Thermal Conductivity Measurements of Liquids with Transient Hot-Bridge Method

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1. INTRODUCTION

Thermal conductivity of heat transfer fluids plays a vital role in the development of high performance heat-exchange devices. In energy applications such as automobile engine and electronic chips, basic fluids like water and ethylene glycol are used for cooling. For these cooling fluids, the thermal conductivity is a very important property which is taken into account in designing and controlling the process.

Generally, measurements of the thermophysical properties of liquids are quite difficult and present some problems. These problems are related to the coupling of conduction with convective and radiative heat transfers.

A comprehensive review of the main methods used for the measurement of the thermal conductivity of fluids is given in [1]. These measured methods can be classified into two types [2]. The first one, named the steady-state techniques, are well developed, but are expensive in equipment and are practically difficult in operation. The second type is named the unsteady-state methods which can be divided in two kinds electrothermal and photothermal techniques. Only few works deal with the use of photothermal methods for characterization of liquids. Schriempf was the first to develop an apparatus dedicated to measurement on liquids [3]. The inconvenience of its method is that it is not adaptable for low thermal conductivity. Farooq et al. [4] proposed a similar approach based on a three-layered cell using an original sample holder made of outside layers brazed to a ring-shaped central spacer. Remy and Degiovanni [5–6] show that liquids present some difficulties since several mode of heat transfer are coupled. In their works, a theoretical study and an experimental approach based on the flash method are developed in order to determine the thermal parameters of liquids.

The Flash method is also adapted by Coquard and Panel [7] for the thermal characterization of liquids and pasty materials. They give a new technique for the computation of the thermal conductivity based on a 3-D simulation of the transient heat transfer in the axisymmetric composite sample.

Among, the unsteady electrothermal methods, the transient hot wire (THW) becomes one of commonly used techniques for the measurement of the thermal conductivity of liquids [8, 17]. In its standard configuration, a thin metal wire introduced in a liquid sample, acts as a resistive heat source and a thermometer. The voltage drop across the wire provides a measure of the thermal conductivity. For this method, to have a significant value of resistance, the hot wire should have a very thin diameter, which is sometimes difficult experimentally.

All the mentioned techniques present some inconvenient even in the experimental set-up or in the theoretical formulation. The intension of this paper is to overcome these problems and to present a method with quick experimental runs. This technique has a robust design suitable for many industrial applications, giving that it doesn’t require specific fluid enclosure and have easy experimental procedure. This method is based on the transient hot-bridge (THB) [18–20], which has previously shown its efficiency with the classical identification procedure based on slope calculation for determining thermal conductivity of solids. In this paper, the THB is adapted to liquids with a new identification procedure, based on the instantaneous slope versus time. It is based on a thermo-electrical sensor composed of four strips; each one is segmented into two parts of different lengths and connected to form a Wheatstone bridge. The bridge is inherently balanced. A constant current is passed through the bridge to heat the resistances, thus resulting in unbalancing of the bridge due to the resistance change.

Different liquid is characterized by this method. The thermal conductivity is measured with a good accuracy. These obtained results are compared with literature values and a good agreement is obtained.

This procedure could find applications for the measurements of numerous materials such as biological tissues or food materials for which other classical measuring
methods are not suitable.

The remainder of this paper is organized as follows: Section 1 introduces the theoretical formulation. Section 2 describes the experimental set-up and the measurement process. In the section 3, we present the obtained results and we finish with a conclusion.

2. THEORETICAL FORMULATION

2.1 Analytical model

The sensor is constituted of four parallel tandem strips. Each tandem strip comes in two individual striplets, a short and a long one. Two of the tandems are placed very close to each other at the centre of the sensor and one tandem on either edge (Figure 1).

The eight striplets are symmetrically switched for an equal-resistance Wheatstone bridge. The electrical circuit is represented on Figure 2.

