Application of super-modular game model on quality and safety management of supply chain based on process control

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ABSTRACT. *None of the existing modelling methods on product quality and safety management takes account of process control. To make up for this gap, this paper investigates the coordination contract and optimal decisions of supplier and manufacturer. Specifically, a supply chain of one supplier and one manufacturer was created, with the supplier producing the key components of the final products and deciding product quality and safety. On this basis, a model of quality and safety management was constructed based on the theory of static game and the super-modular game, the model of quality management is construct. Then, the supplier's and retailer's decisions were analyzed separately under global decision model and local decision model. The research results show that measures like technology innovation, knowledge sharing and well-established legal system promote the stability and development of the supply chain; the linear-cost sharing contract designed by local decision model can only achieve suboptimal equilibrium, which contains at least one pure strategy Nash equilibrium, rather than obtain the global optimal solution; the proposed contract can enhance the product quality and reduce the quality cost of both parties, if there are multiple equilibria. The main contributions of this paper are as follows: proving the existence of pure strategy Nash equilibrium when the supplier and manufacturer are in a quality management game; determining the conditions for reaching Pareto optimality under multiple equilibria; setting up the quality management decision model, in which the supplier learns about quality improvement using manufacturer investment, considering the limited ability of the supplier to improve product quality; constructing the game model of quality management by intervening in the production process. The research findings provide useful reference for the quality and safety management of supply chain.*

RÉSUMÉ. *Aucune des méthodes de modélisation existantes sur la gestion de la qualité et de la sécurité des produits ne prend en compte le contrôle de processus. Pour combler cet écart, cet article examine le contrat de coordination et les décisions optimales du fournisseur et du fabricant. Spécifiquement, une chaîne d'approvisionnement comprenant un fournisseur et un fabricant a été créée, le fournisseur produisant les composants clés des produits finis et décidant de la qualité et de la sécurité du produit. Sur cette base, un modèle de gestion de la qualité et de la sécurité a été construit sur la base de la théorie du jeu statique et du jeu super-modulaire. Ensuite, les décisions du fournisseur et du détaillant ont été analysées séparément* *selon un modèle de décision globale et un modèle de décision locale. Les résultats de la recherche montrent que des mesures telles que l'innovation technologique, le partage des connaissances et un système juridique bien établi favorisent la stabilité et le développement de la chaîne d'approvisionnement; le contrat de partage des coûts linéaires conçu par un modèle de décision locale ne peut atteindre qu'un équilibre sous-optimal, qui contient au moins une stratégie pure d'équilibre de Nash, plutôt que d'obtenir la solution optimale globale; le contrat proposé peut améliorer la qualité du produit et réduire le coût de la qualité pour les deux parties en cas des équilibres multiples. Les principales contributions de cet article sont les suivantes: prouver l'existence d'une stratégie pure d'équilibre de Nash lorsque le fournisseur et le fabricant participent à un jeu de gestion de la qualité; déterminer les conditions pour atteindre l'optimalité de Pareto sous des équilibres multiples; réaliser la mise en place du modèle décisionnel de gestion de la qualité, dans lequel le fournisseur apprend l'amélioration de la qualité en utilisant les investissements du fabricant, en tenant compte de la capacité limitée du fournisseur à améliorer la qualité du produit; construire le modèle du jeu de gestion de la qualité en intervenant dans le processus de production. Les résultats de la recherche constituent une référence utile pour la gestion de la qualité et de la sécurité de la chaîne d'approvisionnement.*

KEYWORDS: *super-modular game, process control, product quality safety problems, supply chain management.*

MOTS-CLÉS: *jeu super-modulaire, contrôle de processus, problèmes de qualité et de sécurité des produits, gestion de la chaîne d'approvisionnement.*

