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L_q Regularization for Sparse Control in Power Grids

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ABSTRACT

Many problems of recent interest in the design of sparse control for power grids can be posed in the structure optimization. Different performance indices and sparsity promoting penalty functions have been proposed for different control goals and structural constrains. We study the application of L_q norm as penalty function for sparse control optimization for bulk power networks. Our emphasis focus on identifying the sparsity patterns of feedback matrix under different value of q in L_q norm, by incorporating our design into the optimization framework of the alternating direction method of multipliers, which is well developed in recent researches on sparse control for power networks. The advantage of the alternating method allows us to exploit the penalty function by separating the function and quadratic performance in each iteration of solving the augmented Lagrangian. Case studies are provided to demonstrate the effectiveness of the proposed algorithm.

Keywords: Alternating direction method of multipliers, L_q norm, Power networks, Sparse control.

1. INTRODUCTION

Due to the large size and high complexity of power systems, it is important to put forward sparsity promoting control models and algorithms for the purpose of economy and robustness of the feedback control system. As a result, the conventional centralized control tactics are neither necessary nor applicable due to their high computational burden and poor economy costs. For identifying the sparse structure and optional design of decentralized controllers for power grids, recent research efforts appeal to the method of alternating correction method of multipliers (ADMM). The ADMM method was developed in 1970s; and is well implemented for solving decentralized optimization problems in industry and academic. The ADMM method is a version of method of multipliers and dual decomposition. The advantage of ADMM is that it can lead to a separating minimization in each iteration.

The interconnection of power grids improves the system economy of operation, and leads to more complex decentralized control of these heterogeneous generations like wind and solar energy source. New challenges are arising due to the increasing integration of renewable power generation in power systems. The clustering structure of systems leads to wide area oscillations on the order of 0.2–2 .0Hz [1]. This inter-cluster swings may cause instability even catastrophic blackouts. However, the damping of low frequency oscillations is still intractable due to the delayed measurement or failure to determine the source frequency instantly [2]. The prevailing method to design the controller by using PSS (power system stabilizer) and FACTS (Flexible AC transmission system). Solutions to automatic design of PSS controller have been proposed recently in [3]. A range of measurements and analyses have been proposed to choose the best control input signal for the sparse control scheme, such as bus voltage, relative rotor angel, generator power flow and current[4-8]. Recently, the rise of the wide-area measurement system (WAMS) technology facilitate the design of rotational synchronization of generator, because the phase information about remote generator is applicable due to the phasor measurement units (PMUs) ,the GPS-based location and synchronization, high speed sampling and communication [9-10]. Most of the literatures mentioned above model the power system at nominal operating point due to the small amplitude of the disturbance in wide-area oscillations, authors of [11] have explored the robustness of power systems around off-nominal operating points, and proposed a coordinated design of multiple model FACTS-based controllers.

The structure of this paper is as follows: In Section 2, a general format for formulating the optimization problem as the linear state-space model is discussed. Section 3 analyzes the separating of the augmented Lagrangian and penalty function for the alternating direction algorithm. In Section 4, we study the application of L_q ($0<q<1$) norm as penalty function for the structure optimization problem. Section 5 evaluates the effects of sparsity coefficient with L_0, L_1 and L_q norms by numerical experiments, and Section 6 concludes the paper.

2. MODEL

Consider the following general format of single machine infinite bus power system:

$$2H\ddot{\theta} = P - \sum\left(\frac{E^2\gamma - E_i E_k \cos(\theta+\alpha)}{p^2}\right) \quad (1)$$

We omit the higher-order terms of the dynamics of every generator, then get the swing equation as follows:

