [36, 37]:

$$Y = A_0 + A_1X_1 + \cdots + A_nX_n \tag{7}$$

In this possibilistic model, the uncertainty to determine an accurate relation between the parameters dealt with the triangular fuzzy numbers $A_i = (r_i, c_i)$, with center r_i that the membership value was equal to one and the half-width c_i. Therefore, the membership function of triangular fuzzy numbers expressed as follows [38]:

$$\mu_A(x) = L\left(\frac{x-r}{c}\right) \tag{8}$$

The parameter L(x) was the reference function of the model which obeys the following axioms:

$$L(x) = L(-x) \tag{9}$$

$$L(0) = 1 \tag{10}$$

L(x) is decreasing in $[0, \infty)$.

As fuzzy triangular numbers were used in this study, the reference function formed with the following equation, which was decreasing in (0, 1):

$$L(x) = \max(0, 1 - |x|) \tag{11}$$

Then, according to the theorem [39], the membership function of the linear regression model expressed as follows:

$$\mu_{Y_j}(y_j) = L\left[\frac{y_j - (r_0 + \sum_{i=1}^{n} r_i x_{ij})}{c_0 + \sum_{i=1}^{n} c_i |x_{ij}|}\right] \tag{12}$$

in which i represented the number of input data and j expressed the different sets. A degree h was also defined with the following form, which determined that the experimental data should be included in the computed output Y_j.

$$\mu_{Yj}(y_j) \geq h \tag{13}$$

In order to define the best fuzzy coefficients for minimizing the whole spread of the outputs, the following objective function was used [40]:

$$J = \min\{mc_0 + \sum_{j=1}^{m}\sum_{i=1}^{n} c_i|x_{ij}|\} \tag{14}$$

which was minimized with the Eqs. (15)-(17):

$$y_j \geq r_0 + \sum_{i=1}^{n} r_i x_{ij} - (1-h)\left(c_0 + \sum_{i=1}^{n} c_i|x_{ij}|\right) \tag{15}$$

$$y_j \leq r_0 + \sum_{i=1}^{n} r_i x_{ij} + (1-h)\left(c_0 + \sum_{i=1}^{n} c_i|x_{ij}|\right) \tag{16}$$

$$c_i \geq 0, \qquad i = 1,2,\ldots,n \tag{17}$$

As a result, taking into account the aforementioned theorems, the linear programming problem was formed with the Eqs. (14)-(17) which was solved with simplex method. In this study, Fuzzy Linear Regression was used to determine the relationship between the input parameters and the compressive strength of sustainable rice husk ash concrete, which contains uncertainties due to variability in the recycled materials.

4. MODEL APPLICATION

Fuzzy Linear Regression (FLR) with triangular fuzzy numbers is a widely used method in solving mathematical problems by minimizing the square error among the experimental and predicted outputs. Its application is valid in the estimated value process as it gives an exact relation between the parameters that handles the perceptual uncertainties of linear programming problems [41]. This is achieved by using fuzzy coefficients which express the ambiguity of the relation between the parameters. In this study, FLR method was used as a new method for evaluating the implication between the amount of cement and the compressive strength of sustainable rice husk ash concrete.

The application of ecofriendly cementitious materials has gained the interest of many researchers in the construction industry for investigating more environmentally friendly substitutes [42, 43]. Rice husk ash concrete is a sustainable concrete material that is used instead of traditional concrete for reducing the emission of CO_2 in the environment. It consists of recycled materials and cements substitutes without sacrificing the final strength of the material [44]. It is well demonstrated that compressive strength plays a vital role in the concrete quality procedure. Thus, the determination of the parameter contribution of the compressive strength evaluation is an important process. In a previous paper, Iftikhar et al. [24] studied the prediction of the compressive strength of sustainable rice husk ash concrete by using gene expression programming and Random Forest Regression method. The parameters that were used as inputs were the age of concrete, the amount of cement, the rice husk ash, water, super plasticizer and aggregate. The data were selected from published literature. In this study, same data were used for estimating the relation between the inputs and the compressive strength of sustainable rice husk ash concrete with FLR method.