DOI:10.3166/JESA.50.569-580

1. Introduction

Product quality safety problems is a major social problem currently. Product safety and quality issues, such as melamine-contaminated milk powder, poisoning nitrite, are occurred frequently. Under such circumstances, in 2016, "No.1 document" of central government of china, puts forward policies related to "food safety regulation", indicating that solving product safety problems have become the most important task of Chinese government (Xinhua Net, 2016). In fact, the safe products are produced step by step, ignoring the quality control in the key components product of upstream process may negatively affect the prevention of quality and safety incidents. Quality management in key components production process is the main point to ensure the quality and safety of the final products. In recent years, experts and scholars mainly focused on the quality and safety risk assessment (Dani, 2010), the sustainability of the product quality of the supply chain (Ting *et al*., 2014), the traceability of qualityand safety information (Cus-Babic *et al*., 2014; Dabbene *et al*., 2014), the quality assurance system (Chiu *et al*., 2015), etc. The above researches have certain reference value for solving the quality management problem of the supply chain.

However, it is still necessary to analyze the quality and safety of supply chain through operational modeling. At present, representative researches on the quality management of the supply chain in view of operational modeling are as follows: Reyniers *et al*. (1995) improved the quality of the final product to achieve a "win-win" situation through contracts, which can pick up suppliers with high quality; Chao *et al*. (2009) adopting accountability theory, designed two kinds of cost sharing contracts according to the proportion of effort and the corresponding responsibility of the node enterprises in production. The two contracts can both achieve the optimal system performance and the best quality; The above research findings have provided a feasible solution to solve the quality and safety problems of product, but lack of research from the source of production process to manage the product quality. Zhu *et al*. (2007) have studied the problem which the product manufacturers outsource product production to suppliers and participate in the quality improvement process, the result shows that the manufacturers' participation in the production process can significantly improve the performance of supply chain and product quality. However, this paper has not yet considered the realistic problems that the supplier's capacity of carry outing quality prevention and the quality improvement is limited.

Based on the above literature review, this paper takes the production process control as the research point, applies the theory of the super-modular game as the basic method of modeling and solving to construct the game model of quality management. Compared with the existing literature research, this paper has the following three characteristics: (1) applying the theory of super-modular game to prove the existence of Pure Strategy Nash equilibrium when manufacture and supplier are in quality management game, and the conditions which can achieve the Pareto Optimality upon the existence of multiple equilibria are given. (2) Have considered the limited ability of supplier to improve the quality level of products, and studied the quality management decision model that manufacture invest to help supplier and supplier take effort to learn; (3) from the production process to intervene, and on this basis construct the game model of quality management.

2. Problem description and model assumptions

The research object is a two-level product supply chain system consisting of a manufacture (M) and a supplier (S), in which the supplier supplies the key component of the product, which decides final products quality level. To improve the quality of products, the manufacture will invest certain information, technology and management level. $I, I \in [0,1]$ is used to characterize the investment level. Beside in charge of production, supplier will meanwhile work hard to learn the invested information, technology, management and other aspects of knowledge and materials. This paper uses $L, L \in [0,1]$ to characterize the learning level. Under the quality management decision mode, both sides have two kinds of decision-making behavior: participation and non-participation. If one party chooses non-participation strategy, they will lose the chance to participate in quality management decision mode forever. At the same time, the chance of sanction and retaliation to non-cooperator in bilateral multiple gaming will be greater than that in single transaction. Therefore, this paper will study the quality management game model of manufacture and supplier under multiple cycles. To further conduct quantitative analysis, the assumptions are made as below.

Assumption 1: In the whole production process, the cost invested by the manufacture and the supplier are respectively $C_M(I)$ and $C_S(L)$. $C_M(I)$, $C_S(L) \in$

$[0,+\infty]$; $C_M(0) = C_S(0) = 0$ means no any additional costs when the two sides fail to make efforts; $\lim_{I\to 1} C_M(I) = \lim_{L\to 1} C_S(L) = +\infty$ means paying a huge cost if any party makes efforts and the degree gets close to 1. $\frac{dC_S(L)}{dL} > 0, \frac{dC_M(I)}{dI} > 0$, indicates that the cost input will increase with the increasing efforts (input and learning) of the manufacture and the supplier; $\frac{d^2C_S(L)}{dL^2} > 0, \frac{d^2C_M(I)}{dI^2} > 0$, indicates that the cost of each side increases rapidly with the increasing efforts of each side.