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_{j=1}^{N}\left|Y_{ij}\right|E_i E_j \sin(\theta_i - \theta_j) \quad (2)$$

where M is the inertia matrix, D is the damping matrix, θ is the generator rotor angle. Within the scope of wide-area oscillations, the system described by (2) can be linearized at the steady operating point, and equivalent to the linear state-space model [12]:

$$\mathrm{M}\ddot{\theta} + D\dot{\theta} + L\theta = 0 \quad (3)$$

where L is the coupling weights matrix named Laplacian matrix. We formulate the states of generator as $x = (\theta, \ldots)$, then have the system model with control factor as follows:

$$\dot{x} = Ax + B_1 d + B_2 u \quad (4)$$

where d denotes the disturbance, u denotes the control input signal. The value of A, B_1, B_2 may depends on the current operating point of system. A two level control scheme using linear time-invariant had been proposed by [7]. With the widespread use of FACTS, controllers based on the device aiming at damping both local and wide-area oscillations are designed by using different measurement, such as generator and bus voltage [8]:

$$\begin{bmatrix} \square i_f \\ \square i_t \end{bmatrix} = \begin{bmatrix} a+bj & c+dj \\ e+fj & g+hj \end{bmatrix}\begin{bmatrix} u_f \\ u_t \end{bmatrix} - F^T\begin{bmatrix} u_f \\ u_t \end{bmatrix} + B_1 d \quad (5)$$

Actually, there are many measurement choices for the damping of low frequency wide-area oscillation. The control model (4) uses the rotor angel and rotating frequency as feedback input. Recently, the robust of FACTS design under multi-working conditions poses was considered [9]. The sparse arrangement of controllers with SISO or MIMO working model also has been proposed [10].

We derive the equation (6) based on the Taylor expansion of (4):

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (6)$$

Let $x = Vz$, then we have:

$$\begin{aligned} \dot{z} &= \Lambda z + B'u \\ y &= C'z + Du \end{aligned} \quad (7)$$

where the modal controllability and observability denote as:

$$B' = V^{-1}B, \quad C' = CV$$

3. OPTIMIZATION MODEL

Consider the Mechanical energy and electrical energy of generator, then [13]

$$x^T Q x = \frac{1}{2}\dot{\theta}^T M\dot{\theta} + \frac{1}{2}\theta^T L\theta \quad (8)$$

In (7), the output is the function of system state and control factor. Consider the H_2 norm of the transfer function from d to z, then we have:

$$H(j\omega)_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)_{HS}^2\, d\omega = trace(X(Q+G^T RG)) \quad (9)$$

Hilbert-Schmidt norms denotes the power spectral distribution as follows:

$$H(j\omega)_{HS}^2 = \mathrm{trace}(H(j\omega)H^*(j\omega)) = \sum_i \delta_i^2\left(H(j\omega)\right) \quad (10)$$

According to the result of Lyapunov equation, we got the controllability Gramian solved by

$$\left(\mathrm{A} - B_2 G\right)X + X\left(A - B_2 G\right)^T = -B_1 B_1^T \quad (11)$$

The H_2 norm of transfer function denotes as follows:

$$\| f \|_F = \left(\sum_{i=1}^{m}\sum_{j=1}^{n}(\mathrm{J}_{ij})^2\right)^{1/2} \quad (12)$$

Combined with the feedback control the norm can be formulated as follows, see [4] for more details:

$$J(F) = \begin{cases} (\bar{B}_1^T F\bar{B})_1 \\ \infty \end{cases} \quad (13)$$

H∞ control may provide robust solution for power systems, however, not stable transient property [14]. Consider our sparse control problem, there are two objects in the whole scheme: minimizing the H_2 norm of transfer function; minimizing the number of the control links and optimizing the effect of control. For the quadratic control problem, ADMM method is preferred [2, 4]. Then the structure of the object function is formulated as follows:

minimize $J(F) + \gamma\, \mathrm{g}G$ subject to

$$F - G = 0 \quad (14)$$

We form the augmented Lagrangian for (14):

