Inverse kinematic tension analysis and optimal design of a cable-driven parallel-series hybrid joint towards wheelchair-mounted robotic manipulator

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ABSTRACT. This paper aims to overcome the inapplicability of the traditional wheelchair-mounted robotic manipulators (WMRMs) to disabled elderly people. To this end, the author proposed a cable-driven parallel-series hybrid joint (CDPSHJ) for the WMRM. The joint is driven by 2 cables between the upper and lower platforms; the two platforms are supported by a middle compression spring, forming the parallel part; the middle of the spring has two rigid shafts with a revolute pair; rigid shaft 1 passes through the upper platform, forming the series part. Then, the inverse kinematic analysis was performed to evaluate the cable length, and the cable tension was analysed through static modelling and lateral buckling modelling of the spring. Then, the correctness of the proposed model was verified by numerical implementations, and the proposed CDPSHJ was proved rational through Matlab simulation. Finally, optimize design based on the inverse kinematics tension analysis. With large work space, smooth motion and light structure, the proposed CDPSHJ is an ideal tool for assistive BCIs.

RÉSUMÉ. Cet article a pour objectif de surmonter l’inapplicabilité des manipulateurs robotiques traditionnels montés en fauteuil roulant (WMRMs, le sigle de « Wheelchair-mounted Robotic Manipulator » en anglais) aux personnes âgées handicapées. À cet effet, l’auteur a proposé un joint hybride en série parallèle entraîné par câble (CDPSHJ, le sigle de « cable-driven parallel-series hybrid joint » en anglais) pour le WMRM. Le joint est entraîné par 2 câbles entre les plates-formes supérieure et inférieure; les deux plates-formes sont supportées par un ressort de compression médian, formant la partie parallèle; le milieu du ressort a deux arbres rigides avec une paire révolutionnaire; l’arbre rigide 1 traverse la plate-forme supérieure et forme la pièce en série. Ensuite, l’analyse cinématique inverse a été réalisée pour évaluer la longueur du câble, et la tension du câble a été analysée en fonction de la modélisation statique et modélisation par flambage latéral du ressort. Ensuite, l’exactitude du
modelé proposé a été vérifié par des implémentations numériques et le CDPSPHJ proposé a été prouvé rationnel par une simulation Matlab. Enfin, l’optimisation de la conception basée sur l’analyse cinématique inverse de la tension est proposée. Avec un grand espace de travail, un mouvement fluide et une structure légère, le CDPSPHJ proposé est un outil idéal pour les BCIs fonctionnels.

KEYWORDS: wheelchair-mounted robotic manipulator (WMRM), cable-driven, hybrid mechanism, spring lateral buckling.

MOTS-CLÉS: manipulateur robotique monté en fauteuil roulant (WMRM), câble, mécanisme hybride, flambage latéral du ressort.

1. Introduction

The statistics of the World Health Organization (WHO) show that the number of people over the age of 60 will increase from 605 million in 2000 to 2 billion in 2050; In developing countries, the number of old people with no self-care ability is expected to quadruple by 2050 (World Health Organization, 2014). According to China Statistical Yearbook on the Work for Persons with Disabilities 2013, China has a total of 37.95 million people of disabilities, among whom 15.64 million (59%) are limb disabled, and the total number is growing at an average annual rate of 6.30% (Sun, 2013). The leading cause of disability among the elderlies is neurodegenerative diseases like stroke, Parkinson’s disease, Alzheimer’s disease, and amyotrophic lateral sclerosis. Relevant symptoms include paralysis, muscle weakness, gait disorder and excessive pain. Once disabled, the elderly people will have immense difficulty in their activities of daily living (ADLs), namely, reaching, gripping and picking up objects from a shelf or the floor. The long-term care (e.g. assisted living) of these people has become a major social issue. A possible way to solve the problem lies in robotic technology (Hersh, 2015). Of course, this technology has not been fully implemented in real life. For instance, the global sales volume of assistive robots for disabled elderly people was merely 6423 in 2017. Against this backdrop, it is very meaningful to develop an advanced assistive robot to help disabled elderly people with their ADLs. A typical type of the said assistive robot is wheelchair-mounted robotic manipulator (WMRM). It blesses the disabled elderly people with greater independence and better quality of life. The research and development (R&D) of WMRM5s can be traced back to the1960s. Over the past 50 years, nearly a dozen WMRM5s products have been developed namely, Weston (Hersh, 2015), KARESI-II (Grigorescu et al., 2012), UCF-MANUSI-II, Human-in-the-loop, PerMMA (Jiang et al., 2016), FRIEND system, ASIBOT, Purdue-JACOI-II, Lightweight Robot III (Vogel et al., 2015). However, the products are not widely available in the market, owing to poor usability and low payload.