Firstly, FLR method was applied with the following form according to the Eq. (7):

$$Y = A_0 + A_1X_1 + A_2X_2 + A_3X_3 + A_4X_4 + A_5X_5 + A_6X_6 \tag{18}$$

in which Y was the compressive strength of RHA concrete, X_1 was the age of concrete (days), X_2 was the amount of cement (Kg/m^3), X_3 was rice husk ash (Kg/m^3), X_4 was water (Kg/m^3), X_5 was super plasticizer (Kg/m^3) and X_6 was aggregate

(Kg/m^3). Therefore, the following objective function was emerged:

$$J = \min\{mc_0 + \sum_{j=1}^{192}\sum_{i=1}^{6} c_i|x_{ij}|\} \tag{19}$$

By solving the linear programming problem with simplex method from the Eqs. (14)-(17), the estimated output values of Y and the fuzzy parameters were resulted, and the Eq. (7) took the following form:

$$Y = -0.804 + 0.339 \cdot X_1 + 0.063 \cdot X_2 + 0.050 \cdot X_3 - 0.050 \cdot X_4 + 2.938 \cdot X_5 + 0.005 \cdot X_6 + 0.011 \cdot X_3 + 2.669 \cdot X_5 + 0.007 \cdot X_6 \tag{20}$$

In Table 2, the calculations of fuzzy parameters were represented.

According to the results of fuzzy triangular numbers, it was worth noting that the variables X_3, X_5 and X_6 contributed to the uncertainty of the Fuzzy Linear Regression ($c_i \neq 0$), while the other variables had c_i values equal to 0. In the rare case where all c_i values turned out to be 0, the model would not contain any fuzziness and would transform into a conventional multiple linear regression model.

Then, the degree of membership function, that specifies the grade to which the experimental output belongs to the fuzzy predicted values between the term [0, 1], was evaluated. Regarding the membership function, the distance of the estimated outputs from the center of the regression plays an important role in the determination of the membership function values. The closer the real demand is to the center of the regression, the higher the degree of participation, taking the value 1 when the real demand coincides with the center of the fuzzy regression and the value 0 when the real demand is on the boundaries of the fuzzy regression. The results of the Fuzzy Linear Regression model were summarized in the Appendix A and the boundaries and the center of the FLR method were shown in Figure 1.

Table 2. The results of the fuzzy triangular numbers

Variable	Estimate R_i	Estimate C_i
A_0	-0.804	0.000
A_1	0.339	0.000
A_2	0.063	0.000
A_3	0.050	0.011
A_4	-0.050	0.000
A_5	2.938	2.669
A_6	0.005	0.007

