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Theoretical and Experimental Analysis and Testing of Medium Length Electric Power Transmission Line

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ABSTRACT

In this paper, a medium length electric line that presents only longitudinal ohmic resistance and longitudinal inductance is modelled, is studied mathematically and is tested experimentally in order to verify the findings.

1. INTRODUCTION

An electric power transmission line is one of the basic parts of an electric power system [1-8]. In a previous paper [1], a short length electric power transmission line was studied in depth. The characteristic element of a short line is the long-wise inductance. The other electric elements of the line such as the long-wise ohmic resistance and the transversal ohmic conductance and capacitance due the shortness of the line have very small value and are not considered in the electric analysis. In this paper, the study is extended to a medium length transmission line where the value of the long-wise ohmic resistance is increased and therefore is taken into account. The equations that are developed from the electric analysis of the line are therefore longer and more complicated than those of the short line that developed in [1].

In section 2, the above line is examined electrically while in section 3 the above line is modelled and the findings are compared with those of section 2.

In section 4, what is expected and stated in section 2 and the equations drawn in section 3 are tested experimentally.

At the end, in section 5, discussion and conclusions follow.

2. ELECTRIC EXAMINATION OF MEDIUM LENGTH LINE

In Figure 1, the electric circuit of the medium length line is presented [1].

Examining electrically the mentioned line, the following are expected [1]:

1) P_1, P_2 and P_{line} must be positive;
2) $P_1=P_{line}+P_2$;
3) Q_{line} must be positive;
4) $Q_1=Q_{line}+Q_2$;
5) $Z_1=Z_{line}+Z_2$;
6) A criterion drawn by either S_2 or Z_2 can be developed to indicate the kind of the load. The criterion logically must be the same whatever element (S_2 or Z_2) is used.

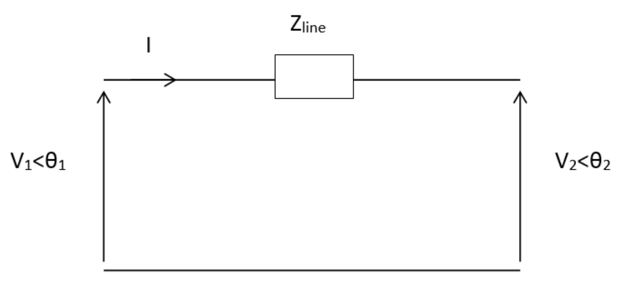

Figure 1. Electric circuit of medium length line

3. MATHEMATICAL ANALYSIS

From Figure 1, using the basic electric laws and carrying out the relative mathematical operations, the following are derived:

$$I=\frac{V_1\angle\theta_1-V_2\angle\theta_2}{Z_{line}}= \\ =\frac{[R(V_1\cos\theta_1-V_2\cos\theta_2)+\omega L(V_1\sin\theta_1-V_2\sin\theta_2)]}{Z^2}+ \\ +\frac{j[R(V_1\sin\theta_1-V_2\sin\theta_2)-\omega L(V_1\cos\theta_1-V_2\cos\theta_2)]}{Z^2} \quad (1)$$

$$S_1=V_1\angle\theta_1 I^*= \\ =\frac{[R[V_1{}^2-V_1V_2\cos(\theta_1-\theta_2)]+\omega LV_1V_2\sin(\theta_1-\theta_2)]}{Z^2}+ \\ +\frac{j[\omega L[V_1{}^2-V_1V_2\cos(\theta_1-\theta_2)]-RV_1V_2\sin(\theta_1-\theta_2)]}{Z^2} \quad (2)$$

$$P_1 = \frac{[R[V_1^2 - V_1V_2\cos(\theta_1-\theta_2)] + \omega L V_1V_2\sin(\theta_1-\theta_2)]}{Z^2} \quad (3)$$

$$Q_1 = \frac{[\omega L[V_1^2 - V_1V_2\cos(\theta_1-\theta_2)] - RV_1V_2\sin(\theta_1-\theta_2)]}{Z^2} \quad (4)$$