Table 1 lists some representative commercial WMRM5s: Raptor, Manus, JACO, LWA 4D, however, some common problems are found.
Table 1. Four representative commercial WMRMs

<table>
<thead>
<tr>
<th>Device</th>
<th>Institutions</th>
<th>DOF</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raptor</td>
<td>Applied Resources, US</td>
<td>4</td>
<td>a planar gripper; control with a joystick or a 10-button controller; It cannot be controlled in Cartesian coordinates; lower cost.</td>
</tr>
<tr>
<td>Manus</td>
<td>Exact Dynamics, Netherlands</td>
<td>6</td>
<td>two-finger gripper; control with 4×4 button keypad or a joystick; It ensures the gripper remains closed in the event of a power failure; a maximum gripping load of 2 kilograms; It can be folded, reach up to 80 cm and weighs 13 kg.</td>
</tr>
<tr>
<td>JACO</td>
<td>Kinova Robotics, Canada</td>
<td>6</td>
<td>a three or a two fingers gripper; It uses carbon fiber to a light-weight (6 kg); a maximum payload of 1.5 kg; It can reach up to 90 cm; control with a three axis joystick; Cartesian controller</td>
</tr>
<tr>
<td>LWA4D</td>
<td>SCHUNK, Germany</td>
<td>7</td>
<td>a two fingers SCHUNK gripping system; a maximum payload of 10 kg; Weight of 17.5kg, weight/load capacity ratio of 2:1; Repeat accuracy of ±0.15mm; Position feedback is pseudo-absolute position measurement; Interface of CANopen(CIA DS402:IEC61800-7-201)</td>
</tr>
</tbody>
</table>

First, the manipulator is too heavy. The movement of the WMRMs are rather clumsy due to the mechanical transmission of the manipulator and hand grasping mechanism. To fit in with the large active space, there is no solution but to increase the length and weight of the manipulator. Second, the joints are too bulky. The size of the joints is bloated with the installation of large power-driven motors, decelerators, and various sensors (e.g. encoders and torque sensors). Third, the flexible motion is too limited. Whereas a human hand can grasp an object in a small activity space, a joint of the WMRM has a limited number of DoFs and needs to go through certain sequence transformation before grabbing.

These problems can be ameliorated by the cable-driven mechanism, in which the motor is fixed onto the base. Since its birth in the US and Japan in late 1980s (Tobias Bruckmann & Pott, 2013), the cable-driven mechanism has been applied in various fields, thanks to its low inertia, large workspace, high payload-to-weight ratio, good transportability, ideal reconfigurability and fully remote actuation. The scope of application ranges from large-scale FAST system for a large radio telescope receiver (Zi et al., 2008), to high-speed assembly and pick-and-place operations. Much research has been done on the mechanism. For example, Yang et al. (2011) developed a 7DoF anthropopathic cable-driven robotic manipulator. Chen et al. (2010) proposed a very similar anthropopathic cable-driven robotic manipulator. Cihat & Pinar, (2017) and Gao et al. (2014) probed deep into the humanoid neck mechanism. Similar research results with spring as the support include a flexible elephant trunk robot, a humanoid neck mechanism, a vertebra-based humanoid torso.
Overall, there are few in-depth reports on the bending and compressive of the coil spring which is a common mechanical element in robots. Based on Castigliano’s theorem, Reference (Cihat & Pinar, 2017) analyses the variable stiffness characteristics of the compression and bending spring. References (Gao et al., 2014; Yang et al., 2006) treat the spring as a flexible rod for modelling and analysis. Reference (Nori et al., 2007) assumes that the length of the spring and the deformation curvature are both constants. Reference (Cafla & Ceccarelli, 2016) performs a dynamic simulation of vertebra-based humanoid torso on ADAMS software, and does not conduct any theoretical modelling analysis.

