
Numerical investigation of the performance of the etoile flow conditioner under different geometric and dynamic configurations

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ABSTRACT. The present work is concerned with a numerical study on the performance of industrial flow conditioners to produce the fully developed flow condition required by international standards for accurate flow metering purposes. The Etoile flow straightener was tested under severe distorted flow generated by a double 90 degrees bend elbows in perpendicular planes, at different Reynolds numbers ranging from 10^3 to 10^6 . Different geometrical settings for the Etoile were tested. The flow field has been simulated using the COMSOL CFD code. The performance of the flow straightener to produce the fully developed flow condition under different geometrical and dynamic conditions was investigated and discussed.

RÉSUMÉ. Le travail présent se focalise sur une recherche numérique sur la performance des redresseurs d'écoulement industriels qui produisent l'écoulement entièrement développé demandé par les normes internationales pour mesurer le débit précisément. Le redresseur d'écoulement Étoile a été testé sous un écoulement déformé sévère généré par un pli du coude à 90 degrés dans les plans perpendiculaires, à nombres différents de Reynolds allant de 103 à 106. Des paramètres géométriques différents pour l'Étoile ont été testés. Le champ d'écoulement a été simulé avec le code COMSOL CFD. La performance du redresseur d'écoulement pour produire la condition d'écoulement pleinement développé dans les différents contextes géométriques et dynamiques, a été étudiée et discutée.

KEYWORDS: computational fluid dynamics, flow conditioner, pipe flow, fully developed flow, flow rate measurements, international standards, industry 4.0.

MOTS-CLÉS: mécanique des fluides numérique, redresseur d'écoulement, flux dans le tube, écoulement entièrement développé, mesures de débit, normes internationales, industrie 4.0.

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1. Introduction

Industry 4.0 embraces the use of new technologies to provide consumers with products and services with high quality and cost-effective manner. It relies heavily on the use of Big Data collected through digital networks and smart sensors. Flow metering devices have always been an integral part of our industrial operations, in petrochemicals, water treatment, food, pharmaceutical industries for process control and fiscal and custody transfer purposes. Smart water pipe networks and district heating networks and cooling use extensively flow metering devices. This important role will also be required to stay within the Industry 4.0 environment Ghernaout *et al.* (2018), OGA (2015) and Turiso *et al.* (2018).

Our capability to measure in accurate way flow rate in pipes has been of a big concern for industrial professionals and academic researchers due to its vital importance when large volumes of fluids are transferred for process purposes and custody transfer issues. In the Gas and Petroleum industry, big companies usually handle billions of barrels or cubic meters of gas and petroleum over a period of a year. Similar amounts of drinkable water are handled annually through thousands of kilometers of pipe lines. The accurate measurements of such huge amounts of precious products are very important from both economic and technological perspectives. Measurement errors in flow rate can have large cost and efficiency implications in our operations Sawchuk, (2016).

According to International Standards such as ISO 5167: 2003 (2014) and ANSI/API 2530 (2000), accurate flow rate measurements can be achieved if the flow meter is operating under fully developed pipe flow characterized by axisymmetric velocity profile and swirl and pulsation free. Such a condition can be achieved if some 80 to 90 pipe diameter of straight length development is allowed without any disturbance. However, industrial settings require the use of pipe fittings such as bends and valves, which are sources of both swirl asymmetries and turbulence distortions. In the recent years, the effect of these flow distortions on the accuracy of flow meters have been investigated by many researchers and industrial professional and metering errors of the order of $\pm 5\%$ and greater have been noticed in situation where tolerated errors should not exceed $\pm 0,1\%$ Turiso *et al.* (2018), Aichouni *et al.* (1996), Merzkirch (2001), Gersten (2008) and Sawchuk *et al.* (2010).

