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Optimizing Budget Allocation Through First-Order Linear Differential Equations and Innovative Transform Techniques

Athraa N. Albukhuttar*, Janaan A. Alshamkhawii, Hayder N. Kadhim

Department of Mathematics, College of Education for Girls, University of Al-Kufa, Najaf 54002, Iraq

Corresponding Author Email: athraa.albukhuttar@yahoo.com

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ABSTRACT

The resolution of a first-order mathematical system effectively tackles a wide range of physical problems. The technique illustrates that the quantity of authorised variables in the system may be depicted by the activities that need to be evaluated for cost, and that the system takes into consideration the connections between these costs. Mathematical systems adhere to primary conditions, resulting in derived solutions that are specific and reliant on a single independent variable representing the temporal aspect in cost computation. This enables the forecast of costs in future years. This study confronts the inadequacies found in traditional cost allocation methods used for organizational budgeting, often leading to a biased allocation of costs to departments, irrespective of their profitability. We introduce an innovative method that employs first-order linear differential equations to model the cost dynamics associated with various activities within an organization. Moreover, an innovative transformation technique is presented to solve these equation systems efficiently, thereby enabling a precise computation of activity-based costs over a three-year projection. The results illustrate that the proposed method provides a more precise and insightful budget allocation, suggesting potential applications for financial planning and management across diverse sectors.

1. INTRODUCTION

Traditional cost calculation methods adopted by organizations typically assign expenses to each department based on their incurred costs within the year, often neglecting an analysis of departmental profitability [1, 2]. An alternative method, Activity-Based Costing (ABC), has been applied with some success, but it presents challenges in linking costs between activities and products, and lacks scalability [3, 4].

Kaplan and Anderson [3] proposed the Time-Driven Activity-Based Costing (TDABC) system, a derivative of the ABC method, in 1998. The TDABC system allocates indirect costs based on time but is limited to a specific period [5-7]. This study explores the use of ordinary differential equation systems for cost calculation, considering the relationship between costs and activities, and accommodating an indefinite time span. This approach provides accountants with a modern tool for developing practical and effective future budgets for project costs [8].

Integral transformations are accurate and efficient methods for solving mathematical systems, including ordinary differential equation systems. Various transformations exist, such as Laplace, Elzaki, Novel, and SEE transformations [9]. Introduced in 2016, the Novel transformation is an effective method for solving first-order linear mathematical systems and is derived from the Laplace transform [10].

Albukhuttar and Almasoudi [11] utilized systems of Euler's equations to model cost calculation problems in bank auditing. Meanwhile, Alshibley et al. [12] introduced first-order fuzzy linear systems to calculate costs when the initial project cost is fuzzy over a certain time period.

Several other integral transforms have been introduced and utilized to solve linear ordinary differential equations with constant coefficients, including the Aboodh transform by Aboodh [13] and the Kushare transform by Kushare et al. [14]. Jafari [15] proposed a new general integral transform that encompasses all classes of integral transforms in the Laplace transform class and explored its application in solving ordinary differential equations (ODEs) with constant and variable coefficients. Jafari's transform is particularly adept at handling fractional-order differential equations and integral equations.

The Anuj transforms, a recent development by Patil et al. [16], has been used to solve linear Volterra integral equations of the first kind, demonstrating its practicality and efficiency. Similarly, Patil and Khakale [17] introduced and applied the Soham transform to solve linear ordinary differential equations with constant coefficients.

In the realm of solving linear Volterra integral equations of the first kind, Aggarwal et al. [18] utilized the Kamal transform and provided applications to demonstrate its usefulness. They also applied the Mohand transform to solve convolution-type linear Volterra integral equations of the second kind, providing practical examples of its application [19].

Aiming to solve partial differential equations, Ahmed et al.

[20] introduced the double Laplace - Sumudu transform (DLST), providing an alternative to other double transforms. The study establishes theorems on the modern properties of the DLST, discusses the convolution theorem, and demonstrates the solution of partial differential equations using these results. This investigation suggests that the DLST is superior in terms of effectiveness and utility when dealing with such equations.

For the purpose of solving differential equations in the time domain, Maitama and Zhao [21] presented the Shehu transform, a generalisation of the Laplace and Sumudu integral transforms. To demonstrate its ease of use, effectiveness, and high accuracy, the suggested integral transform was successfully deduced from the traditional Fourier integral transform. It was then applied to both ordinary and partial differential equations.