This paper focuses on analyzing the performance of a cable-driven parallel-series hybrid elbow joint (CDPSHEJ) towards WMRA inspired by the three types of humanoid joints. CDPSHEJ is a cable-driven, compression spring supported hybrid mechanism. To the best knowledge of the authors, there are only a few studies to analyze the lateral bending and compression of spring. In this paper the spring lateral bending and compression model is introduced. This model is based on the elliptic integral solution. This method provides better accuracy compared with other for solving the large deflection problems in compliant mechanisms (Zhang & Chen, 2013). Then, the simulation results were analysed in great details to prove the proposed CDPSHEJ rational. In order to decrease the size of actuators, optimal CDPSHEJ structure is performed based on the inverse kinematics analysis and tension analysis.

This paper is organized as follows. The “Design of CDPSHEJ” section presents the concept of CDPSHEJ. Then, the kinematic modelling is presented in “Inverse Kinematics Analysis” section. Next, the “Tension Analysis” section is reported through static modelling and lateral compression and bending modelling of the spring. Then, the MATLAB simulation results are at “Results Analysis” section. Afterwards, optimal joint structure is investigated in “Optimal CDPSHEJ” section. Finally, conclusions stemming from the results are presented in the last section.

2. Design of cdpshej

2.1. Musculoskeletal mechanism of elbow

The compound movement in human upper limb is driven by multiple skeletal muscles around the shoulder, elbow, wrist and hand joints. Previous anatomic studies have shown that the elbow joint is a hinge joint, i.e. the elbow can complete flexion and extension movements. Hence, the elbow joint was identified as a 1DoF joint. As shown in Figure 1, the elbow joint flexes at the contraction of musculus biceps brachii and the relaxation of musculus triceps brachii; the joint extends at the relaxation of musculus biceps brachii and the contraction of musculus triceps brachii.
2.2. Cable-driven elbow joint

Based on the musculoskeletal mechanism of the human elbow joint, an elbow joint driven by 2 cables was designed for WMAMs, considering the advantages of parallel-series hybrid mechanism (e.g. flexible, rigid, accurate, spacious and load-resistant). As shown in Figure 2(a), cable 1, a mimic of muscle biceps brachii, drives the upper platform (moving platform), while cable 2, a mimic of muscle triceps brachii, drives the upper platform; the two platforms are supported by a middle compression spring, forming the parallel part; the middle of the spring has two rigid shafts with a revolute pair; rigid shaft 1 passes through the upper platform, forming the series part. Overall, the CDPSHEJ has a total of 2DoFs.

3. Inverse kinematics analysis
The diagram of the CDPSHEJ is illustrated in Figure 2(b). The upper and lower platforms are designed as thin round plates with a radius of \( b \) and \( a \), respectively. Then, two Cartesian coordinate systems \( O_1x_1y_1 \) and \( O_2x_2y_2 \) are fixed to the lower and upper platforms, respectively, with \( O_1x_1y_1 \) being the global coordinate system. The connecting point of cables 1 and 2 are denoted as \( A_1, B_1, A_2 \) and \( B_2 \), respectively; the distance from \( O_1 \) to the revolute pair centre is denoted as \( d \). The spring is simplified as an arc.

As mentioned before, the CDPSHEJ has a total of 2DoFs. One DoF refers to the rotation around the axis \( Z \), and the other the translational motion on the \( X-Y \) plane. Under the rigid restraint effect of the middle rotation pair, the relationship of the translational motion relative to \( O_2 \) along the \( X \) axis and that along the \( Y \) axis in the global coordinate system can be expressed as:

\[
x = (y - d) \tan \theta
\]

Therefore, the generalized independent variable of the CDPSHEJ can be defined as \((y, \theta)\) and the joint variable as \((l_1, l_2)\), with \( \theta \) being the rotation angle of the upper platform around the \( Z \) axis, and \( l_1 \) and \( l_2 \) being the lengths of cables 1 and 2, respectively.