To overcome this situation, flow distortions caused by pipefittings such elbows, valves, expanders and reducers, can be reduced using flow conditioners and straighteners. These elements have been designed and used to control the quality of the flow approaching the metering device and to reduce the effect of the flow distortions. Considerable research effort has been devoted to the optimum design of a

flow conditioner-meter package that minimizes installation effects and increases the quality of the metering device.

Most of these studies considered experimentally and computationally the “flow conditioner-meter” package within geometric and dynamic settings. Basically, the effect of flow conditioner location with respect to the metering device on its calibration coefficients was always the case. Major conclusions showed that flow disturbances in the approaching flow generated by pipe fittings upstream flow meter are still sources of significant shifts in the meter's calibration coefficient and leading to considerable errors in flow metering even with the use of different flow conditioner devices. This would lead to conclude that even with such a worldwide research effort, there is still a need for fundamental understanding of the flow behaviour within this industrial metering setting. The ongoing research project falls within this international scientific effort towards the understanding of the flow development and the efficiency of flow conditioners to remove flow distortions and to produce the standard fully developed condition recommended by international standards to achieve accurate flow metering.

In a recent study conducted by Aichouni *et al.* (2016), Computational Fluid Dynamic techniques have been used to investigate the performance of the Etoile flow straightener described in the international standards ISO 5167(R:2014) (2014) towards the production of the fully developed flow condition. Different design configurations of the flow straightener were tested at one specific Reynolds number ($Re = 2.5 \times 10^5$). The effect of the length of the Etoile flow straightener with ($2D$ as described by the ISO 5167 standards), D , $D/2$, $D/4$ and $D/8$) on the flow performance was investigated. The predictions show that the mean flow distortions were not removed after $12D$ of flow development for the 5 straightener configurations ($2D$, D , $D/2$, $D/4$ and $D/8$) while the swirl tends to be removed by the straightener within the ISO limits of ± 2 degrees. This indicates that the length of the Etoile flow straightener would not have a great effect on the flow performance, suggesting that the standard configuration of the Etoile flow straightener would be recommended for accurate industrial flow metering purposes. In an early research work carried by Barton, (2002) at the National Engineering Laboratory, U.K, the performance of flow conditioners at different Reynolds numbers was assessed. The results showed some dependence of the flow conditioners performance at high Reynolds numbers. This led to raise the question of what would be the performance of the Etoile flow conditioner under different geometric and dynamic flow conditions; this research question will be considered in the present paper. We performed numerical study of the downstream development of the mean flow and swirl intensity towards the fully developed pipe flow condition. It is hoped that such study will provide valuable information to better understand and optimize the design of flow conditioners used in the industry for metering purposes

2. Numerical method

The prediction method is based on the numerical solution of the time mean averaged equations for conservation of mass and momentum. These equations have

been used together with the standard k- ε turbulence model to set a closed system of partial differential equations for the turbulent pipe flow. The flow is supposed to be steady and the fluid is incompressible. The general equation can be written as:

$$\frac{\partial(\rho\phi)}{\partial t} + \text{div}(\rho\phi\vec{U}) = \text{div}(\Gamma_\phi \cdot \text{grad } \phi) + S_\phi \quad (1)$$

Where ϕ is the general dependent variable which can be the mean velocity, the turbulent kinetic energy k or the rate of dissipation ε of the turbulent kinetic energy k ; S_ϕ is the term source of the variable ϕ and Γ_ϕ is the coefficient of diffusion of the flow parameter ϕ .