The scientific study of differential equations and their applications has garnered significant attention in recent years. The Sumudu transform, for instance, has been employed by Eltayeb and Kilicman [22] to solve linear ordinary differential equations, both without constant coefficients and with them. These equations, notably, included convolution terms. The authors optimally utilized the Sumudu transform for addressing various issues related to Spring-Mass systems, Population Growth, and finance.

In parallel, the exploration of the Shehu transform has also proven fruitful. Bokhari [23] introduced novel characteristics of the Shehu transform, applying this transformation to Atangana-Baleanu derivatives in the Caputo and Riemann-Liouville senses. Their work involved the resolution of various fractional differential equations.

In a pivotal study, Asiru [24] validated the convolution theorem for the Sumudu transform for functions expressible either as a polynomial or a convergent infinite series. Consequently, this theorem can be used to resolve integral equations of the convolution type. Arikoglu and Ozkol [25] resorted to the differential transform method for analysing integro-differential systems and integral equation systems. Their method, enriched with a formulation that addresses Fredholm integrals, has expanded the horizon for differential equation solutions. The method's robustness and competence have been exemplified through its application to various integral and integro-differential equations.

Atangana and Akgül [26] conducted a comparative analysis of various derivatives - the classical derivative, the Caputo derivative, the Caputo-Fabrizio derivative, and the Atangana-Baleanu derivative – to discern the differences between the Laplace and Sumudu transforms. Their findings illustrated that the Sumudu transform-derived Bode diagrams essentially function as a low-pass filter, while those derived from the Laplace transform mainly function as a high-pass filter.

Emphasizing the effectiveness of integral transforms, Patil et al. [27] successfully applied the Kushare integral transform to solve the Faltung Type Volterra Integro-Differential Equation of the First Kind. The results achieved provided evidence of the integral transform's efficacy in solving first-class integro-differential equations.

In a recent advancement, Saadeh et al. [28] introduced a new integral operator transform, the ARA transform. Their research validated the existence of this transform, as well as its inverse. They also demonstrated its applicability in solving ordinary and partial differential equations, which are frequently encountered in various research fields, including physics and engineering.

In the realm of cost dynamics, the current study proposes a novel method for modelling various organizational activities using first-order linear differential equations. This method is expected to enable the accurate calculation of activity-based costs over a three-year period. In the context of current research, the present study aims to apply this proposed method to answer questions concerning cost calculation and budget preparation.

2. BASIC DEFINITIONS AND PROPERTIES OF NOVEL TRANSFORM

The following defines the Novel transform for the function $\mathcal{H}(\mathrm{t})$, t>0.

$$\mathcal{N}_{\mathrm{i}}\big(\mathcal{H}(\mathrm{t})\big) = \frac{1}{\mathcal{P}}\int_0^\infty \mathrm{e}^{-\mathcal{P}\mathrm{t}}\ \mathcal{H}(\mathrm{t})\mathrm{dt}\ \mathrm{t} > 0 \tag{1}$$

where, $\mathcal{H}(\mathrm{t})$ is a real function, $\frac{\mathrm{e}^{-\mathcal{P}\mathrm{t}}}{\mathcal{P}}$ is the kernel function, and $\mathcal{N}_{\mathrm{i}}$ is the operator of Novel transform.

The inverse of Novel integral transform is given by:

$$\mathcal{N}_{\mathrm{i}}^{-1}\{\mathcal{N}_{\mathrm{i}}\big(\mathcal{H}(\mathrm{t})\big)\} = \mathcal{H}(\mathrm{t}) \text{ for } \mathrm{t} > 0 \tag{2}$$

where, $\mathcal{N}_{\mathrm{i}}^{-1}$ returns the transformation to the prime function.

Property: If $\mathcal{H}_1(\mathrm{t}\ )$, $\mathcal{H}_2(\mathrm{t}), \ldots\ldots, \mathcal{H}_{\mathrm{n}}(\mathrm{t})$ have Novel transform then:

$$\begin{gathered}\mathcal{N}_{\mathrm{i}}(\mathrm{b}_1\mathcal{H}_1(\mathrm{t}) \pm \mathrm{b}_2\mathcal{H}_2(\mathrm{t}) \pm \mathrm{b}_{\mathrm{n}}\mathcal{H}_{\mathrm{n}}(\mathrm{t})) \\ =\mathrm{b}_1\mathcal{N}_{\mathrm{i}}(\mathcal{H}_1(\mathrm{t})) \pm \mathrm{b}_2\mathcal{N}_{\mathrm{i}}(\mathcal{H}_2(\mathrm{t})) \pm \cdots \pm \mathrm{b}_{\mathrm{n}}\mathcal{N}_{\mathrm{i}}(\mathcal{H}_{\mathrm{n}}(\mathrm{t})\end{gathered} \tag{3}$$

where, b_1, b_2, …, b_n are constants.