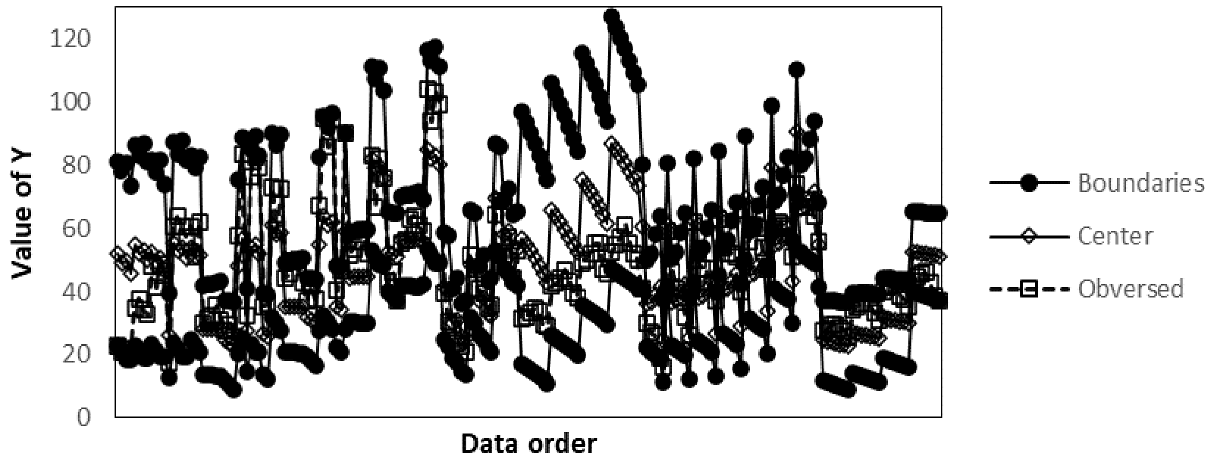

Figure 1. The parameters of FLR method

For assessing the accuracy of the regression and therefore the fuzzy implication, Theil's inequality coefficient [45] was evaluated. The range of this coefficient is between 0 to 1. The smaller the coefficient is, the closer the predicted and experimental outputs are. For instance, when the Theil's inequality coefficient is calculated to be 0, then the Fuzzy Linear Regression exhibits perfect predictive capability. However, in the case where the Theil's inequality coefficient is estimated to be 1, the fitted Fuzzy Linear Regression model lacks any predictive capability. This coefficient was expressed as follows:

$$\text{Theil's } U = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(F_t - A_t)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(F_t)^2} + \sqrt{\frac{1}{n}\sum_{t=1}^{n}(A_t)^2}} = 0.1 \tag{21}$$

in which A_t and F_t were the experimental and predicted variables, respectively, and n was the number of observations. By evaluating the result, it was concluded that the application of Fuzzy Linear Regression method yielded accurate outputs, valid fuzzy parameter and reliable values of membership function, as the value of Theil's inequality coefficient was low, and hence, the implication provided satisfactory results.

As it proved, Fuzzy Linear Regression can be used not only as prediction method but also as an algorithmic process for evaluating the truth value of every variable. More specifically, the aforementioned method uses a specific equation with fuzzy coefficients for determining a fuzzy output Y for every experimental output, contrary to predictive black box methods based on machine learning [46, 47]. These observed parameters belong to the fuzzy output with a specific degree of truth which represents the truth value of the total fuzzy implication.

Also, the application of Fuzzy Linear Regression as implication method deals with the uncertainties that are involved in dividing data into linguistic parameters. In this study, the determination of linguistic parameters was unnecessary, as the truth values that were used for the implication process were resulted directly from evaluating the fuzzy outputs. Thus, the evaluation of the membership function emerged from applying FLR method without having to divide data into ascending order and set language variables. Another advantage of this method is that it evaluates the truth value between the output and many different inputs, in contrast to the other implication methods in which the degree of truth is among the output and one input. Thus, FLR method is a valid mathematical process not only for prediction problems, but it can also be used as a successful tool for developing approximate reasoning.

5. CONCLUSIONS

The estimation of an appropriate fuzzy implication is a simplex procedure as it includes many parameters for applying the best mathematical process. In this paper, Fuzzy Linear Regression with triangular fuzzy numbers was used for evaluating the relation between the parameters that affect concrete and the compressive strength of sustainable rice husk ash concrete. Particularly, six inputs such as the age of concrete, the amount of cement, the rice husk ash, water, super plasticizer and aggregate were applied for determining the concrete's compressive strength. By developing FLR method, each experimental output, that came out from previous publication [24], belonged to a fuzzy output with a specific truth value, which turned out to be the degree of truth of the entire fuzzy implication. This approach was based on the fuzzy implication axiom, named neutrality of truth.