Assumption 2: After the end of production, the fraction defective of products is $P(I,L) \in [0,1]$, $P(I,L)$ is continuous and differentiable in the interval $[0,1] \times [0,1]$. In addition, $\frac{\partial P(I,L)}{\partial I} < 0, \frac{\partial P(I,L)}{\partial L} < 0$, this indicates that with the increase of efforts (input, learning) level from each side, it will improve the quality of products; $\frac{\partial^2 P(I,L)}{\partial I^2} > 0, \frac{\partial^2 P(I,L)}{\partial L^2} > 0$, this indicates that with the increase of efforts (input, learning) level from each side, the speed of improving products quality will slow down; $\frac{\partial^2 P(I,L)}{\partial I \partial L} > 0$, this indicates that the efforts level manufacture and supplier choose to pay (input, learning) are complementary, i.e., if manufacture (supplier) increase investment (learning) level, then supplier (manufacture) will pay a higher level of learning (input) and vice versa.

Assumption 3: To reduce the check cost of products, manufacture will outsource this item to the third-party checking manufacture. It is assumed that the problem (unqualified) products can be completely detected. The unit sampling probability is $\bar{P} = \bar{P}\big(P(L,I)\big)$, $\frac{d\bar{P}}{dp} > 0, \frac{d^2\bar{P}}{dp^2} = 0$, which indicates that the probability of sampling inspection will decline with the rising of product quality, and there is a linear relationship between them.

The description of other related symbols:

C_S: The production cost of unit component of supplier;

C_M: The total purchase and produce cost of unit product of manufacture;

c_1: The loss cost of unqualified unit product;

c_2: The test cost of unit product;

δ_1: The discount factor of supplier;

δ_2: The discount factor of manufacture;

M_S: The expected income of gaining competitive advantage due to higher quality of products after supplier participation in the decision model of quality management.

M_M: The expected income generated by gaining the competitive advantage and reputation due to higher quality of products after manufacture participation in the decision model of quality management.

N_S: The expected risk cost that supplier undertake when they do not participate in quality management decision model.

N_M : The expected risk cost that manufacture undertake when they do not participate in quality management decision model.

3. Model analysis

3.1. Stability of the game model of the quality management

This paper studies the game of quality management between manufacture and supplier in different periods (the decisions of the two sides in n periods are shown in Figure 1). From T=0 to T=1, figure 1 depicts a completely production process, in which manufacture and supplier participate in.

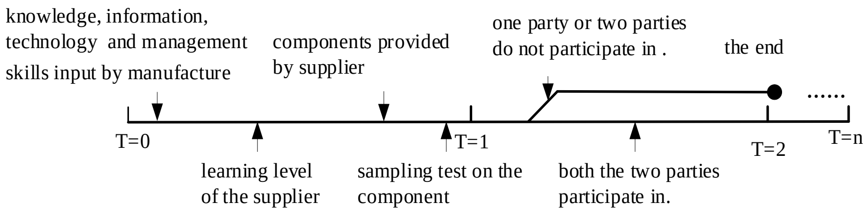

Figure 1. Decisions of the manufacture and the supplier under the decision-making model of quality management

In practice, when *T=1* manufacture and supplier decide their strategies based on the costs of participating and not participating in the quality management game. The cost when *T=1* as well as the long-term cost when *T=n* should be taken into consideration.

The expected costs of supplier participation in quality when $T = 2$ to $T = n$ is $C_S^L = \sum_{i=2}^{n}(c_S + c_1 P(L, I) + C_S(L) - M_S){\delta_1}^{i-1}$, L is the effort of supplier S to learn the information and technology of the manufacture when taking part in the quality management game.

The expected cost when the supplier does not participate in quality management game from period *2* to period n is $C_S^{NL} = \sum_{i=2}^{n}(c_S + c_1 P(NL, NI) + N_S){\delta_1}^{i-1}$. *NL* means the supplier does not participate in the game of quality management.