Theorem (2.1) [1]: If the function $\mathcal{H}^{\mathrm{n}}\ (\mathrm{t})$is the derivative of the function $\mathcal{H}(\mathrm{t})$ then its Novel transform is defined by the following where Table 1 shows the summary of novel transformation:

$$\mathcal{N}_{\mathrm{i}}(\mathcal{H}'(\mathrm{t}))=\mathcal{P}\ \mathcal{N}_{\mathrm{i}}(\mathcal{H}(\mathrm{t}))-\tfrac{\mathcal{H}(0)}{\mathcal{P}} \tag{4}$$

$$\mathcal{N}_{\mathrm{i}}\big(\mathcal{H}''(\mathrm{t})\big) = \mathcal{P}^2\mathcal{N}_{\mathrm{i}}(\mathcal{H}(\mathrm{t}))- \mathcal{H}(0) - \frac{\mathcal{H}'(0)}{\mathcal{P}} \tag{5}$$

$$\mathcal{N}_{\mathrm{i}}(\mathcal{H}'''(\mathrm{t}))=\mathcal{P}^3\mathcal{N}_{\mathrm{i}}\big(\mathcal{H}(\mathrm{t})\big) - \mathcal{P}\mathcal{H}(0) - \mathcal{H}'(0) - \tfrac{\mathcal{H}''(0)}{\mathcal{P}} \tag{6}$$

$$\begin{gathered}(\mathcal{H}^{(\mathrm{n})}(\mathrm{t}))=\mathcal{P}^{\mathrm{n}}\mathcal{N}_{\mathrm{i}}(\mathcal{H}(\mathrm{t}))-\mathcal{P}^{\mathrm{n}-2}\mathcal{H}(0) - \\ \mathcal{P}^{\mathrm{n}-3}\mathcal{H}'(0) - \mathcal{H}^{(\mathrm{n}-2)}(0) - \tfrac{1}{\mathcal{P}}\mathcal{H}^{(\mathrm{n}-1)}\end{gathered} \tag{7}$$

Table 1. Novel transformation for several function

ID	Function $\mathcal{H}(\mathrm{t})$	$\mathcal{N}_{\mathrm{i}}(\mathcal{H}(\mathrm{t})) = \frac{1}{\mathcal{P}}\mathrm{L}\ (\mathcal{H}(\mathrm{t}))$	
1	a	$\frac{\mathrm{a}}{\mathcal{P}^2}$	$\mathrm{a}\in R, \mathcal{P} > 0$
2	t^{n}	$\frac{\mathrm{n}!}{\mathcal{P}(\mathcal{P}^{\mathrm{n}+1})}$	$\mathrm{n}\in \mathrm{N}, \in \mathcal{P} > 0$
3	e^{at}	$\frac{1}{\mathcal{P}(\mathcal{P} - \mathrm{a})}$	$\mathrm{a}\neq \mathcal{P}, \mathcal{P} > 0$
4	sin at	$\frac{\mathrm{a}}{\mathcal{P}(\mathcal{P}^2 + \mathrm{a})}$	$\mathcal{P} > 0$
5	cos at	$\frac{1}{(\mathcal{P}^2 - \mathrm{a}^2)}$	$\mathrm{a} \neq \mathcal{P}$
6	sinh at	$\frac{\mathrm{a}}{\mathcal{P}(\mathcal{P}^2 - \mathrm{a}^2)}$	$\mathrm{a}\neq \mathcal{P}, \mathcal{P} \neq 0$
7	cosh at	$\frac{1}{(\mathcal{P}^2 - \mathrm{a}^2)}$	$\mathrm{a}\neq \mathcal{P}$

3. METHOD

In this section, the general formula for the homogeneous and non-homogeneous systems of first order in dimension n×n is obtained.