Replacing Eqns (2, 3) in the equation (1), we obtain:

\[
\Delta T = T_{\text{ins}} - T_{\text{out}}
\]  

where the temperature rises on the inner and outer strips are the sum of the temperatures rises seen from the strips themselves and from other strips with distance Di. In this case, we can write:

\[
T_{\text{ins}} = T(D_1) + T(D_2) + T(D_3) + T(D_4)
\]  

and

\[
T_{\text{out}} = T(D_1) + T(D_2) + T(D_3) + T(D_4)
\]  

The conductivity identification is based on the calculation of the output signal slope, which can be written as [19]:

\[
\Delta T = T(D_1) - T(D_2)
\]  

The conductivity is calculated from the slope of the output-signal U_B versus time in a logarithmic scale, ln(t). Another identification method, using the instantaneous slope m_i and time t_i values allows for sufficiently precise determination of the thermal conductivity.

In this case, the voltage drop of a long segment is reduced to that of a short. The output signal U_B of the sensor with supply current I_B is formed only from the middle segments of the four strips. So, the difference of the temperature rises between inner and outer strips is defined by:

\[
\Delta T = T_{\text{ins}} - T_{\text{out}}
\]  

where the temperature rises on the inner and outer strips are the sum of the temperatures rises seen from the strips themselves and from other strips with distance Di. In this case, we can write:

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\[
\frac{d}{d(t)} \Delta T(t) = \frac{q}{4\pi \lambda} \left[ Ei\left( -\frac{D^2}{4at} \right) - Ei\left( -\frac{D^2}{4at} \right) \right]
\]  

with \( m(t) = m_i \) is the adimentioned instantaneous slope.

Based on Eq. (7), the maximum slope is obtained for \( t = t_{\text{max}} \), so one can write:

\[
\frac{d}{d\left( \ln(t_{\text{max}}) \right)} \Delta T(t_{\text{max}}) = \frac{q}{4\pi \lambda} \times m_{\text{max}}
\]
The governing equation of the 2D axial symmetry unsteady heat conduction problem is given by [18]:

$$\nabla(\lambda_i \nabla T_i) = \rho_i C_{pi} \frac{\partial T_i}{\partial t}$$

(12)

where \(i\) is the material layer.

Coupled with initial and boundary conditions:

$$T(t = 0) = T_0$$

$$-\lambda \frac{\partial T}{\partial x} = h(T - T_0) \quad x = l, y \in [0,d], x \in [0,l], y = d$$

The problem is solved using a finite element method. A domain of interest is represented as an association of finite elements. Approximating functions in finite elements are determined in terms of nodal values. The continuous physical problem is then converted into a discretized finite element problem with unknown nodal values. In the case of a linear problem, a system of linear algebraic equations is obtained. The obtained linear system is solved with step time of \(\Delta t=10^{-3}\) s. An example of the obtained temperature distribution is represented on Figure 5.

![Figure 5. Distribution of temperature](image)

3. EXPERIMENTAL SET-UP

The experimental setup illustrated in Figure 7 is constituted by a cell filled with liquid. The sensor, placed in this cell, is connected with a current generator and an Agilent acquisition card. The recorded signal from the output of the Agilent card is transferred to a computer using the RS 232 serial interface. The sensor, constituted by eight resistances, has the following dimensions 120×60×0.055 mm³. All the resistances are...
symmetrically switched for an equal-resistance Wheatstone bridge. As long as the sample temperature is uniformed, the bridge is inherently balanced. A constant current is passed through the bridge to heat the resistances, thus resulting in unbalancing of the bridge due to the resistance change. The input $V_{in}$ and output $V_{out}$ voltages are measured using a computerized data acquisition system (Agilent card). The sensor system has been fabricated in such a way that the data acquisition system can be easily connected and disconnected for measurement. The bridge voltage output and the acquisition time are measured and stored simultaneously. Post-processing of the acquired data is then performed in order to calculate the resistance change, temperature change and then the thermal conductivity of the test fluid.

![Figure 7. Experimental setup](image)

4. RESULTS AND DISCUSSION

In order to investigate the performance of the presented setup, different liquids have been analysed. Measurements are performed on water, starch-water mixture (volume fraction of water 0.75), mineral oil, olive oil, engine oil, ethanol and ethylene glycol samples. The current values supplied to the sensor are about 200 mA and 150 mA relatively for a duration of 25s. The acquisition time step used in all experiments is 0.1s. At each temperature, five individual tests were carried out to check the repeatability of the measurement results.

Figure 8 shows an example of the output signal of the sensor. This figure illustrates the variation of tension versus acquisition time. In order to calculate the instantaneous slope, the sensor output signal is represented versus time logarithm (Figure 9).