The expected cost of participating in the quality management game from $T = 2$ to $T = n$ is $C_M^I = \sum_{i=2}^{n}(c_M + c_2 \bar{P}(P(L, I)) + C_M(I) - M_M){\delta_2}^{i-1}$. I represents the input level of efforts of manufacture when taking part in the quality management game. It strives to deliver information, technology and other knowledge to supplier.

From $T = 2$ to $T = n$, the expected cost of not participating in the game of quality management is $C_M^{NI} = \sum_{i=2}^{n}\left(c_M + c_2 \bar{P}\left(P(NL, NI)\right) + N_M\right){\delta_2}^{i-1}$, in which *NI* indicates that the manufacture does not participate in the game of quality management.

Above all, a game model of static quality management of manufacture and supplier in multi-period ($n > 2$) can be constructed, as shown in the following table 1.

Table 1. Static quality management game model of manufacture and supplier when considering long-term cost

		Manufacture M	
		I	NI
Supplier S	L	$c_S + c_1P(L,I) + C_S(L) - M_S + C_S^L$ $c_M + c_2\bar{P}\big(P(L,I)\big) + C_M(I) - M_M + C_M^I$	$c_S + c_1P(L,NI) + C_S(L) + C_S^L$ $c_M + c_2\bar{P}\big(P(L,NI)\big) + N_M + C_M^{NI}$
	NL	$c_S + c_1P(NL,I) + N_S + C_S^{NL}$ $c_M + c_2\bar{P}\big(P(NL,I)\big) + C_M(I) + C_M^I$	$c_S + c_1P(NL,NI) + N_S + C_S^{NL}$ $c_M + c_2\bar{P}\big(P(NL,NI)\big) + N_M + C_M^{NI}$

To analyze the equilibrium solution of the game, the total cost function of the manufacture and the supplier under different strategy combinations are assumed to be $C_S(i,j), C_M(i,j)$, in which $i \in \{L, NL\}, j \in \{I, NI\}$.

The preference value of supplier' long-term participation in the game of quality management is

$$\omega_S = C_S^{NL} - C_S^L = \sum_{i=2}^{n} \left[\begin{array}{l} c_1\big(P(NL,NI) - P(L,I)\big) \\ +M_S + N_S - C_S(L) \end{array}\right] \delta_1^{i-1} \qquad (1)$$

The preference value of manufacture' long-term participation in the game of quality management is

$$\omega_M = C_M^{NI} - C_M^I = \sum_{i=2}^{n} \left[\begin{array}{l} c_2\left(\bar{P}\big(P(NL,NI)\big) - \bar{P}\big(P(L,I)\big)\right) \\ +M_M + N_M - C_M(I) \end{array}\right] \delta_2^{i-1} \qquad (2)$$

The minimum cost of unqualified products saved by the supplier's study is:

$$\Delta C_S^* = min \left\{\begin{array}{l} c_1\big(P(NL,I) - P(L,I)\big), \\ c_2\big(P(NL,NI) - P(L,NI)\big) \end{array}\right\} \qquad (3)$$

The minimum cost of testing products saved by the manufacture's investment in helping supplier to learn is:

$$\Delta C_M^* = min \left\{\begin{array}{l} c_2\left(\bar{P}\big(P(L,NI)\big) - \bar{P}\big(P(L,I)\big)\right), \\ c_2\left(\bar{P}\big(P(NL,NI)\big) - \bar{P}\big(P(NL,I)\big)\right) \end{array}\right\} \qquad (4)$$

Preposition 1: When $\omega_S + N_S + \Delta C_S^* > C_S(L)$, $\omega_M + N_M + \Delta C_M^* > C_M(I)$, the supplier and the manufacture will choose to take part in the quality management game, which means there exits dominant strategy equilibrium (L, I) in quality management game..