3.1 Formula of general solution of non-homogeneous system of first-order

The formula of non-homogeneous system is:

$$H^{\wedge\prime} = CH + W \tag{8}$$

$$\mathcal{H}' = \begin{pmatrix} \frac{d\mathcal{H}_1(t)}{dt} \\ \frac{d\mathcal{H}_2(t)}{dt} \\ \vdots \\ \frac{d\mathcal{H}_3(t)}{dt} \end{pmatrix}, \mathrm{C}=\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix},$$

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_1(t) \\ \mathcal{H}_2(t) \\ \vdots \\ \mathcal{H}_n(t) \end{pmatrix}, \mathrm{W}=\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}, \tag{9}$$

$$\text{so} \begin{pmatrix} \mathcal{H}_1(t) \\ \mathcal{H}_2(t) \\ \vdots \\ \mathcal{H}_n(t) \end{pmatrix}' = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{pmatrix} \begin{pmatrix} \mathcal{H}_1(t) \\ \mathcal{H}_2(t) \\ \vdots \\ \mathcal{H}_n(t) \end{pmatrix} + \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}$$

After taking Novel transform for both sides, yields:

$$\begin{cases} \mathcal{P}\mathcal{N}_i(\mathcal{H}_1) - \dfrac{\mathcal{H}_1(0)}{\mathcal{P}} \\ = c_{11}\mathcal{N}_i(\mathcal{H}_1) + c_{12}\mathcal{N}_i(\mathcal{H}_2) + \cdots + c_{1n}\mathcal{N}_i(\mathcal{H}_n) + \mathcal{N}_i(w_1) \\ \mathcal{P}\mathcal{N}_i(\mathcal{H}_2) - \dfrac{\mathcal{H}_2(0)}{\mathcal{P}} \\ = c_{21}\mathcal{N}_i(\mathcal{H}_1) + c_{22}\mathcal{N}_i(\mathcal{H}_2) + \cdots + c_{2n}\mathcal{N}_i(\mathcal{H}_n) + \mathcal{N}_i(w_2) \\ \mathcal{P}\mathcal{N}_i(\mathcal{H}_n) - \dfrac{\mathcal{H}_n(0)}{\mathcal{P}} \\ = c_{n1}\mathcal{N}_i(\mathcal{H}_1) + c_{n2}\mathcal{N}_i(\mathcal{H}_2) + \cdots + c_{mn}\mathcal{N}_i(\mathcal{H}_n) + \mathcal{N}_i(w_n) \end{cases} \tag{10}$$

Also, $\mathcal{H}_1(0), \ldots., \mathcal{H}_n(0)$ are initial condition.

$$\begin{cases} (\mathcal{P} - c_{11})\ \mathcal{N}_i(\mathcal{H}_1) - c_{12}\mathcal{N}_i(\mathcal{H}_2) - c_{1n}\mathcal{N}_i(\mathcal{H}_n) = \dfrac{\mathcal{H}_1(0)}{\mathcal{P}} + \mathcal{N}_i(w_1) \\ (\mathcal{P} - c_{22})\ \mathcal{N}_i(\mathcal{H}_2) - c_{21}\mathcal{N}_i(\mathcal{H}_1) - c_{2n}\mathcal{N}_i(\mathcal{H}_n) = \dfrac{\mathcal{H}_2(0)}{\mathcal{P}} + \mathcal{N}_i(w_2) \\ (\mathcal{P} - c_{mn})\ \mathcal{N}_i(\mathcal{H}_n) - c_{m1}\mathcal{N}_i(\mathcal{H}_1) - c_{m2}\mathcal{N}_i(\mathcal{H}_n) = \dfrac{\mathcal{H}_n(0)}{\mathcal{P}} + \mathcal{N}_i(w_n) \end{cases} \tag{11}$$

Additionally, a straightforward calculation to obtain $\mathcal{N}_i(\mathcal{H}_1), \cdots, \mathcal{N}_i(\mathcal{H}_n)$.

$$\Delta = \begin{vmatrix} (\mathcal{P} - c_{11}) & -c_{12} & \cdots & -c_{1n} \\ -c_{21} & (\mathcal{P} - c_{22}) & \cdots & -c_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ -c_{m1} & -c_{m2} & \cdots & (\mathcal{P} - c_{mn}) \end{vmatrix} \tag{12}$$