$$S_2 = V_2 < \theta_2 \; I^* = \\ = \frac{[R[V_1V_2\cos(\theta_1-\theta_2) - V_2^2] + \omega L V_1V_2\sin(\theta_1-\theta_2)]}{Z^2} \\ + \frac{j[\omega L[V_1V_2\cos(\theta_1-\theta_2) - V_2^2] - RV_1V_2\sin(\theta_1-\theta_2)]}{Z^2} \quad (5)$$

$$P_2 = \frac{[R[V_1V_2\cos(\theta_1-\theta_2) - V_2^2] + \omega L V_1V_2\sin(\theta_1-\theta_2)]}{Z^2} \quad (6)$$

$$Q_2 = \frac{[\omega L[V_1V_2\cos(\theta_1-\theta_2) - V_2^2] - RV_1V_2\sin(\theta_1-\theta_2)]}{Z^2} \quad (7)$$

$$V_{line} = V_1 < \theta_1 - V_2 < \theta_2 = \\ = (V_1\cos\theta_1 - V_2\cos\theta_2) + j(V_1\sin\theta_1 - V_2\sin\theta_2) \quad (8)$$

$$V^2_{line,magn.} = V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2) \quad (9)$$

$$S_{line} = V_{line}\, I^* = \frac{V^2_{line,magn.}}{Z^*_{line}} = S_1 - S_2 \\ = \frac{V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2)}{Z^2}(R + j\omega L) \quad (10)$$

$$P_{line} = \frac{V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2)}{Z^2} R \quad (11)$$

$$Q_{line} = \frac{V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2)}{Z^2} \omega L \quad (12)$$

$$Z_1 = \frac{V_1 < \theta_1}{I} \\ = Z_{line}\frac{[V_1^2 - V_1V_2\cos(\theta_1-\theta_2)] - jV_1V_2\sin(\theta_1-\theta_2)}{V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2)} \quad (13)$$

$$Z_2 = \frac{V_2 < \theta_2}{I} \\ = Z_{line}\frac{[V_1V_2\cos(\theta_1-\theta_2) - V_2^2] - jV_1V_2\sin(\theta_1-\theta_2)}{V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2)} \quad (14)$$

$$Z_{line} = Z_1 - Z_2 = Z_{line} \quad (15)$$

Furthermore, the following inequalities must always be valid:

$$\text{a) } P_1 > 0 \rightarrow [R[V_1^2 - V_1V_2\cos(\theta_1-\theta_2)] + \\ \omega L V_1V_2\sin(\theta_1-\theta_2)] > 0 \rightarrow \\ \rightarrow \frac{\cos(\theta_1-\theta_2+\varphi_L)}{\cos\varphi_L} < \frac{V_1}{V_2} \quad (16)$$

$$\text{b) } P_2 > 0 \rightarrow [R[V_1V_2\cos(\theta_1-\theta_2) - V_2^2] + \\ \omega L V_1V_2\sin(\theta_1-\theta_2)] > 0 \rightarrow \\ \rightarrow \frac{\cos(\theta_1-\theta_2-\varphi_L)}{\cos\varphi_L} > \frac{V_2}{V_1} \quad (17)$$

$$\text{c) } P_{line} > 0 \rightarrow V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2) > 0 \rightarrow \\ \rightarrow V_1^2 + V_2^2 > 2V_1V_2\cos(\theta_1-\theta_2)\ ? \quad (18)$$

$$\text{d) } Q_{line} > 0 \rightarrow V_1^2 + V_2^2 - 2V_1V_2\cos(\theta_1-\theta_2) > 0 \rightarrow \\ \rightarrow V_1^2 + V_2^2 > 2V_1V_2\cos(\theta_1-\theta_2)\ ? \quad (19)$$

The above (c) and (d) are always true due to Eq. (20).

$$(V_1 - V_2)^2 \geq 0 \rightarrow V_1^2 + V_2^2 - 2V_1V_2 \geq 0 \rightarrow \\ \rightarrow V_1^2 + V_2^2 \geq 2V_1V_2 \geq \\ \geq 2V_1V_2\cos(\theta_1-\theta_2) \rightarrow \\ \rightarrow V_1^2 + V_2^2 \geq 2V_1V_2\cos(\theta_1-\theta_2) \quad (20)$$