Proof 1: The selection of supplier' participation in quality management game should be first considered. According to Table 1, formula (1) and (3):

$$C_S(NL,I) - C_S(L,I) = c_1\big(P(NL,I) - P(L,I)\big) - C_S(L) + M_S + N_S + \omega_S$$

$$\geq \Delta C_S^* + M_S + N_S - C_S(L) + \omega_S \tag{5}$$

$$C_S(NL,NI) - C_S(L,NI)$$
$$= c_1\big(P(NL,NI) - P(L,NI)\big) - C_S(L) + N_S + \omega_S$$
$$\geq \Delta C_S^* + N_S + \omega_S - C_S(L) \tag{6}$$

When $\omega_S + N_S + \Delta C_S^* > C_S(L)$, it can be inferred from formula (5) and (6) that $C_S(NL,j) - C_S(L,j) > 0, j \in \{I, NI\}$. The cost of the supplier who make effort to learn the knowledge provided by the manufacture is lower than those who do not. Therefore, striving to learn is a dominant strategy for supplier. Considering the selection of the manufacture's participation in the quality management game, according table 1 and formula (2) and (4), it can be inferred that

$$C_M(L,NI) - C_M(L,I)$$
$$= c_2\left(\bar{P}\big(P(L,NI)\big) - \bar{P}\big(P(L,I)\big)\right) - C_M(I) + M_M + N_M + \omega_M$$
$$\geq \Delta C_M^* + M_M + N_M + \omega_M - C_M(I) \tag{7}$$

$$C_M(NL,NI) - C_M(NL,I)$$
$$= c_2\left(\bar{P}\big(P(NL,NI)\big) - \bar{P}\big(P(NL,I)\big)\right) + N_M + \omega_M - C_M(I)$$
$$\geq \Delta C_M^* + N_M + \omega_M - C_M(I) \tag{8}$$

When $\omega_C + N_C + \Delta C_C^* > C_C(I)$, it can be inferred from formula (7) and (8) that $C_C(i,NI) - C_C(i,I) > 0, j \in \{L, NL\}$, indicating that investing supplier is the dominant strategy for the manufacture when participating in the quality management game.

Above all, when $\omega_S + N_S + \Delta C_S^* > C_S(L)$, $\omega_M + N_M + \Delta C_M^* > C_M(I)$, (L, I) is the equilibrium solution of dominant strategy of $G = \big\{\{L, NL\}, \{I, NI\}; C_S, C_M\big\}$,namely, (participate, participate), which is the equilibrium solution. Till now, the proof is completed.

When the above conditions are satisfied, manufacture and supplier are motivated to actively participate in the game of quality management. When the conditions cannot be satisfied, if the nodes in the supply chain of products compete for horizontal competitive advantage, they will also participate in the game of quality management.

3.2. Optimal decision-making of game model of quality management

3.2.1. Optimal decision analysis in global decision model

The global decision is designed to study the overall performance of the supply chain system. The manufacture and the supplier make joint decisions, namely, L and I, to minimize the total cost of the system. The total cost of the supply chain system is as follows:

$$\underset{L,I}{Min}\Pi_{SC} = c_S + c_M + c_1 P(L,I) + c_2 \bar{P}\big(P(L,I)\big) + C_S(L) + C_M(I) - M_S - M_M.$$

Proposition 2: In the global decision-making model, for any$(L, I) \in [0,1] \times [0,1]$, the only optimal solution.(L_{SC}^*, I_{SC}^*) of the supply chain system which consist of a manufacture and a supplier satisfy the following first order conditions:

$$\frac{\partial \Pi_{SC}}{\partial L} = c_1 \frac{\partial P(L_{SC}^*, I_{SC}^*)}{\partial L} + \frac{dC_S(L_{SC}^*)}{dL} + c_2 \frac{\partial \bar{P}\big(P(L_{SC}^*, I_{SC}^*)\big)}{\partial P(L,I)} \frac{\partial P(L_{SC}^*, I_{SC}^*)}{\partial L} = 0 \tag{9}$$

$$\frac{\partial \Pi_{SC}}{\partial I} = c_1 \frac{\partial P(L_{SC}^*, I_{SC}^*)}{\partial I} + \frac{dC_M(I_{SC}^*)}{dI} + c_2 \frac{\partial \bar{P}\big(P(L_{SC}^*, I_{SC}^*)\big)}{\partial P(L,I)} \frac{\partial P(L_{SC}^*, I_{SC}^*)}{\partial I} = 0 \tag{10}$$