For each material, we made a series of measurements of the thermal conductivity and studied the distribution of the measured values. The thermal conductivity is calculated from the maximum slope using Eq. (11). Then, mean value is calculated and the obtained results at 300 K are presented in Table (1). The relative uncertainty is calculated using the same equation.

From Table 1, one can see that the thermal conductivity is measured with high accuracy; the relative uncertainty is relatively less than 4% expect for water, it is about 5.6%. The comparison of these relative uncertainties allows us to confirm the fact that the measurement of the thermal conductivity for water is slightly less precise than for other liquids. This is due to the presence of air bubbles in water which make some problems essentially for the measurement with the sensor. These air bubbles may cause errors in the measurement of the thermal parameters as the thermal heat transfer is reduced locally into the fluids.

![Figure 8. Sensor output signal versus time](image)

![Figure 9. Sensor output signal versus time logarithm](image)

<table>
<thead>
<tr>
<th>Sensor values</th>
<th>Relative uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.585</td>
</tr>
<tr>
<td>Starch-water mixture</td>
<td>0.528</td>
</tr>
<tr>
<td>Distilled water</td>
<td>0.566</td>
</tr>
<tr>
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<td>0.135</td>
</tr>
<tr>
<td>Engine oil</td>
<td>0.145</td>
</tr>
<tr>
<td>Olive oil</td>
<td>0.165</td>
</tr>
<tr>
<td>Ethanol</td>
<td>0.182</td>
</tr>
<tr>
<td>Ethylene glycol</td>
<td>0.250</td>
</tr>
</tbody>
</table>

Table 1. Illustration of the average thermal conductivities measured for different liquids

<table>
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<th>Literature values</th>
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Table 2. Comparison between sensor measurements and literature values

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5. INFLUENCE OF VARIABLE FLUID PROPERTIES AND THERMAL RADIATION

The ideal model assumes that the fluid properties, such as heat capacity, density, viscosity and thermal conductivity are constant. But in the real experimental process, they will change with the increase of the fluid temperature depending on the temperature coefficients of each property. This will influence the measurement results. The main problems come from the compressibility of fluids and the finite cell dimensions. In our case, the increase of temperature by the sensor is small, so we can neglect the corrections induced by properties variation.

When the temperature of the sensor increases, part of the thermal energy can be transferred into the fluid and its surroundings by thermal radiation, which reduces the energy transferred by heat conduction. In our case, the temperature rise is too small, so the effect of radiation can be unimportant.

We can conclude from the measurements presented above that our device proves to give coherent values of the thermal conductivity.

6. CONCLUSION

Throughout this paper, we have presented an electrothermal method for measuring the thermal conductivity of liquids. It is based on a Wheatstone bridge constituted by four tandem strips on foil sensor. Numerical model is compared to analytical one and the thermal conductivity is calculated using the instantaneous slope of the sensor output signal. The obtained results are estimated with small relative uncertainty and show a good agreement with literature data. Then, our method proves to be very practical for the measurement of thermal characteristics of materials used in various technological fields such as biology and nanofluids.

ACKNOWLEDGMENTS

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REFERENCES

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**NOMENCLATURE**

- \( a \) thermal diffusivity (m\(^2\)s\(^{-1}\))
- \( c \) heat capacity (J kg\(^{-1}\) K\(^{-1}\))
- \( D_i \) distance between sensor strips (m)
- \( \text{Nu} \) local Nusselt number along the heat source
- \( I_B \) sensor current (A)
- \( L_{eff} \) effective strip length (m)
- \( L_L \) length of the long sensor resistance (m)
- \( L_s \) length of the short sensor resistance (m)
- \( m_i \) reduced instantaneous slope
- \( q \) linear heat flux density (W m\(^{-1}\))
- \( \text{Reff} \) effective sensor electrical resistance (Ω)
- \( R_i \) electrical resistance of a striplet (Ω)
- \( T \) temperature (K)
- \( t \) time (s)
- \( T_a \) ambient temperature (K)
- \( U_B \) sensor output signal (V)

**Greek symbols**

- \( \rho \) density (kg m\(^{-3}\))
- \( \lambda \) thermal conductivity (W m\(^{-1}\)K\(^{-1}\))