In addition, the following inequalities are always valid when the character of the respective load is ohmic-inductive:

$$\text{a) } Q_1 > 0 \rightarrow \Big[\omega L[V_1^2 - V_1V_2\cos(\theta_1-\theta_2)] - \\ -RV_1V_2\sin(\theta_1-\theta_2)\Big] > 0 \rightarrow \\ \rightarrow \frac{\sin(\theta_1-\theta_2+\varphi_L)}{\cos\varphi_L} < \frac{V_1}{V_2}\,\frac{\omega L}{R} \quad (21)$$

$$\text{b) } Q_2 > 0 \rightarrow [\omega L[V_1V_2\cos(\theta_1-\theta_2) - V_2^2] - \\ -RV_1V_2\sin(\theta_1-\theta_2)] > 0 \rightarrow \\ \rightarrow \frac{\sin(\theta_1-\theta_2-\varphi_L)}{\cos\varphi_L} < \frac{V_2}{V_1}\left(-\frac{\omega L}{R}\right) \quad (22)$$

If the opposite inequalities are true, the character of the respective load is ohmic-capacitive. If the equality is true, the character of the respective load is pure ohmic.

4. EXPERIMENTAL TESTING OF THE EQUATIONS

Using a laboratory model of the electric line, the following measurements were taken:

Z_{line}=400<90° Ω
V_1=142.4<22°V
V_2=132<0°V
I=0.133<0°A
P_1=18W
Q_1=6.5Var
P_2=18W
Q_2=0Var

Then, calculating the following equations, we find:

Eq. (1): I=0.133<-0.033°A
Eq. (3): P_1=17.6W
Eq. (4): Q_1=7.12Var
Eq. (6): P_2=17.6W
Eq. (7): Q_2=0.01Var
Eq. (11): $P_{line}=P_1 - P_2 = 0$
Eq. (12): Q_{line}=7.11Var
Eq. (16): -∞<1.0788
Eq. (17): +∞>0.927
Eq. (18): 37701.76>34856.18
Eq. (19): 37701.76>34856.18
Eq. (21): 0.927=0.927
Eq. (22): 0.927=0.927

The value of Eq. (1) is very close to that of the experimental result. The values of Eqs. (3) and (6) are the same and very close to that of the experimental result implying that the line has no resistance. Their positive value is verified by inequalities (16) and (17). The values of Eqs. (4) and (7) are very to those of experimental results and indicate the inductive character of the line. Their positive value is verified by Eqs. (21) and (22) that also prove the ohmic character of the load. The value of Eq. (11) is the same to that of the experimental result indicating a line without resistance and is verified by inequality (18). The value of Eq. (12) is almost the same to that of the experimental result indicating a line with inductance and is verified by inequality (19). Any small differences are due to the rounding of numbers, the precision of the instruments and the fact that the meters measuring power and phase are not digital.

5. DISCUSSION AND CONCLUSIONS

If you compare the analysis and findings in sections 2, 3 and 4, the following can be drawn:

1) the P_1 and P_2 are positive as expected,

2) the P_{line} was proved to be always positive and equal to P_1 minus P_2 as stated,

3) the Q_{line} proved to be positive and equal to Q_1 minus Q_2 as suggested,

4) it was proved that the Z_2 plus the Z_{line} is equal to the Z_1,

5) a criterion, inequality (22), was developed and tested successfully to indicate the kind of the load,

6) the inequalities (16) and (17) regarding that active power P_1 and P_2 are always positive were developed and tested successfully,

7) the inequalities (21) and (22) regarding reactive power Q_1 and Q_2 were developed and tested successfully.

At the end, the experimental values come to verify the theoretical findings.

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NOMENCLATURE

$\mathbf{Z_{line}}=R_{line}+j\omega L_{line}=Z<\varphi_L$

For all other symbols see [1].