It was also reported that FLR method provides an equation with fuzzy parameters that demonstrates the way that the truth value emerged, which proves the accuracy of the method. Moreover, by applying this method the separation of inputs into language variables is unnecessary which is a useful property in case that there is no information about the division of parameters. Since Fuzzy Linear Regression establishes a direct relationship between inputs and outputs and the membership functions are calculated from the fuzzy outputs, it eliminates the need to partition the inputs into linguistic variables as required in some other fuzzy implication methods. In addition, the use of Fuzzy Linear Regression allows the evaluation of the degree of truth between one output and more than one inputs contrary to the other implication methods that determine the relation among the output and one input.

To sum up, FLR method led to accurate predictions between the coefficients that are involved in concrete construction and the compressive strength of sustainable rice husk ash concrete. According to the results, Fuzzy Linear Regression could effectively handle multiple inputs and determined the degree of truth between the inputs and output. The low value of the Theil's inequality coefficient (Theil's U=0.1) proved the capability of this method to be used as fuzzy implication for determining the truth value of the consequence and provided an accurate modelling tool in the approximate reasoning process.

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NOMENCLATURE

I	fuzzy implication
Y	dependent output
X	independent input
A_i	triangular fuzzy numbers
r_i	center of triangular number
c_i	range of values
L	reference function
h	level of confidence
J	objective function
Theil's U	inequality coefficient
F_t	predicted outputs
A_t	experimental outputs

Greek Symbols

μ_A	membership function

Subscripts

FLR	Fuzzy Linear Regression
RHA	Rice Husk Ash

APPENDIX

Appendix A. Compressive strength of sustainable rice husk ash concrete as evaluated by the FLR method across 192 observations

Age (days)	Cement (Kg/m^3)	RHA (Kg/m^3)	Water (Kg/m^3)	Superplasticizer (Kg/m^3)	Aggregate (Kg/m^3)	Compr. Strength (Kg/m^3)	FLR (Left)	FLR (Center)	FLR (Right)	$\mu_A(y_i)$
1	495	55	165	5.8	1819	22.7	22.5	52.0	80.2	0.0
1	500	0	160	5.5	1891	20.9	20.7	49.3	76.6	0.0
1	400	100	160	6.22	1859	22	18.6	49.9	80.0	0.1
1	425	75	170	5	1843	19.7	18.4	46.1	72.5	0.0
1	495	55	165	6.8	1819	34.7	22.8	55.0	85.8	0.4
1	500	0	160	6.5	1891	37.8	21.0	52.2	82.2	0.5
1	400	100	160	7.36	1859	34.2	18.9	53.3	86.4	0.4
1	425	75	170	6.4	1843	32.6	18.8	50.3	80.4	0.4
3	495	55	165	5.8	1819	47.9	23.2	52.7	80.9	0.8
3	500	0	160	5.5	1891	41.3	21.4	50.0	77.2	0.7
3	400	100	160	6.22	1859	48.7	19.3	50.6	80.7	0.9
3	425	75	170	5	1843	45.2	19.1	46.8	73.2	0.9
3	378	42	189	0	1810	17.6	12.6	26.5	38.9	0.4
3	495	55	165	6.8	1819	60.8	23.5	55.6	86.5	0.8
3	500	0	160	6.5	1891	63.9	21.7	52.9	82.9	0.6
3	400	100	160	7.36	1859	60.7	19.6	54.0	87.1	0.8
3	425	75	170	6.4	1843	57.3	19.4	50.9	81.1	0.8
7	495	55	165	5.8	1819	60.6	24.6	54.0	82.2	0.8
7	500	0	160	5.5	1891	51	22.8	51.3	78.6	1.0
7	400	100	160	6.22	1859	61.8	20.6	52.0	82.1	0.7
7	375	0	150	0	1970	30	13.8	28.1	41.3	0.9
7	356.25	18.75	142.5	0	1970	31.5	13.7	28.2	41.7	0.8
7	337.5	37.5	135	0	1970	32.5	13.6	28.3	42.0	0.7
7	318.75	56.25	127.5	0	1970	35.5	13.5	28.4	42.3	0.5
7	300	75	120	0	1970	31	13.5	28.5	42.7	0.8
7	364	19	203	0	1725	27.6	11.6	24.7	36.2	0.8
7	306	77	203	0	1725	29.7	10.3	24.0	36.1	0.5
7	249	134	203	0	1725	25.7	8.9	23.2	36.0	0.8
7	425	75	170	5	1843	57.6	20.4	48.2	74.6	0.6
7	495	55	165	6.8	1819	83.6	24.8	57.0	87.8	0.1
7	391	29	189	0	1810	32.4	14.3	28.0	40.2	0.6
7	500	0	160	6.5	1891	76.4	23.0	54.3	84.2	0.3
7	400	100	160	7.36	1859	82.8	20.9	55.3	88.4	0.2
7	425	75	170	6.4	1843	79.2	20.8	52.3	82.4	0.1
14	345	38	203	0	1725	35.3	13.6	26.9	38.5	0.3
14	287	96	203	0	1725	36.1	12.2	26.1	38.4	0.2
28	495	55	165	5.8	1819	72.8	31.7	61.2	89.3	0.6
28	500	0	160	5.5	1891	59.6	29.9	58.4	85.7	1.0
28	400	100	160	6.22	1859	72.7	27.7	59.1	89.2	0.5