$$L = J(F) + \lambda g(G) + y^T(F-G) + \frac{\rho}{2}\|F-G\|_F^2 \quad (15)$$

ADMM consists of the iterations

$$F^{k+1} = \arg\min L(\mathrm{F}, \mathrm{G}^k, y^k) \quad (16a)$$

$$G^{k+1} = \arg\min L(\mathrm{F}^{K+1}, G, \mathrm{y}^k) \quad (16b)$$

$$y^{k+1} = y^k + \rho(F^{k+1} - G^{k+1}) \tag{16c}$$

The advantage of the ADMM is the separating of F-minimizing step and G-minimizing step. For the (16a), interior point method, Newton method both have good performance. For the (16b), soft threshold operators are used for solving. The value of F is not equals G in the process. When $|F - G| < \varepsilon$ ($\varepsilon = 1.\text{e}-2$), iteration exits.

4. PENALTY FUNCTION

4.1 L0 norm (q=0)

For the L_0 Norm, it acts as the Card function, which counts the number of nonzero elements

$$g_{L0}(F) = \|F\|_{L0} = Card(F) \tag{17}$$

For the structure sparse problem, the L_0 norm is the straight answer. However, the nonconvex property of L_0 norm needs the L_1 norm as an alternative convex-release version in practices.

4.2 L_q norm (0<q<1)

For the L_q norm, it denoted as

$$\|F\|_q = (\sum_{i,j} |F_{i,j}|^q)^{1/q} \tag{18}$$

Because the Lq norm is non-convex, non-smooth, and non-Lipschitz, there is no general theoretical algorithms for solving the (16b). The shrinkage threshold operators provide the analytic solution for some special point [15]. For q=1/2,

$$H_\beta(z) = \begin{cases} \frac{2}{3}|z_i|(1+\cos(\frac{2\pi}{3} - \frac{2\varphi(z_i)}{3})) & z_i > p(\lambda) \\ 0 & |z_i| \le p(\lambda) \\ -\frac{2}{3}|z_i|(1+\cos(\frac{2\pi}{3} - \frac{2\varphi(z_i)}{3})) & z_i < -p(\lambda) \end{cases} \tag{19}$$

For q=2/3,

$$H_\beta(z) = \begin{cases} (\frac{m+\sqrt{\frac{2|z_i|}{m-m^2}}}{2})^3 & z_i > p(\lambda) \\ 0 & |z_i| \le p(\lambda) \\ -(\frac{m+\sqrt{\frac{2|z_i|}{m-m^2}}}{2})^3 & z_i < -p(\lambda) \end{cases} \tag{20}$$

By using the threshold operator to (16b), we have

$$G^{k+1} = Shrink(V^{k+1} + \frac{y_k}{\rho}, \frac{\lambda}{\rho}) \tag{21}$$

L_q (0<q<1) norm proves a good method to obtain the sparse structure of object function, as Fig 1 shows. In a 2-axis data space, only the solutions of L_q (q=1/2) and log norm obtain sparse structures. The solutions from other norms, L_1 (q=1), weighted L_1, L_2 (q=2), L_p (p=1.5), which are the intersections of constrains set and minimal concentric circles of J (F), are not located on the axis.

4.3 L_1 norm (q=1)

For the L_1 Norm, it acts a weighted version of L_0 norm, a convex release version, which counts the absolute value number of nonzero elements.

