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Effect of temperature dependent viscosity on ferrothermohaline convection saturating an anisotropic porous medium with Soret effect using the Galerkin technique

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ABSTRACT

In the present paper, the effect of temperature dependent viscosity on a Soret driven ferrothermohaline convection heated from below and salted from above subjected to a transverse uniform magnetic field in the presence of an anisotropic porous medium using Brinkman model is studied. For the case of two free boundaries, an exact solution is obtained using a linear stability analysis and normal mode technique is applied. The effect of salinity has been included in magnetization and density of the fluid. The critical thermal magnetic Rayleigh number N_{sc} for the onset of instability is calculated numerically for sufficiently large values of the buoyancy magnetization parameter M_1 using the method of computational Galerkin technique. It is found that non-buoyancy magnetization parameter, permeability of the porous medium, anisotropy effect and temperature dependent viscosity stabilizes the system.

1. INTRODUCTION

For centuries, many fascinating materials have been attracting the scientists and researchers due to their extraordinary physical properties and technological usage. Magnetic fluids, also called ‘ferrofluids’, are electrically non-conducting colloidal suspensions of tiny particles of solid ferromagnetic material in a non-electrically conducting carrier fluid like water, hydrocarbon fuels, etc. These fluids behave as a homogeneous continuum and exhibit a variety of interesting phenomena. Ferromagnetic fluids are not found in nature but are artificially synthesized. The viscosity of a ferrofluid as a function of the applied magnetic field, direction of magnetic field with respect to the flow direction and temperature.

Ferrofluids are widely used in magnetic inkjet printers, heat transfer, nanomotors, nanogeneratores, inertial dampers, switches, sensors, transformer cooling, loudspeakers, micro- and nanofluidic devices, magnetic targeted drug delivery, etc. In the biomedical field, they have been found very useful. These can be used to deliver drugs to a certain area of human body and also used for cancer treatment by heating the tumor soaked in ferrofluids by means of an alternating magnetic field [1-3]. An authoritative introduction to the research on magnetic fluids has been given in the monograph by Rosensweig [4], which reviews several applications of heat transfer through ferrofluids, such as enhanced convective cooling having a temperature dependent magnetic moment due to magnetization of the fluid. This heat transfer through ferrofluids is called ferroconvection, which is similar to Bénard convection (Chandrasekhar [5]). Convective instability of ferromagnetic fluids has been predicted by Finlayson [6]. Schwab et al. [7] experimentally investigated Finlayson’s problem under a strong magnetic field and detected the onset of convection by plotting the Nusselt number versus the magnetic Rayleigh number. Later, Stiles and Kagan [8] examined the experimental problem reported by Schwab et al. [7] and generalized Finlayson’s model assuming that under a strong magnetic field, the rotational viscosity augments the shear viscosity.

In many investigations, porous medium is taken to be isotropic for geological and pedological process rarely it forms isotropic media, as is usually assumed in transport studies. Processes such as frost action, sedimentation, compaction and reorientation of solid matrix are responsible for the creation of anisotropic natural porous media. Ursino et al. [9] studied upscaling of anisotropy in unsaturated Miller-similar porous media. In this, analytical expressions for the anisotropic conductivity tensor are derived based on the dynamic law that governs the flow problem at the pore scale and the effects of anisotropy on transport parameters are estimated by numerical modeling. In chemical engineering processes, anisotropy can be characteristic of artificial porous like fiber materials. Epherre [10] was the first attempt to study the onset of convection in a horizontal porous layer with anisotropic thermal conductivity.