Age (days)	Cement (Kg/m³)	RHA (Kg/m³)	Water (Kg/m³)	Superplasticizer (Kg/m³)	Aggregate (Kg/m³)	Compr. Strength (Kg/m³)	FLR (Left)	FLR (Center)	FLR (Right)	$\mu_A(y_i)$
28	375	0	150	0	1970	44.5	20.9	35.3	48.5	0.3
28	356.25	18.75	142.5	0	1970	45.5	20.8	35.4	48.8	0.2
28	337.5	37.5	135	0	1970	49.5	20.7	35.5	49.5	0.0
28	318.75	56.25	127.5	0	1970	50	20.6	35.6	50.0	0.0
28	300	75	120	0	1970	43	20.6	35.7	49.8	0.5
28	383	0	203	0	1725	37.1	19.2	32.1	43.4	0.6
28	326	57	203	0	1725	41.8	17.8	31.4	43.3	0.1
28	268	115	203	0	1725	37.6	16.5	30.6	43.1	0.4
28	425	75	170	5	1843	67.2	27.5	55.3	81.7	0.5
28	495	55	165	6.8	1819	95.2	32.0	64.1	95.2	0.0
28	500	0	160	6.5	1891	85.7	30.2	61.4	91.3	0.2
28	400	100	160	7.36	1859	94.3	28.0	62.4	95.6	0.0
28	420	0	189	0	1810	40.3	22.1	35.5	47.4	0.6
28	357	63	189	0	1810	46.9	20.6	34.7	47.3	0.0
28	425	75	170	6.4	1843	90.3	27.9	59.4	90.3	0.0
56	375	0	150	0	1970	51.5	30.4	44.8	57.9	0.5
56	356.25	18.75	142.5	0	1970	53.5	30.3	44.9	58.3	0.4
56	337.5	37.5	135	0	1970	56	30.2	45.0	58.6	0.2
56	318.75	56.25	127.5	0	1970	59.5	30.1	45.1	59.5	0.0
56	300	75	120	0	1970	52	30.1	45.2	59.3	0.5
90	495	55	165	5.8	1819	83.2	52.7	82.2	110.3	1.0
90	500	0	160	5.5	1891	66.8	50.9	79.5	106.7	0.6
90	400	100	160	6.22	1859	82.2	48.8	80.1	110.2	0.9
90	425	75	170	5	1843	75.8	48.6	76.3	102.7	1.0
90	364	19	203	0	1725	43.3	39.8	52.9	64.3	0.3
90	306	77	203	0	1725	46	38.4	52.1	64.2	0.6
90	249	134	203	0	1725	37.2	37.0	51.4	64.1	0.0
90	375	0	150	0	1970	55.5	41.9	56.3	69.5	0.9
90	356.25	18.75	142.5	0	1970	56.5	41.8	56.4	69.8	1.0
90	337.5	37.5	135	0	1970	63	41.7	56.5	70.1	0.5
90	318.75	56.25	127.5	0	1970	64	41.7	56.6	70.5	0.5
90	300	75	120	0	1970	61	41.6	56.7	70.8	0.7
90	378	42	189	0	1810	59	42.1	56.0	68.4	0.8
90	495	55	165	6.8	1819	104.1	53.0	85.1	116.0	0.4
90	500	0	160	6.5	1891	94	51.2	82.4	112.3	0.6