Therefore,

$$\begin{cases} \mathcal{N}_i(\mathcal{H}_1) = \dfrac{1}{\Delta} \begin{vmatrix} \dfrac{\mathcal{H}_1(0)}{\mathcal{P}} + \mathcal{N}_i(w_1) & -c_{12} & \cdots & -c_{1n} \\ \dfrac{\mathcal{H}_2(0)}{\mathcal{P}} + \mathcal{N}_i(w_2) & (\mathcal{P} - c_{22}) & \cdots & -c_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \dfrac{\mathcal{H}_n(0)}{\mathcal{P}} + \mathcal{N}_i(w_n) & -c_{mn} & \cdots & (\mathcal{P} - c_{mn}) \end{vmatrix} \\ \mathcal{N}_i(\mathcal{H}_n) = \dfrac{1}{\Delta} \begin{vmatrix} (\mathcal{P} - c_{11}) & -c_{12} & \cdots & \dfrac{\mathcal{H}_1(0)}{\mathcal{P}} + \mathcal{N}_i(w_1) \\ -c_{21} & (\mathcal{P} - c_{22}) & \cdots & \dfrac{\mathcal{H}_2(0)}{\mathcal{P}} + \mathcal{N}_i(w_2) \\ \vdots & \vdots & \cdots & \vdots \\ -c_{m1} & -c_{m2} & \cdots & \dfrac{\mathcal{H}_n(0)}{\mathcal{P}} + \mathcal{N}_i(w_n) \end{vmatrix} \end{cases} \tag{13}$$

Then, taking the inverse of the Novel transform for $\mathcal{N}_i(\mathcal{H}_i), i = 1,2,3 \dots, n$, yields the set solution of problem (3.1).

3.2 A homogeneous system which has the formula $\mathcal{H}' = C\mathcal{H}$

From the previous formula (3.1), the vector $\vec{W} = 0$, so the solution of (3.1) has the from:

$$\begin{cases} \Delta = \begin{vmatrix} (\mathcal{P} - c_{11}) & -c_{12} & \cdots & -c_{1n} \\ -c_{21} & (\mathcal{P} - c_{22}) & \cdots & -c_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ -c_{m1} & -c_{m2} & \cdots & (\mathcal{P} - c_{mn}) \end{vmatrix} \\ \mathcal{N}_i(\mathcal{H}_n) = \dfrac{1}{\Delta} \begin{vmatrix} (\mathcal{P} - c_{11}) & -c_{12} & \cdots & \dfrac{\mathcal{H}_1(0)}{\mathcal{P}} \\ -c_{21} & (\mathcal{P} - c_{22}) & \cdots & \dfrac{\mathcal{H}_2(0)}{\mathcal{P}} \\ \vdots & \vdots & \cdots & \vdots \\ -c_{m1} & -c_{m2} & \cdots & \dfrac{\mathcal{H}_n(0)}{\mathcal{P}} \end{vmatrix} \end{cases} \tag{14}$$

Accordingly, in similar way by taking the inverse of the Novel transform for $\mathcal{N}_i(\mathcal{H}_i), i = 1,2 \dots, n$ yields the set solution of problem (3.2).

4. RESULTS AND DISCUSSION

In this section, some application in budget for project costs are introduced. Based on a study done in 2016, we used an example from one of the hospitals in Najaf and the TDABC system to figure out the costs of a set of activities. Where their accounts were closed and fixed for a period of time in 2016 only. As for the use of the mathematical system, the study of preparing the budget for costs is moving and open. It is evident that the hospital has two types of activities, the first being service and the second being medical, and that the sum of them constitutes 100% of the allocations for each activity and is distributed among the hospital departments as in the following system:

$$\begin{cases} \mathcal{N}_i(\mathcal{H}_1') = 0.75\,\mathcal{H}_1 + 0.25\,\mathcal{H}_2 \\ \mathcal{H}_1(0) = \mathcal{H}_2(0) = 35 \\ \mathcal{N}_i(\mathcal{H}_2') = 0.8\,\mathcal{H}_1 + 0.2\,\mathcal{H}_2 \end{cases} \tag{15}$$

Then, solution here by using formal (3.2) and obtain:

$$(\mathcal{P} - 0.75)\,\mathcal{H}_1 - 0.25\,\mathcal{H}_2 = \frac{35}{\mathcal{P}} \tag{16}$$

$$(\mathcal{P} - 0.2)\,\mathcal{H}_2 - 0.8\,\mathcal{H}_1 = \frac{35}{\mathcal{P}} \tag{17}$$

$$\begin{cases} \mathcal{N}_i(\mathcal{H}_1) = \dfrac{\begin{vmatrix} \dfrac{35}{\mathcal{P}} & -0.25 \\ \dfrac{35}{\mathcal{P}} & \mathcal{P} - 0.2 \end{vmatrix}}{(\mathcal{P} - 1)(\mathcal{P} - 0.05)} \\ \mathcal{N}_i(\mathcal{H}_1) = \dfrac{35\mathcal{P} - 1.75}{\mathcal{P}(\mathcal{P} - 1)(\mathcal{P} + 0.05)} \end{cases} \tag{18}$$

$$\frac{35\mathcal{P} + 1.75}{\mathcal{P}(\mathcal{P} - 1)(\mathcal{P} + 0.05)} = \frac{1}{\mathcal{P}}\left(\frac{A}{\mathcal{P} - 1} + \frac{B}{\mathcal{P} + 0.05}\right) \tag{19}$$