For these flow conditions, the steady three-dimensional differential equations governing the phenomenon can be expressed by:

The continuity equation of mass conservation:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2)$$

Momentum conservation equation:

$$\frac{\partial(\rho u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \mu_t \right) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \delta_{ij} \frac{2}{3} \frac{\partial u_k}{\partial x_k} \right] - \frac{\partial(\delta_{ij} p)}{\partial x_j} \quad (3)$$

Turbulent kinetic energy equation:

$$\frac{\partial(\rho u_j K)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{Pr_k} \right) \frac{\partial K}{\partial x_j} \right] + \mu_t \frac{\partial u_j}{\partial x_i} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \rho \varepsilon \quad (4)$$

Dissipation rate of turbulent kinetic energy:

$$\frac{\partial(\rho u_j \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{Pr_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + \frac{C_1 \varepsilon \mu_t}{K} \frac{\partial u_i}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{C_2 \rho \varepsilon^2}{K} \quad (5)$$

The turbulent viscosity is defined by:

$$\mu_t = \frac{C_\mu \rho \varepsilon^2}{K} \quad (6)$$

Where the standard turbulence model coefficients are taken as suggested by Launder and Spalding (1974):

$$C_\mu = 0.09, C_1 = 1.47, C_2 = 1.92, Pr_K = Pr_T = 1, Pr_\varepsilon = 1.3$$

The boundary conditions were set as:

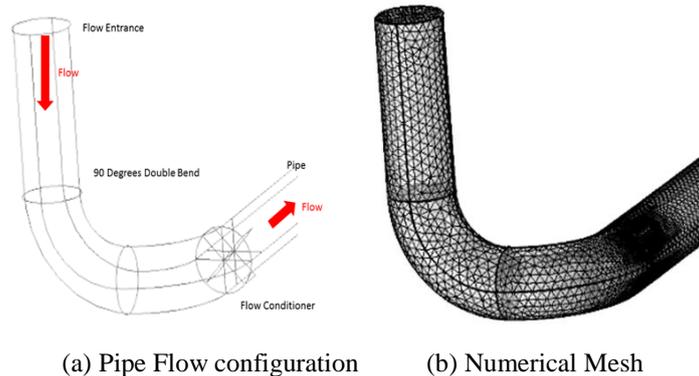
$$u = 0, v = 0, w = 0, \text{ at all tube walls}$$

$$w = U_i, K = K_i, \varepsilon = \varepsilon_i \text{ at the inlet;}$$

$$\frac{\partial}{\partial n}(u_i, T, K, \varepsilon) = 0 \text{ at the outlet.}$$

The flow domain is shown in Figure (1-a), which consists of double bend elbow placed three diameters upstream of the Etoile flow straightener followed by a long straight pipe. A typical numerical mesh is shown in Figure (1-b).

The numerical simulations were carried out using commercial code COMSOL Multi-physics CFD, which solves the set of the Navier-Stokes equations using the finite volume method.



(a) Pipe Flow configuration (b) Numerical Mesh

Figure 1. Flow configuration and computational mesh

3. Results and discussions

As stated before, flow conditioners and straighteners are used in industrial pipelines to reduce the developing length between pipe fittings and flow metering devices, and to produce a free swirl fully developed flow condition within manageable distances; Such flow condition is required by international standards to ensure accurate metering operations.

In the present study, numerical predictions of the flow development downstream the Etoile flow conditioner will be presented and discussed for different design configurations and different Reynolds number ranging from 10^3 to 10^6 . Predictions of the mean flow and turbulent structures downstream the flow conditioner was obtained

and analyzed. Numerical predictions were tested at different mesh sizes and grid-independent solutions were obtained with the numerical grids shown in Table 1 for a typical Reynolds number 10^5 .

Validation of the numerical procedure was carried out by comparing the predicted axial velocity profile with the standards experimental data of Laufer (1954). Calculations carried out up to 90 diameters from the exit of the flow conditioner. At 90 pipe diameters the flow has reached completely the fully developed condition as shown in figure 1 where the computed profiles (the axial mean velocity) at Reynolds $Re=10^5$ are compared to the standards experimental results of Laufer (1954). The initial conditions do not have any effect at this late stage of flow development.