90	400	100	160	7.36	1859	103.3	49.1	83.5	116.6	0.4
90	425	75	170	6.4	1843	99.1	48.9	80.4	110.6	0.4
7	481	48.1	169.312	3.367	1040	39.5	24.1	41.6	57.7	0.9
7	427	85.4	163.968	3.416	1040	30.5	22.4	40.4	57.1	0.4
7	416	41.6	183.04	1.1232	1041	29.7	18.5	29.9	40.0	1.0
7	370	74	177.6	1.85	1041	23.6	17.3	31.0	43.4	0.5
7	367	36.7	201.85	1.101	1041	22.7	14.2	25.7	35.5	0.7
7	327	65.4	196.2	1.308	1041	20.8	13.2	25.5	36.2	0.6
28	481	48.1	169.312	3.367	1040	51.4	31.2	48.7	64.8	0.8
28	427	85.4	163.968	3.416	1040	47.4	29.6	47.6	64.2	1.0
28	416	41.6	183.04	1.1232	1041	40.8	25.6	37.1	47.1	0.6
28	370	74	177.6	1.85	1041	39.4	24.4	38.2	50.5	0.9
28	367	36.7	201.85	1.101	1041	34.5	21.4	32.8	42.6	0.8
28	327	65.4	196.2	1.308	1041	35.9	20.3	32.6	43.3	0.7
90	481	48.1	169.312	3.367	1040	64.5	52.2	69.7	85.8	0.7
90	427	85.4	163.968	3.416	1040	68.5	50.6	68.6	85.3	1.0
90	416	41.6	183.04	1.1232	1041	51.5	46.6	58.1	68.1	0.4
90	370	74	177.6	1.85	1041	57.3	45.4	59.2	71.5	0.9
90	367	36.7	201.85	1.101	1041	44.4	42.4	53.8	63.6	0.2
90	327	65.4	196.2	1.308	1041	52.9	41.3	53.6	64.3	0.9
1	450	0	238	11.25	1405	31.5	16.2	57.0	95.9	0.4
1	427.5	21.375	238	10.6875	1405	32.1	15.5	55.0	92.7	0.4
1	405	40.5	238	10.125	1405	33.3	14.6	52.9	89.3	0.5
1	382.5	57.375	238	9.5625	1405	34.5	13.7	50.7	85.7	0.6
1	360	72	238	9	1405	33.6	12.7	48.3	82.0	0.6
1	337.5	84.375	238	8.4375	1405	29.3	11.6	45.9	78.2	0.5
1	315	94.5	238	7.875	1405	29	10.5	43.3	74.3	0.6
28	450	0	238	11.25	1405	41.7	25.4	66.2	105.1	0.4
28	427.5	21.375	238	10.6875	1405	42.7	24.6	64.2	101.8	0.5
28	405	40.5	238	10.125	1405	44.2	23.8	62.1	98.4	0.5
28	382.5	57.375	238	9.5625	1405	46.8	22.9	59.8	94.9	0.6
28	360	72	238	9	1405	43.5	21.9	57.5	91.2	0.6
28	337.5	84.375	238	8.4375	1405	39.5	20.8	55.0	87.4	0.5
28	315	94.5	238	7.875	1405	38.2	19.6	52.5	83.4	0.6