It is obvious that the \((l_1, l_2)\) is the output and the translation and rotation of the upper platform \((y, \theta)\) is the input of the inverse kinematics analysis. The kinematic relationship between the input and output can be obtained by the closed vector method:

\[
\vec{l}_m = A_m B_m = O_1 \vec{O}_2 + \vec{O}_2 B_m - \vec{O}_1 A_m = O_1 \vec{O}_2 + o_{1}R \vec{O}_2 B_m - \vec{O}_1 A_m
\]

where \( o_{1}R = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \).

Since \( m = 1,2 \), the above formula can be expanded as:

\[
\vec{l}_1 = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} -b \\ a \end{bmatrix} - \begin{bmatrix} a \\ -b \end{bmatrix} = \begin{bmatrix} x - b \cos \theta + a \\ y + b \sin \theta \end{bmatrix}
\]

\[
\vec{l}_2 = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} - \begin{bmatrix} a \\ -b \end{bmatrix} = \begin{bmatrix} x + b \cos \theta - a \\ y - b \sin \theta \end{bmatrix}
\]

Hence, the lengths of the cables are \( l_1 = \|\vec{l}_1\|, l_2 = \|\vec{l}_2\| \).

4. Tension analysis

The driving cables only work in the tensioning state. Thus, each cable is applied with a tension force. Figure 3 shows the static analysis model of point \( O_2 \) point on the upper platform.
Figure 3. Static analysis model of point O₂

Note: G is the gravity of the upper platform; N is the positive pressure of rigid shaft 1 at point O₂; F₁, F₂ and M are the force/moment with the compression spring acting on point O₂; T₁ and T₂ are tension forces on cables 1 and 2, respectively.

The effects of gravity, positive pressure and friction are negligible due to the small mass of the upper platform. Thus, the static equilibrium equations can be expressed as:

\[ \Sigma_{m=1}^{2} T_m(-\bar{l}_m) = -\bar{F}_{ext} \]  \hspace{1cm} (3)

\[ \Sigma_{m=1}^{2} \alpha \bar{R}_{O2B_m}^R \bar{O}_2 \times T_m(-\bar{l}_m) = -\bar{M}_{ext} \]  \hspace{1cm} (4)

The two formulas can be expanded and merged into:

\[ ST = -W \]  \hspace{1cm} (5)

Where

\[ \bar{l}_m = \bar{l}_m \]  \hspace{1cm} \[ \bar{F}_{ext} = \bar{F}_{ext} \]  \hspace{1cm} \[ \bar{M}_{ext} = \bar{M}_{ext} \]

4.1. Modelling of lateral spring compression and bending

Concerning the elastic stability under lateral buckling, the coil compression spring can be treated as an elastic bar, but it must be taken into account the change in the length of the spring due to buckling since the change is not negligible as in the case of compressed bars. Following this train of thought, the compression and bending spring of the CDPSHEJ is subjected to the force analysis as shown in Figure 4.
Figure 4. Force analysis diagram of the compression spring

Note: The end forces of the spring satisfy $F_1' = -F_1$, $F_2' = -F_2$ and $M' = -M$.

Based on the previous static analysis, three equilibrium equations were derived with $T_1$, $T_2$, $F_1$, $F_2$ and $M$ being unknown quantities. To determine these unknown quantities, the author performed the following modelling analysis of the compression spring.

It is generally agreed in mechanics that the bending moment of an elastic bar equals the product between the bending stiffness and the curvature of bending moment of the bar; the curvature of the elastic bar with large deformation is $\frac{d\alpha}{ds}$, that is:

$$\beta \frac{d\alpha}{ds} = -F_1'y - F_2'x - M' = F_2x + F_1y + M \quad (6)$$

where $\beta$ is the bending stiffness of the spring.

Find the differential of $s$ in formula (6):

$$\beta \frac{d^2\alpha}{ds^2} = F_1 \frac{dy}{ds} + F_2 \frac{dx}{ds} \quad (7)$$

where $\frac{dy}{ds} = \cos \alpha$ and $\frac{dx}{ds} = \sin \alpha$.