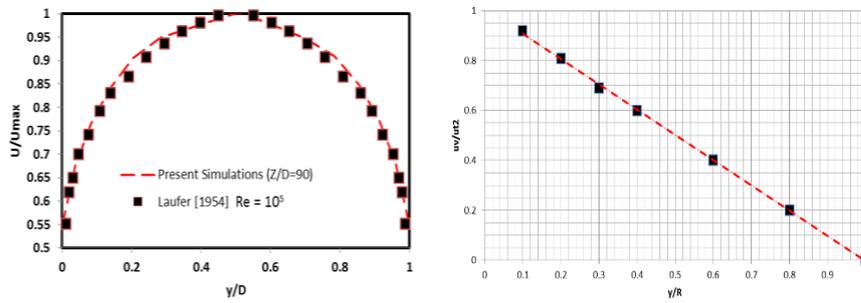


Figure 2. Comparison of simulated profiles and experimental results for the normalized axial mean velocity and the Reynolds shear Stress obtained at the fully developed condition

Table 1. Typical numerical meshing used for $Re=10^5$

| Etoile Length | Meshing Elements |
|---------------|------------------|
| $L = 2 D$ | 956340 |
| $L = D$ | 915919 |
| $L = D / 2$ | 803954 |
| $L = D / 4$ | 755123 |

The discussion will focus on the flow development downstream the flow conditioner for different geometric configurations with varying the Reynolds number. The attenuation of the flow distortion will be examined and a comparison with the fully developed flow condition will be made. Axial mean velocity contours downstream the Etoile flow conditioner with three geometrical configurations (i.e. the standard 2 diameters long flow conditioner ($L = 2D$) and $L = D$ and $L = D/2$) are presented in figure 2 for different Reynolds number. In these figures we are mainly concerned only by the entrance region of the pipe (10 pipe diameters). From these figures it can be seen that for all Reynolds number, the double bend fitting generates

a highly distorted flow characterized by a strong and highly sheared flow profile. Early experimental investigations showed that these pipe fittings generate both highly sheared flow associated with high swirl angles and high turbulence levels Gersten, (2008) and Laribi *et al.* (2003). The solid part of the flow conditioner at the central region causes a strong wake at the central region of the pipe. The wake which characterizes the flow distortion seems to take several pipe diameters to be eliminated for all geometric configurations and for all Reynolds number. The mixing process seems to be faster for the lower Reynolds number of $Re = 10^3$ than for the higher values of the Reynolds number (Figure 3-a, d).

From Figure (3), it can be seen that the flow just after the flow conditioner depends on the conditioner geometry (i.e its length), and at a downstream distance of 10 pipe diameters it is still in its developing process and the fully developed flow condition does not seem to be obtained. These observations are true for lower and higher Reynolds numbers considered in the present study. The effect of the flow distortion and the swirl caused by the double bend still exist after 10 pipe diameters of flow development. Previous experimental research conducted by Laws and Ouazzane, (1994), Gersten, (2008) and Brown and Griffith, (2013) showed that some flow conditioner design configurations were capable to produce the fully developed flow condition within such manageable and economical distances. However, the authors believe that a flow conditioner with such an excellent performance is yet to be designed and manufactured. For this reason, it is believed that more research work incorporating new tools and technologies of Industry 4.0 such as cloud computing and Big Data technologies would be necessary before optimum flow conditioner design can be achieved.

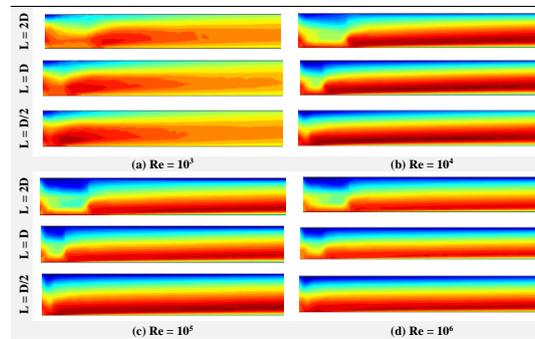


Figure 3. Flow contours downstream three flow conditioner configurations at different Reynolds number