After use partition fractional (A=35, B=0):

$$\mathcal{N}_i(\mathcal{H}_1) = \frac{35}{\mathcal{P}(\mathcal{P} - 1)} \tag{20}$$

Then, taking the inverse novel transform's presently to $\mathcal{N}_i(\mathcal{H}_1)$ produce:

$$\mathcal{H}_1(t) = 35e^{t} \tag{21}$$

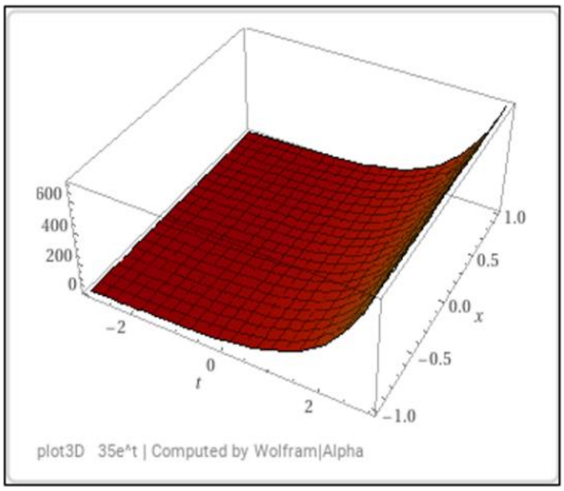
$\mathcal{N}_i(\mathcal{H}_2)$, in similar:

$$\begin{cases} \mathcal{N}_i(\mathcal{H}_2) = \dfrac{\begin{vmatrix} \mathcal{P} - 0.75 & \dfrac{35}{\mathcal{P}} \\ -0.8 & \dfrac{35}{\mathcal{P}} \end{vmatrix}}{\mathcal{P}(\mathcal{P} - 1)(\mathcal{P} + 0.005)} \\ \mathcal{N}_i(\mathcal{H}_2) = \dfrac{35}{\mathcal{P}(\mathcal{P} - 1)} \end{cases} \tag{22}$$

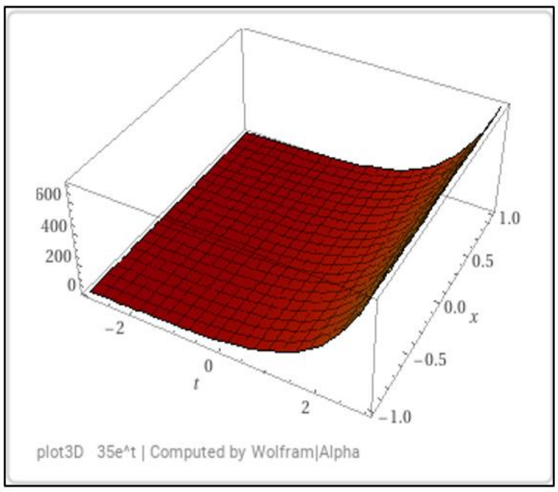
Taking the inverse novel transform's presently to $\mathcal{N}_i(\mathcal{H}_2)$ produce.

$$\mathcal{H}_2(t) = 35e^{t} \tag{23}$$

The set solution of system 1 displays in Table 2 and Figure 1.



(a)



(b)

Figure 1. Set solution of a system 1: (a) solution of equation $\mathcal{H}_1(t)$; and (b) solution of equation $\mathcal{H}_2(t)$

Table 2. Solution of system 1

t	$\mathcal{H}_1(\mathrm{t})$	$\mathcal{H}_2(\mathrm{t})$
t=1	95.139864	95.139864
t=2	258.6169635	258.6169635
t=3	702.9937923	702.9937923

Example 1 didn't take into account how long it would take to make the budget, but if the study was limited to a time function, it would only take one or two years, as shown in Example 2.