Multiply both sides of formula (7) with $\frac{d\alpha}{ds}$ and find the integral:

$$\int \beta \frac{d^2\alpha}{ds^2} \frac{d\alpha}{ds} ds = \int (F_1 \cos \alpha + F_2 \sin \alpha) d\alpha \quad (8)$$

Let $\cos \varphi = F_2/\sqrt{F_1^2 + F_2^2}$, $\sin \varphi = F_1/\sqrt{F_1^2 + F_2^2}$. Formula (8) can be simplified as:

$$\left(\frac{d\alpha}{ds}\right)^2 = -\frac{2}{\beta} \sqrt{F_1^2 + F_2^2} \cos (\alpha + \varphi) + C \quad (9)$$

where $C$ is the integral constant determined by the top condition of the bar. Since the bending moment is 0 at top the bar, $C = \frac{2}{\beta} \sqrt{F_1^2 + F_2^2} \cos (\theta + \varphi)$ if $\alpha = \theta$ and $\frac{d\alpha}{ds} = 0$. 
Meanwhile, we have:

\[ M = -F_2x - F_1y \]  

(10)

As shown in Figure 4, \( \frac{da}{ds} \) is negative:

\[ \frac{da}{ds} = -\frac{2}{B} \sqrt{F_1^2 + F_2^2} \left( \cos(\theta + \phi) - \cos(\alpha + \varphi) \right) \]  

(11)

Let \( x_\alpha \) be the deflection of an elastic buckling bar in the horizontal direction. Since \( \frac{dx}{ds} = \sin \alpha \), we have \( dx = \sin \alpha ds \). Thus \( x_\alpha = \int \sin \alpha ds \). When \( \alpha = \theta \), the horizontal deflection at the top of the bar (O2):

\[ x = \int_0^\theta \sin \alpha \frac{da}{ds} ds \]  

(12)

Let \( \frac{2}{B} \sqrt{F_1^2 + F_2^2} = k^2 \), \( \theta + \varphi = \phi \) and \( \alpha + \varphi = \gamma \). Thus, \( \alpha = \gamma - \varphi \). According to formula (12), we have:

\[ x = \int_0^\theta \sin(\gamma - \varphi) \frac{1}{\sqrt{\cos \gamma - \cos \phi}} dy \]  

(13)

Let \( p = \sin \frac{\phi}{2} \), and introduce the new variable \( \varepsilon \). Hence, \( \sin \frac{\varphi}{2} = psin \varepsilon = \sin \frac{\phi}{2} \sin \varepsilon \). If \( \alpha \) falls between 0 and \( \theta \), \( \sin \varepsilon \) ranges from 0 to 1. Thus, \( \varepsilon \in [0, \frac{\pi}{2}] \). Substituting \( p \) and \( \varepsilon \) into formula (13), we have:

\[ x_k = x_1 + x_2 \]  

(14)

where,

\[ x_1 = \cos \phi \int_0^\frac{\pi}{2} 2\sqrt{2} p \sin \varepsilon \varepsilon d\varepsilon = 2\sqrt{2} p \cos \phi; \]

\[ x_2 = -\sin \phi \int_0^\frac{\pi}{2} \sqrt{2-2p^2} \varepsilon^2 \sin \varepsilon^2 \varepsilon d\varepsilon. \]

\( x_2 \) is the combination of complete elliptic integral of the first kind and completely elliptic integral of the second kind. Obvious, the variable has no analytical solution and only applies to numerical solution.

At this point, the three equilibrium equations obtained from the static analysis were substituted into formulas (10) and (14) to derive the values of \( T_1, T_2, F_1, F_2 \) and \( M \).