Proof 2: To ensure the existence and uniqueness of product supply chain system in relating to L and I, we only need to verify the total cost function of supply chain system is the convex function on the input level L and I, that is, the second order partial derivatives are non-negative. The first step is to analyze the concavity of the cost function of the supply chain system Π_{SC}, to solve the second order partial derivative of Π_{SC} on the input level I, we can obtain that.

$$\frac{\partial^2 \Pi_{SC}}{\partial I^2} = c_1 \frac{\partial^2 P(L,I)}{\partial I^2} + c_2 \frac{\partial \bar{P}\big(P(L,I)\big)}{\partial P(L,I)} \frac{\partial^2 P(L,I)}{\partial I^2} + c_2 \frac{\partial^2 \bar{P}\big(P(L,I)\big)}{\partial P^2(L,I)} \left(\frac{\partial P(L,I)}{\partial I}\right)^2 + \frac{d^2 C_M(I)}{dI^2} \tag{11}$$

According to the assumption about the model, we can see that:

$$\frac{\partial^2 P(L,I)}{\partial I^2} > 0, \frac{\partial \bar{P}\big(P(L,I)\big)}{\partial P(L,I)} > 0, \frac{\partial^2 P(L,I)}{\partial I^2} > 0, \frac{d^2 C_M(I)}{dI^2} > 0$$

$$d^2 \bar{P}\big(P(L,I)\big)/dP^2(L,I) = 0 \tag{12}$$

Since $c_1 > 0, c_2 > 0$,it is easy to prove that $\frac{\partial \Pi_{SC}}{\partial I^2} > 0$,which means Π_{SC} is the convex function of I According to the properties of the convex function, there exits an optimal and unique input level Π_{SC}^* for Π_{SC}, which satisfies the following first order optimal conditional formula (12), Similar to the solution of the optimal input level Π_{SC}^*,now we prove that there exist an optimal and unique learning level L in the product supply chain system, we just need to verify the Π_{SC}^* is the convex function on L. The second order partial derivative of L is as follows:

$$\frac{\partial^2 \Pi_{SC}}{\partial L^2} = c_1 \frac{\partial^2 P(L,I)}{\partial L^2} + c_2 \frac{\partial \bar{P}\big(P(L,I)\big)}{\partial P(L,I)} \frac{\partial^2 P(L,I)}{\partial L^2} + c_2 \frac{\partial^2 \bar{P}\big(P(L,I)\big)}{\partial P^2(L,I)} \left(\frac{\partial P(L,I)}{\partial L}\right)^2 + \frac{d^2 C_S(L)}{dL^2} \tag{13}$$

According to the assumptions of the model, we can see that:

$$\frac{\partial^2 P(L,I)}{\partial L^2} > 0, \frac{\partial \bar{P}(P(L,I))}{\partial P(L,I)} > 0,$$

$$\frac{\partial^2 P(L,I)}{\partial L^2} > 0, \frac{d^2 \bar{P}(P(L,I))}{dP^2(L,I)} = 0,$$

$$d^2 C_S(L)/dL^2 > 0 \tag{14}$$

Since $c_1 > 0, c_2 > 0$,it is easy to prove that$\frac{\partial \Pi_{SC}}{\partial L^2} > 0$,which means Π_{SC} is the convex function of L. According to the properties of the convex function, there exits an optimal and unique input level Π_{SC}^* for Π_{SC}, which satisfies the following first order optimal conditional formula (14). Till now, the proof is completed.

According to Proposition 2, under the global decision model, the supply chain system has the optimal and unique input level (L_{SC}^*, I_{SC}^*), and at the same time, the optimal product quality level (qualified rate) $1 - P(L_{SC}^*, I_{SC}^*)$ and the lowest total cost of the supply chain system $\Pi_C(L_{SC}^*, I_{SC}^*)$ are obtained. The optimal input level under global decision-making is the optimal solution to minimize the cost of the supply chain of products and to optimize the solution of the product quality (trade-off) problem.