$$L_1 = \sum_{i,j} |G_{ij}| \tag{22}$$

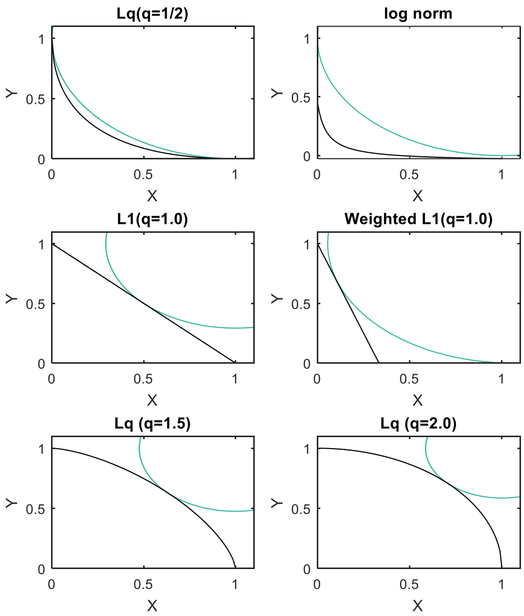

Figure 1. The advantage of L_q (q=1/2) norm

5. RESULTS

We use MATLAB software as the tools for our simulation. We solve the optimal control problem (16) for about 20 iterations with line spaced values of γ in the interval. Our sparse structure results are reported in Fig. 2.

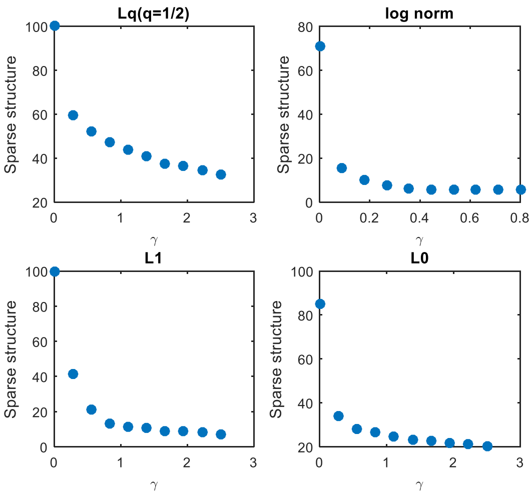

Figure 2. Sparse structure for L_0,L_q,L_1,and log norms

Our performance results are reported in Figs. 3. For γ =0, the optimal feedback gain is fully populated, thereby requiring centralized implementation. As γ increases, the off-diagonal elements of the feedback matrix become significantly sparser whereas the relative cost increases only slightly; see Figs. 3. Especially, for the L_q norm (q=1/2), when r>=2, the sparse process works in a low effect, while there is a persistent deterioration of the performance. For the performance analyze of the logarithmic norm, as γ increases, sparse structure and performance all have a bad discrimination. Compared with L_0 norm, the performance of L_1 norm has an increasing worsen due to its more sparse structure. Additionally, as γ increases, the performance of L_q norm shows a nearly linear power law changing, which is suitable for analyze and design of the control problem.

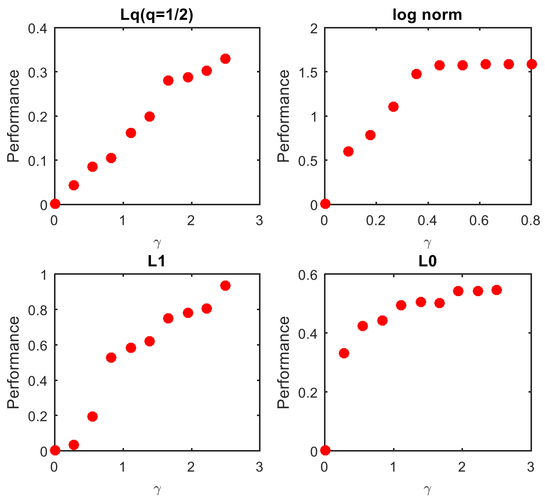

Figure 3. Performance for L_0 norm

6. CONCLUSIONS

In this paper, the effect of L_q (0<q<1) norm played as penalty function of an optimization scheme was presented. The optimization scheme uses the ADMM algorithm to minimize the steady-state variance of the sparse controlled system. The designed ADMM scheme consists two separating steps in which the penalty functions played as L_0, L_1 and L_q regularized term. The simulation results showed that *r* has a nearly linear relationship with sparse structure and performance when at low value, and saturation effect appeared with the increasing of *r*. The results and analysis in this paper have potential applications to wide-area control in actual power systems.

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