4.2. Spring parameters

The spring is made of carbon steel. Its parameters are listed in Table 2 below.
Table 2. Spring parameters

<table>
<thead>
<tr>
<th>E (Gpa)</th>
<th>G (Gpa)</th>
<th>d (mm)</th>
<th>D (mm)</th>
<th>l0 (mm)</th>
<th>n0</th>
<th>K (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>196</td>
<td>78.5</td>
<td>5</td>
<td>40</td>
<td>105</td>
<td>8</td>
<td>23840</td>
</tr>
</tbody>
</table>

Note: E is the elastic modulus; G is the Young’s modulus; d is the diameter of spring wire; D is the outer diameter of the spring; l0 is the initial length of the compression spring; n0 is the active coils number of the compression spring. K is the spring constant.

The spring bending stiffness $\beta$ satisfies the following relationship (Timoshenko & Gere, 1961):

$$\beta = \beta_0 \frac{l}{l_0}$$  \hspace{1cm} (15)

where $l_0$ is the initial length of the spring; $l$ is the compressed length of the spring. Since the spring was treated as an elastic bar of variable length and large deformation, it is assumed that $l = \sqrt{x^2 + y^2} = \|O_1O_2\|$.

Let $I$ be the inertia moment of the cross-section of the spring, and $\beta_0$ be the initial bending stiffness of the spring. Then, the two parameters can be expressed as:

$$I = \frac{\pi d^4}{64} = 3.068 \times 10^{-11} \text{ and } \beta_0 = \frac{4EGl_0}{\pi dn_0(E + 2G)} = 0.559$$

5. Results analysis

5.1. Inverse kinematics analysis

In the inverse kinematics analysis, the $(l_1, l_2)$ is the output and the translation and rotation of the upper platform $(y, \theta)$ is the input. The variable $y$ is introduced to adjust the preload of each cable to $(0.085m, 0.095m)$. The normal ADLs require an elbow extension/flexion of 110° (Raiss et al., 2007). Thus, the value of $\theta$ is $(-55^\circ, 55^\circ)$. Because the platform motion is symmetric between $(-55^\circ, 0^\circ)$ and $(0^\circ, 55^\circ)$, the $\theta$ value is determined as $(0^\circ, 55^\circ)$.
The upper and lower platform diameters of cable-driven joint are \( b=0.05m \) and \( a=0.07m \), respectively; the distance between rotation pair and the lower platform is \( d=0.04m \). These values are inputted into Matlab for simulation. The results are presented in Figure 5.

### 5.2. Tension analysis

When \( \theta=0^\circ \), the spring is linear compression spring. Hence, \( F_1=0 \), \( F_2=K(l_0-y) \), \( M=0 \). The cable tension in Figure 5 is analysed through Matlab simulation, aiming to verify the correctness of the spring modelling analysis. Figure 6 shows the results of inverse kinematics analysis and tension analysis at \( y=0.085m \).

As mentioned before, the musculoskeletal mechanism of the elbow joint corresponds to the contraction/relaxation of the musculus biceps brachii and musculus triceps brachii. According to Figures 5 and 6, variation tendency of cables length and tension shows antagonistic characteristics, indicating that the two cables act in a similar way to the muscles of human elbow joint.

The results in Figures 5 and 6 verify the rationality of the CDPSHEJ.

In addition, determine whether ADL are possible, are elbow flexion and extension. As expected, large elbow flexion and extension are needed for most ADLs (Magermans et al., 2005). In other words, the larger the range of motion (RoM) of elbow is, the greater the ability of humanoid arm with shoulder and wrist to perform ADL. With a RoM of 110°, the proposed mechanism satisfies the ADL requirements on elbow extension/flexion. This means the proposed CDPSHEJ is suitable for the WMRM that assists the disabled elderly people in performing ADLs.

As can be seen from Figures 5 and 6, the cable force is positive, indicating that the cables are not slack. When the joint rotated clockwise, the tension of cable 2 is greater than the tension of cable 1. This agrees with the rule of thumb. The above analysis demonstrates the correctness and feasibility of the lateral compression and bending modelling of the spring.
In order to decrease the size of actuators, optimal CDPSHEJ is performed based on the inverse kinematics analysis and tension analysis. The optimization variables should be \(a, b, d\), which represent the structure variables. The other variables \(\theta, y\) should be specified. Since the optimization objective is to seek smaller motor torque to meet minimal energy consumption, the optimized objective function is \(T_{\text{max}} = \max(T_1, T_2)\), which should be scalar function of \(a, b, d\). Because different \(y\) can be obtained by pretightening all the two cables simultaneously, and cable tension decreases with the increase of \(y\) from Figures 5 (b), (d). Therefore, we assume \(y = 0.095\) m in the following optimization. When \(\theta=0^\circ\), \(F_1=0\), \(F_2=K(l_0-y)\), \(M=0\), denote \(\theta\) as \((0^\circ, 55^\circ)\), which doesn’t include \(0^\circ\). Denote such function as \(f(a, b, d)\), then the minimization problem can be generally stated as:

\[
\begin{align*}
\text{minimize} & \ f(a, b, d) \\
\text{subject to} & \ a_l \leq a \leq a_u, \ b \leq b_u, \ d \leq d_u \text{ and } T_i > 0 \ (i = 1, 2)
\end{align*}
\]

where \(a_l, a_u, b, b_u, d, d_u\) are bound for \(a\), \(b\) and \(d\), respectively, which can be derived from the size of the human upper limb and the spring. Let \(a_l = b_l = 0.035\) m, \(a_u = b_u = 0.08\) m, \(d = 0.025\) m, \(d_u = 0.055\) m. \(f(a, b, d)\) is a nonlinear function. As a result, use the nonlinear optimization algorithm in MATLAB Optimization Toolbox to minimize \(f\). Choose two different initial points \((0.045, 0.035, 0.045)\) and \((0.07, 0.05, 0.04)\). The optimization final point is all \((0.08, 0.0603, 0.025)\). The final objective function value is all 95.3. Analyse structural parameters \(a, b, d\) on optimization objective in order to check the optimization results as the following sections.

6.1. Structural parameters \(d\) on objective function

Discretize \(a \times b\) into \(n \times n\) points, as the same \(\theta, d\). Traverse \(n \times n\) points of \((a, b)\), to figure out the corresponding objective function values for \(d\) (n points) as shown in Figure 7. Each value in Figure 7 is calculated like this. When \(a, b\) takes a value and
then traverse $d$ ($n$ points) to calculate $f(a, b, d)$ corresponding to the $n$ points of $\theta$ from $0^\circ$ to $55^\circ$. If $T_1$ or $T_2$ is negative, $f(a, b, d)$ is $\text{Inf}$. Obviously, when $d=0.025\text{m}$, the objective function value of each curve in Figure 7 is minimum.

$$
\text{Figure 7. Objective function at traversing } a, b
$$

6.2. Structural parameter $a, b$ on objective function

It is known from the previous section that when $d=0.025\text{m}$, $f(a, b, d)$ is minimum. Therefore, let $d=0.025\text{m}$ in this part and analyse structural parameters $a, b$ on optimization objective as Figure 8. Traverse $a$ ($n$ points) to figure out the corresponding objective function values for $n$ points of $b$, as shown in Figure 9. Apparently, when $a=0.08\text{m}$, $f(a, b, d)$ is minimized.

Traverse $b$ ($n$ points) to figure out the corresponding objective function values for $n$ points of $a$, as shown in Figure 10. Apparently, when $b=0.0603\text{m}$, $f(a, b, d)$ is minimized.

So far, the above analysis results are consistent with the optimization results.

$$
\text{Figure 8. Objective function at } d=0.025\text{m}
$$
7. Conclusions

This paper puts forward the CDPSHEJ for WMAMs. After introducing the design, inverse kinematic analysis of the joint is carried out to calculate cable length. Then, the cable tension is analysed through static modelling and lateral compression and bending modelling of the spring. Then, the correctness of the proposed model is verified by numerical implementations, and the proposed CDPSHEJ is proved rational through MATLAB simulation. Finally, based on the inverse kinematics tension analysis optimize design CDPSHEJ. With large workspace, smooth motion and light structure, the proposed CDPSHEJ is an ideal tool for WMRA. The future research will apply variable stiffness properties analysis and closed-loop stiffness control to the joint mechanism, and develop the shoulder and wrist joints (3 or 4 cables-driven) of the anthropopathic manipulator.
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References


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