3.2.2. Optimal decision analysis in local decision model

In the local decision-making model, the manufacture and the supplier choose their own optimal decision to minimize the cost. Substandard products are likely to harm the reputation, reduce the market share, and make the manufacture and the supplier be confronted with severely punishment from the government. Due to the limited ability of the supplier to improve the quality of products, the two parties have intrinsic motivation to further improve the quality of products under the local decision model. To share the handling cost and the testing cost brought by unqualified products can encourage a closer cooperation and improve the efficiency of the operation. Based on this, the linear cost-sharing contract is designed, under which the cost of the manufacture and the supplier are:

$$\underset{L}{Min}\Pi_S = c_S + \alpha c_1 P(L,I) + (1-\beta) c_2 \bar{P}(P(L,I)) + C_S(L) - M_S \tag{15}$$

$$\underset{I}{Min}\Pi_M = c_M + (1-\alpha) c_1 P(L,I) + \beta c_2 \bar{P}(P(L,I)) + C_M(I) - M_M \tag{16}$$

$\alpha, \alpha \epsilon [0,1]$ is the handling cost of unqualified products borne by supplier; $\beta, \beta \in [0,1]$ is the testing cost born by manufacture. When α, β is determined, the two parties choose their own input levels to achieve local optimization.

Preposition 3: The quality management game model under the local decision mode has the following properties:

i. at least one pure strategy Nash equilibrium exists.

ii. the optimal response function $L^*(I)(I^*(L))$ is an increasing function when $I\epsilon[0,1](L\epsilon[0,1])$;

iii. if there is a multiple pure strategy Nash equilibrium, then the equilibrium is ordered, that is, to any equilibrium$(\bar{L},\bar{I})$, $(\tilde{L},\tilde{I})$, $\tilde{L} \leq \bar{L}, \tilde{I} \leq I$

vi. if there is a multiple Nash equilibrium, then for any equilibrium$(\bar{L},\bar{I})$, $(\tilde{L},\tilde{I})$, and $\tilde{L} \geq \bar{L}, \tilde{I} \geq I$, then $\Pi_S(\bar{L},\bar{I}) \leq \Pi_S(\tilde{L},\tilde{I})$, $\Pi_M(\bar{L},\bar{I}) \leq \Pi_M(\tilde{L},\tilde{I})$.

Proof 3: To prove the item (I) of the quality management game model and the item (II) in the local decision-making mode, we first analyze the properties of the cost function of the manufacture and the supplier under the linear cost sharing contract. According to (15), the mixed partial derivative of the cost function of L, I is as follows:

$$\frac{\partial^2 \Pi_S}{\partial L\,\partial I} = (1-\beta)c_2 \begin{bmatrix} \frac{\partial \bar{P}(P(L,I))}{\partial P(L,I)} \frac{\partial^2 P(L,I)}{\partial L\,\partial I} + \\ \frac{\partial^2 \bar{P}(P(L,I))}{\partial P^2(L,I)} \frac{\partial P(L,I)}{\partial L} \frac{\partial P(L,I)}{\partial I} \end{bmatrix} + \alpha c_1 \frac{\partial^2 P(L,I)}{\partial L\,\partial I} \tag{17}$$

When the parameter α and β are determined, according to model assumptions:

$\frac{\partial P(L,I)}{\partial L} < 0, \frac{\partial P(L,I)}{\partial I} < 0, \frac{d\bar{P}}{dP} > 0,$

$\frac{d^2\bar{P}}{dP^2} = 0, \frac{\partial^2 P(L,I)}{\partial L \partial I} > 0, \frac{d\bar{P}}{dP} > 0,$

$d^2\bar{P}/dP^2 = 0.$

Then, $\frac{\partial^2 \Pi_S(I,L)}{\partial I \partial L} > 0$. Similar to the analysis of the mixed partial derivative property of the cost function of supplier, it can be obtained from formula (17), the mixed partial derivative of decision variables L, I of the cost function of the manufacture is:

$$\frac{\partial^2 \Pi_M}{\partial I \partial L} = (1-\beta)c_2 \begin{bmatrix} \frac{\partial \bar{P}(P(L,I))}{\partial P(L,I)} \frac{\partial^2 P(L,I)}{\partial I \partial L} \\ + \frac{\partial^2 \bar{P}(P(L,I))}{\partial P^2(L,I)} \frac{\partial P(L,I)}{\partial I} \frac{\partial P(L,I)}{\partial L} \end{bmatrix} + \alpha c_1 \frac{\partial^2 P(L,I)}{\partial I \partial L}.$$