Predicted axial component of the velocity profiles downstream the flow straightener and the swirl angle are presented for the different geometric ($L = 2D, D, D/2$) and dynamic configurations ($Re = 10^3 - 10^6$) on Figures 3 and 4 at different axial distance ($Z/D = 4, 8, 12$). To assess the performance of the flow conditioner the profiles are compared to the fully developed profile predicted after 90 pipe diameters

of development. These figures confirm the early observation about the flow distortion generated by the double bend which persists even after 12 pipe diameters of flow development for the tested straightener configurations (2D, D and D/2) at different Reynolds numbers. The strong sheared flow associated with the high swirl generated by the double elbow does not seem to be attenuated by the flow conditioner. The asymmetrical axial velocity distribution persists with a core velocity region displaced from the central region of the pipe and with a peak velocity which moves from the pipe centreline to the wall as the Reynolds number increases (Figure 3-a, and 4-a).

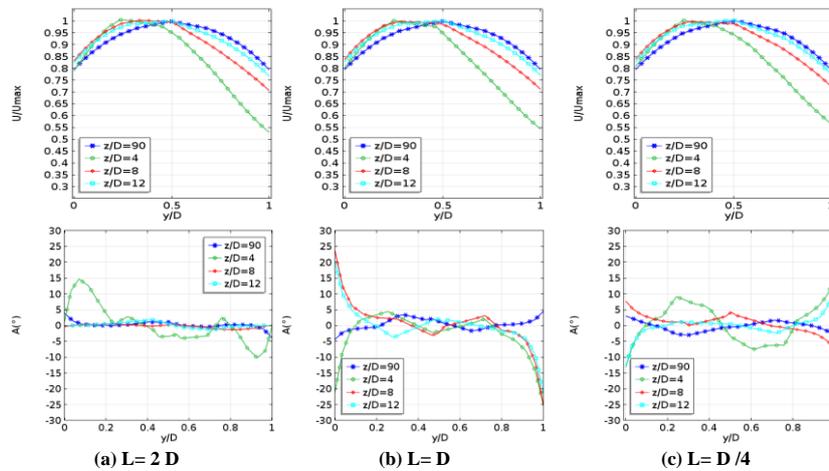


Figure 3. Axial velocity profiles and Swirl Angle at different position Z/D for different Length of the Etoile Straightener (Reynolds Number $Re = 10^3$)

The swirl angle development downstream the Etoile flow straightener seem to attained zero values at lower and higher Reynolds number for the $L = 2D$ configuration, indicating near fully developed condition, while for the shorter configurations the swirl angle is still indicating that the fully developed condition is not attained even at 90 pipe diameters of flow development. It is important to note here that these observations have been noted recently by Turiso *et al.* (2018) in their Laser Doppler velocimetry LDV experimental data at $Re = 4.0 \times 10^4$. Early experimental work by Laribi *et al.* (2003) and Sawchuk *et al.* (2010 and 2016), are entirely consistent with this observation.

The capability of the Etoile flow straightener to produce the fully developed flow condition can be examined through the comparison of the axial velocity profiles and the swirl angle simulated at different axial positions downstream the flow conditioner (figures 3 and 4, a, b and c) at two Reynolds number ($Re = 10^3$ and $Re = 10^6$). It can be seen from these figures that the swirl angle caused by the highly distorted velocity profile at the early stage of flow development presents higher values with an irregular profile. As the flow distortion is attenuated by the mixing process caused by the flow conditioner, the velocity profile becomes more symmetric and the swirl decreases, to

reach a zero value at $z/D = 90$ downstream. This situation corresponds to the fully developed pipe flow required by international standards for accurate flow metering purposes in circular pipe [ISO 5167]. This observation is in agreement with early experimental studies Parchen *et al.* (1993); Laribi *et al.* (2003).