$$\begin{cases} \mathcal{N}_{\mathrm{i}}(\mathcal{H}_1) - 0.75\,\mathcal{H}_1 - 0.25\,\mathcal{H}_2 = \mathrm{t} \quad \mathcal{H}_1(0) = 20 \\ \mathcal{N}_{\mathrm{i}}(\mathcal{H}_2) - 0.8\,\mathcal{H}_1 - 0.2\,\mathcal{H}_2 = 2\mathrm{t} \quad \mathcal{H}_2\,(0) = 45 \end{cases} \tag{24}$$

Depending on 3.1:

$$\begin{cases} (\mathcal{P} - 0.75)\mathcal{H}_1 - 0.25\,\mathcal{H}_2 = \dfrac{1}{\mathcal{P}^3} + \dfrac{20}{\mathcal{P}} \\ (\,\mathcal{P} - 0.2)\mathcal{H}_2 - 0.8\mathcal{H}_1 = \dfrac{2}{\mathcal{P}^3} + \dfrac{45}{\mathcal{P}} \\ \Delta = (\mathcal{P} - 1)(\mathcal{P} + 0.05) \end{cases} \tag{25}$$

$$\begin{cases} \mathcal{N}_{\mathrm{i}}\,(\mathcal{H}_1) = \dfrac{\begin{vmatrix} \dfrac{1 + 20\mathcal{P}}{\mathcal{P}^3} & -0.25 \\ \dfrac{2 + 45\mathcal{P}^2}{\mathcal{P}^3} & \mathcal{P} - 0.2 \end{vmatrix}}{(\mathcal{P} - 1)(\mathcal{P} + 0.05)} \\ \mathcal{N}_{\mathrm{i}}\,(\mathcal{H}_1) = \dfrac{20\mathcal{P}^3 + 7.25\mathcal{P}^2 + \mathcal{P} + 0.3}{\mathcal{P}^3(\mathcal{P} - 1)(\mathcal{P} + 0.05)} \\ \mathcal{N}_{\mathrm{i}}\,(\mathcal{H}_1) = \dfrac{1}{\mathcal{P}}\left[\dfrac{\mathrm{A}}{\mathcal{P}} + \dfrac{\mathrm{B}}{\mathcal{P}^2} + \dfrac{\mathrm{C}}{\mathcal{P} - 1} + \dfrac{\mathrm{D}}{\mathcal{P} + 0.05}\right] \end{cases} \tag{26}$$

By using partition fractional:

$$\begin{gathered} \mathrm{A}=94,\ \mathrm{B}=-6,\ \mathrm{C}=27.19047619,\ \mathrm{D}=-101.1904762 \\ \mathcal{N}_{\mathrm{i}}\,(\mathcal{H}_1) = \frac{94}{\mathcal{P}^2} - \frac{6}{\mathcal{P}^3} + \frac{27.19047619}{\mathcal{P}(\mathcal{P}-1)} - \frac{101.1904762}{\mathcal{P}(\mathcal{P}+0.05)} \end{gathered} \tag{27}$$

Taking the inverse novel transform's presently to $\mathcal{N}_{\mathrm{i}}\,(\mathcal{H}_1)$ produce.

$$\mathcal{H}_1(\mathrm{t}) = 94 - 6\mathrm{t} + 27.19047619\mathrm{e}^{\mathrm{t}} - 101.1904762\mathrm{e}^{-0.05\mathrm{t}} \tag{28}$$

$\mathcal{N}_{\mathrm{i}}(\mathcal{H}_2)$, in similar:

$$\begin{cases} \mathcal{N}_{\mathrm{i}}\,(\mathcal{H}_2) = \dfrac{\begin{vmatrix} \mathcal{P} - \mathrm{o}.75 & \dfrac{1 + 20\mathcal{P}^2}{\mathcal{P}^3} \\ -0.8 & \dfrac{2 + 45\mathcal{P}^2}{\mathcal{P}^3} \end{vmatrix}}{(\mathcal{P} - 1)(\mathcal{P} + 0.005)} \\ \mathcal{N}_{\mathrm{i}}\,(\mathcal{H}_2) = \dfrac{45\mathcal{P}^3 - 17.75\mathcal{P}^2 + 2\mathcal{P} - 0.7}{\mathcal{P}^3(\mathcal{P} - 1)(\mathcal{P} + 0.05)} \end{cases} \tag{29}$$