Age (days)	Cement (Kg/m^3)	RHA (Kg/m^3)	Water (Kg/m^3)	Superplasticizer (Kg/m^3)	Aggregate (Kg/m^3)	Compr. Strength (Kg/m^3)	FLR (Left)	FLR (Center)	FLR (Right)	$\mu_A(y_i)$
56	450	0	238	11.25	1405	49.1	34.8	75.7	114.6	0.3
56	427.5	21.375	238	10.6875	1405	50.2	34.1	73.7	111.3	0.4
56	405	40.5	238	10.125	1405	52.1	33.3	71.5	107.9	0.5
56	382.5	57.375	238	9.5625	1405	55.3	32.4	69.3	104.4	0.6
56	360	72	238	9	1405	55.2	31.4	67.0	100.7	0.7
56	337.5	84.375	238	8.4375	1405	47	30.3	64.5	96.9	0.5
56	315	94.5	238	7.875	1405	45.9	29.1	62.0	92.9	0.5
90	450	0	238	11.25	1405	52.6	46.4	87.2	126.1	0.2
90	427.5	21.375	238	10.6875	1405	54.9	45.6	85.2	122.8	0.2
90	405	40.5	238	10.125	1405	57.3	44.8	83.1	119.4	0.3
90	382.5	57.375	238	9.5625	1405	61.2	43.9	80.8	115.9	0.5
90	360	72	238	9	1405	55.5	42.9	78.5	112.2	0.4
90	337.5	84.375	238	8.4375	1405	51.9	41.8	76.1	108.4	0.3
90	315	94.5	238	7.875	1405	50.2	40.6	73.5	104.4	0.3
1	783	87	212	3.6	1277	41	40.1	60.4	79.1	0.0
1	571	0	219	1	1566	30	21.7	36.2	49.0	0.6
1	514	57	218	1.4	1541	27	20.5	36.6	50.8	0.4
1	457	114	216	2.6	1515	26	19.6	39.3	57.2	0.3
1	400	171	215	3.7	1490	19	18.7	41.7	63.0	0.0
1	383	42	221	0.3	1670	16	11.0	24.8	36.9	0.4
3	783	87	212	3.6	1277	59	40.7	61.1	79.8	0.9
3	571	0	219	1	1566	46	22.4	36.9	49.6	0.3
3	514	57	218	1.4	1541	41	21.2	37.2	51.5	0.7
3	457	114	216	2.6	1515	38	20.3	40.0	57.9	0.9
3	400	171	215	3.7	1490	32	19.3	42.4	63.7	0.5
3	383	42	221	0.3	1670	26	11.7	25.5	37.6	1.0
7	783	87	212	3.6	1277	62	42.1	62.5	81.1	1.0
7	571	0	219	1	1566	50	23.7	38.2	51.0	0.1
7	514	57	218	1.4	1541	47	22.6	38.6	52.9	0.4
7	457	114	216	2.6	1515	47	21.7	41.3	59.3	0.7
7	400	171	215	3.7	1490	43	20.7	43.7	65.1	1.0
7	383	42	221	0.3	1670	37	13.0	26.9	38.9	0.2
14	783	87	212	3.6	1277	63	44.5	64.8	83.5	0.9
14	571	0	219	1	1566	54	26.1	40.6	54.0	0.0
14	514	57	218	1.4	1541	52	24.9	41.0	55.2	0.2
14	457	114	216	2.6	1515	52	24.0	43.7	61.6	0.5
14	400	171	215	3.7	1490	51	23.1	46.1	67.4	0.8
14	383	42	221	0.3	1670	40	15.4	29.2	41.3	0.1
28	783	87	212	3.6	1277	66	49.2	69.6	88.2	0.8
28	571	0	219	1	1566	56	30.8	45.4	58.1	0.2
28	514	57	218	1.4	1541	61	29.7	45.7	61.0	0.0
28	457	114	216	2.6	1515	60	28.8	48.5	66.4	0.4
28	400	171	215	3.7	1490	54	27.8	50.9	72.2	0.9
28	383	42	221	0.3	1670	47	20.1	34.0	47.0	0.0
56	783	87	212	3.6	1277	69	58.7	79.1	97.7	0.5
56	571	0	219	1	1566	60	40.3	54.8	67.6	0.6
56	514	57	218	1.4	1541	62	39.2	55.2	69.5	0.5
56	457	114	216	2.6	1515	61	38.3	57.9	75.9	0.8
56	400	171	215	3.7	1490	60	37.3	60.4	81.7	1.0
56	383	42	221	0.3	1670	51	29.6	43.5	55.5	0.4
90	783	87	212	3.6	1277	74	70.2	90.6	109.3	0.2
90	571	0	219	1	1566	67	51.9	66.4	79.1	1.0
90	514	57	218	1.4	1541	67	50.7	66.7	81.0	1.0
90	457	114	216	2.6	1515	69	49.8	69.5	87.4	1.0
90	400	171	215	3.7	1490	64	48.8	71.9	93.2	0.7
90	383	42	221	0.3	1670	56	41.2	55.0	67.1	0.9
7	364	19	203	0	1725	27.6	11.6	24.7	36.2	0.8
7	345	38	203	0	1725	28	11.2	24.5	36.2	0.7
7	326	57	203	0	1725	29.3	10.7	24.2	36.1	0.6
7	306	77	203	0	1725	29.7	10.3	24.0	36.1	0.5
7	287	96	203	0	1725	28.7	9.8	23.7	36.1	0.6
7	268	115	203	0	1725	27.4	9.3	23.5	36.0	0.7
7	249	134	203	0	1725	25.7	8.9	23.2	36.0	0.8
14	364	19	203	0	1725	34.2	14.0	27.1	38.6	0.4
14	345	38	203	0	1725	35.3	13.6	26.9	38.5	0.3
14	326	57	203	0	1725	36	13.1	26.6	38.5	0.2
14	306	77	203	0	1725	39.3	12.6	26.4	39.3	0.0
14	287	96	203	0	1725	36.1	12.2	26.1	38.4	0.2
14	268	115	203	0	1725	33.5	11.7	25.9	38.4	0.4