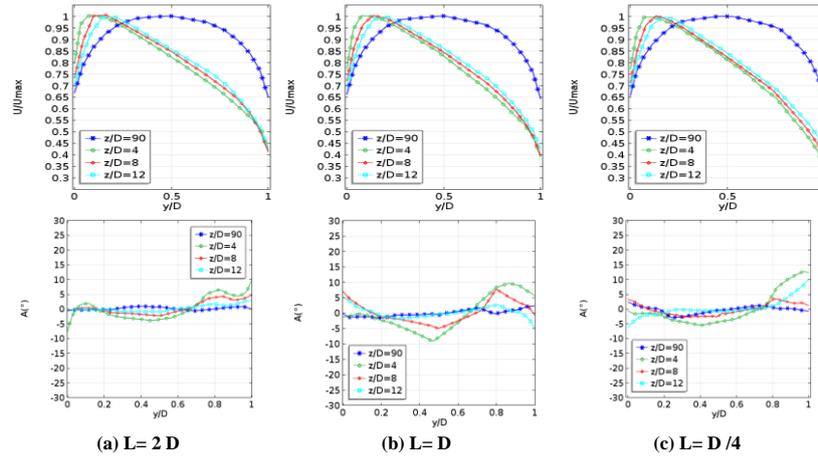


Figure 4. Axial velocity profiles and swirl angle at different position Z/D for different length of the etoile straightener (Reynolds number $Re = 10^6$)

The present simulations would confirm early observations made by Aichouni *et al.* (2016) that the length of the Etoile flow straightener would not have an effect on its performance in reducing flow distortions and producing the fully developed flow condition within short distances of development at the tested Reynolds numbers. Hence it is recommended that the standards Etoile flow straightener would be used in industrial settings with flow meters.

4. Conclusions

Flow rate measurements in non-standards conditions are considered as a big challenge for academic researchers and industrial professionals. A big amount of experimental and computational investigations has been devoted to understand the fundamentals of flow development in metering stations and designing the optimum piping settings that ensure the quality of the metering devices. In the present research work, computational fluid dynamics (C.F.D) techniques are used to investigate the flow development downstream flow conditioners operating under severe flow distortions. The performance of the Etoile flow straightener described in the international standards to produce fully developed flow condition was investigated numerically using the COMSOL CFD code.

The performance of the Etoile with the standards geometry and different other settings at different Reynolds number ($Re = 10^3-10^6$) was investigated; This was

assessed with respect to its capacity to produce the fully developed flow condition in term of velocity profile and swirl free flow within short distances (12 pipe diameters).

The predictions show that the mean flow distortions were not removed after 12 pipe diameters for the straightener configurations at any Reynolds number. This indicates that the length of the Etoile flow straightener would not have an effect on the flow performance, suggesting that the standard configuration of the Etoile flow straightener would be recommended for industrial flow metering purposes.

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NOMENCLATURE

| | |
|-----------------|----------------------------|
| C_1, C_2, C_3 | Turbulence model constants |
|-----------------|----------------------------|

| | |
|----------------------|--|
| D | Pipe diameter, m |
| K | Turbulent kinetic energy, m^2/s^2 |
| P | pressure, Pa |
| R | Pipe Radius, m |
| Re | Reynolds number |
| u_i | Velocity component in i-direction, m/s |
| U | Velocity component in axial direction, m/s |
| V | Velocity component in radial direction, m/s |
| W | Velocity component in circumferential direction, m/s |
| x_i | Cartesian coordinate in i-direction |
| Greek symbols | |
| Pr_ϵ | inverse effect Prandtl number for ϵ |
| Pr_K | inverse effect Prandtl number for K |
| ϵ | dissipation ratio of turbulent kinetic energy, m^2/s^2 |
| δ_{ij} | Dirac delta function |
| μ | dynamic viscosity of fluid, kg/(m s) |
| ρ | density, kg/m ³ |
| Subscripts | |
| i | inlet condition |
| o | outlet condition |
| t | turbulent quantity |
| w | wall condition |