By using partition fractional:

$$\begin{gathered} \mathrm{A}=-306,\ \mathrm{B}=14,\ \mathrm{C}=27.19047619,\ \mathrm{D}=323.8095238 \\ \mathcal{N}_{\mathrm{i}}\,(\mathcal{H}_2) = \frac{-306}{\mathcal{P}^2} + \frac{14}{\mathcal{P}^3} + \frac{27.19047619}{\mathcal{P}(\mathcal{P}-1)} + \frac{323.8095238}{\mathcal{P}(\mathcal{P}+0.05)} \end{gathered} \tag{30}$$

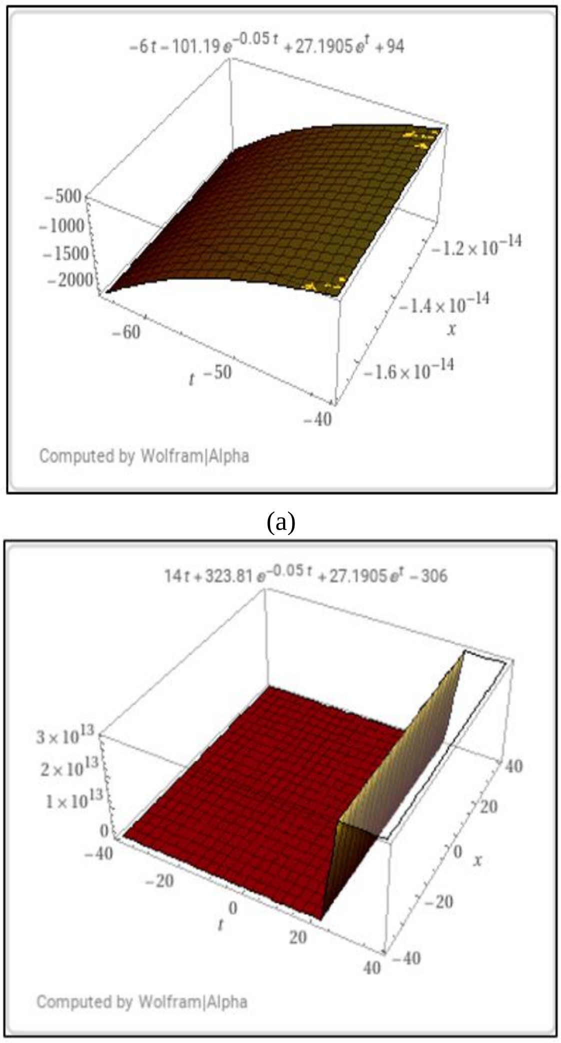
Taking the inverse novel transform's presently to $\mathcal{N}_{\mathrm{i}}\,(\mathcal{H}_2)$ produce.

$$\mathcal{H}_2(\mathrm{t}) = -306 + 14\mathrm{t} + 27.19047619\,\mathrm{e}^{\mathrm{t}} + 323.8095238\,\mathrm{e}^{-0.05\,\mathrm{t}} \tag{31}$$

The set solution of system 2 displays in Table 3 and Figure 2.

Table 3. Solution of system 2

t	$\mathcal{H}_1(\mathrm{t})$	$\mathcal{H}_2(\mathrm{t})$
t=1	65.65601889	89.92852431
t=2	191.3510247	215.9069274
t=3	535.0398635	560.8407535



(a)

(b)

Figure 2. Set solution of a system 2: (a) Solution of equation $\mathcal{H}_1(\mathrm{t})$; (b) Solution of equation $\mathcal{H}_2(\mathrm{t})$

5. CONCLUSIONS

The resolution of the first-order mathematical system effectively addresses numerous physical problems. The procedure demonstrates that the number of approved variables in the system can be represented by the activities to be costed, and that the system accounts for the interrelationships between these costs. Given that the mathematical systems are subject to primary conditions, the derived solutions are special, based on a single independent variable representing the temporal element in cost calculation, allowing for cost prediction in subsequent years.

One challenge faced by auditors in implementing these systems is the identification of the ratio representing the interrelationship between costs, which contributes 100% of the total costs for each activity costed. This study utilized systems of linear differential equations to calculate the activity costs of a specific institution, dependent on each activity's initial cost. Several practical examples are provided to substantiate this approach. An innovative transformation technique was used to derive general solution formulas for these systems. The results show the capability of preparing a budget to compute the costs of activities for the next three years (i=1, 2, 3), enhancing the potential for future budget preparation for a specific institution, irrespective of the initial cost or the interactivity relationships. The main hindrance to this work is the difficulty in accessing some institutions' data and applying the study, as it is considered proprietary information of the institution.

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