Age (days)	Cement (Kg/m^3)	RHA (Kg/m^3)	Water (Kg/m^3)	Superplasticizer (Kg/m^3)	Aggregate (Kg/m^3)	Compr. Strength (Kg/m^3)	FLR (Left)	FLR (Center)	FLR (Right)	$\mu_A(y_i)$
14	249	134	203	0	1725	31.1	11.3	25.6	38.4	0.6
28	364	19	203	0	1725	40	18.8	31.9	43.3	0.3
28	345	38	203	0	1725	41.3	18.3	31.6	43.3	0.2
28	326	57	203	0	1725	41.8	17.8	31.4	43.3	0.1
28	306	77	203	0	1725	42.5	17.4	31.1	43.2	0.1
28	287	96	203	0	1725	38.8	16.9	30.9	43.2	0.4
28	268	115	203	0	1725	37.6	16.5	30.6	43.1	0.4
28	249	134	203	0	1725	35.1	16.0	30.4	43.1	0.6
90	364	19	203	0	1725	43.3	39.8	52.9	64.3	0.3
90	345	38	203	0	1725	44.8	39.3	52.6	64.3	0.4
90	326	57	203	0	1725	45.7	38.9	52.4	64.3	0.5
90	306	77	203	0	1725	46	38.4	52.1	64.2	0.6
90	287	96	203	0	1725	43	37.9	51.9	64.2	0.4
90	268	115	203	0	1725	38.7	37.5	51.6	64.2	0.1
90	249	134	203	0	1725	37.2	37.2	51.4	64.1	0.0