Therefore, $\frac{\partial^2 \Pi_M(I,L)}{\partial I \partial L} \geq 0$. According to, $\frac{\partial^2 \Pi_M(I,L)}{\partial I \partial L} \geq 0$ $\frac{\partial^2 \Pi_S(I,L)}{\partial I \partial L} \geq 0$, The cost function of the manufacture and the supplier has the supermodel property (i.e. the supermodel function) on the grid $[0,1] \times [0,1]$. Therefore, the game of quality management is a supermodel game. According to the supermodel game theorem (Vives, 1999), property (I) and the property (II) are proved.

(III) When there exists a multiple pure strategy Nash equilibrium, suppose $(\bar{L},\bar{I})$ and $(\tilde{L},\tilde{I})$ are two Nash equilibrium of quality control game. Without loss of generality, when we assume that $\tilde{L} \leq \bar{L}$ according to item (II),

the optimal reaction function of the company is a monotone increasing function on the learning level of the farmer, therefore, $\tilde{I} < \tilde{I}^*(\tilde{L}) \leq \tilde{I}^*(\bar{L}) = \bar{I}$. In addition, if assuming$\tilde{L} \leq \bar{L}$, $\tilde{I} = \tilde{I}^*(\tilde{L}) \geq \tilde{I}^*(\bar{L}) = \bar{I}$ can be obtained in the similar way.

(IV) Supposing that $(\tilde{L},\tilde{I}),(\bar{L},\bar{I})$ and are a pair of pure strategy Nash equilibria and $\tilde{L}<\bar{L},\tilde{I}<\bar{I}$.According to model assumptions, we can see $\frac{\partial P(I,L)}{\partial I}<0,\frac{d\bar{P}}{dP}>0$, therefore:

$$\frac{\partial \Pi_F}{\partial I}=\alpha c_1\frac{\partial P(L,I)}{\partial I}+(1-\beta)c_2\frac{\partial \bar{P}\big(P(L,I)\big)}{\partial P(L,I)}\frac{\partial P(L,I)}{\partial I}<0.$$

According to $L^*(\tilde{I})=\tilde{L},\Pi_F(\bar{L},\bar{I})\le\Pi_F(\bar{L},\tilde{I})\le\Pi_F(L^*(\tilde{I}),\tilde{I})=\Pi_F(\tilde{L},\tilde{I})$. Similarly, the monotonicity of the equilibrium of the cost function of the company is $\Pi_C(\bar{L},\bar{I})\le\Pi_C(\bar{L},\tilde{I})\le\Pi_C(L^*(\tilde{I}),\tilde{I})=\Pi_C(\tilde{L},\tilde{I})$. Till now, the proof is completed.

According to proposition 3, the cost functions of the manufacture and the supplier are supermodel functions, which can ensure that at least one pure strategy Nash equilibrium is existed, but it does not exclude the existence of multiple Nash equilibria. If the multiple Nash equilibrium is existed, the equilibrium is ordered. the manufacture and the supplier are more inclined to choose higher level of Nash equilibrium. Therefore, if the multiple Nash equilibrium is existed, the Pareto optimal decision (L^*,I^*) and Pareto optimal quality $1-P\big((L^*,I^*)\big)$ can be obtained.

4. Conclusions

The frequent occurrence of all kinds of quality and safety accidents of products inevitably causes people to rethink profoundly. The research shows that investing in knowledge sharing, perfection of the legal system, innovation of production technology, and the enhancement of the government's constraints will contribute to the stability of the quality management game. According to the supermodel game theory analysis, although the local optimal decision under the linear cost sharing contract cannot achieve the optimal global decision, but it can obtain suboptimal investment decision for the supply chain system. Moreover, if the multiple Nash equilibrium is existed, the Pareto optimal decision and the quality of products can be obtained.

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