Vaidyanathan et al. [11] studied convective instability of ferromagnetic fluid in a porous medium of large permeability using Brinkman model. This investigation has been analyzed for the effect of temperature dependent viscosity by Ramanathan and Muchikel [12] using Galerkin technique. In this, temperature dependent viscosity is studied for stabilizing effect for different behaviors, which is not much pronounced. Govindan et al. [13] studied numerical analysis of ferroconvection with temperature dependent viscosity and an anisotropic porous medium. Nanjundappa et al. [14] introduced magnetic field dependent viscosity on Marangoni-Bénard ferroconvection without a porous medium under microgravity conditions in a horizontal ferrofluid layer in the presence of a uniform vertical magnetic field. This work has been analyzed to the effect temperature dependent viscosity in the absence of magnetic field dependent viscosity by Nanjundappa et al. [15]. They used the Rayleigh Ritz method with Chebyshev polynomials of second kind as trial function. Siddheshwar [16] studied the thermorheological effect of magnetoconvection in fluids with weak electrical conductivity.

The study of convection in two component ferrofluids will throw light on convective instability. This is referred to as a type of convection known as ferrothermohaline convection studied by Baines and Gill [17]. Vaidyanathan et al. [18-19] investigated the presence and absence of a porous medium on ferrothermohaline. Furthermore, Vaidyanathan et al. [20] attempted to study the Soret effect due to ferrothermohaline convection of a sparse distribution and the condition of a porous medium of ferroconvective instability of multi-component fluid heated from below and salted from above was analyzed by Sekar et al. [21-22] for isotropic and anisotropic models. The effect of rotation on thermohaline convection in a ferromagnetic fluid saturating an anisotropic porous medium with Soret effect was obtained by Sekar et al. [23] and further investigation was carried out for magnetic field dependent viscosity by Sekar and Raju [24].

In the present work, it is attempted to analyze the effect of an anisotropic porous medium and temperature dependent viscosity on Soret driven ferrothermohaline convection, subjected to a vertical magnetic field using the Brinkman model and the free boundaries are considered. The resulting eigen value problem is solved numerically using the Galerkin method. Besides, an analytical formula is obtained for the critical magnetic Rayleigh number by a regular perturbation method.

2. MATHEMATICAL FORMULATION

An infinite spread horizontal layer of an Oberbeck-Boussinesq ferromagnetic fluid of thickness "*d*" saturating a sparsely distributed anisotropic porous medium heated from below and salted from above is considered. The temperature and salinity at the bottom and top surfaces are $z = \pm d/2$ $T_0 \pm \Delta T/2$ and $S_0 \pm \Delta S/2,$ respectively. Both the boundaries are assumed to be free and perfect conductors of heat and salt. The system is assumed to be anisotropy along the vertical direction and isotropy along the horizontal direction and the fluid viscosity is assumed to be temperature-dependent in the following form [16,12]

$$\mu(T) = \mu_1\left[1-\delta(T-T_a)^2\right] \tag{1}$$

Considering the Soret effect on the temperature gradient the mathematical equations governing the above investigation are as follows with the porous medium k = (k_1, k_1, k_2).

The continuity equation for an incompressible fluid is

$$\nabla.\,\mathbf{q} = 0 \tag{2}$$

The corresponding momentum equation is

$$\rho_0 \frac{D\mathbf{q}}{Dt} = -\nabla p + \rho\,\mathbf{g} + \nabla\cdot(\mathbf{HB}) + \nabla\left[\mu(T)\left(\nabla\mathbf{q} + \nabla\mathbf{q}^{Tr}\right)\right] - \frac{\mu(\mathrm{T})}{k}\,\mathbf{q} \tag{3}$$

The temperature equation for an incompressible ferromagnetic fluid is

$$\left[\rho_0 C_{v,H} - \mu_o \mathbf{H}(\partial\mathbf{M}/\partial T)_{v,H}\right](dT/dt) + \mu_0 T(\partial\mathbf{M}/\partial T)_{v,H}\cdot(d\,\mathbf{H}/dt) = K_1\nabla^2 T + \phi \tag{4}$$

The conservation of mass flux equation is given by

$$\rho_0(\partial/\partial t + \mathbf{q}?\, S = K_S\nabla^2 S + S_T\nabla^2 T \tag{5}$$

The density equation of state for a Boussinesq two-component fluid is

$$\rho = \rho_0[1-\alpha_t(T-T_0) + \alpha_S(S-S_0)] \tag{6}$$

Maxwell's equations, simplified for a non-conducting fluid with no displacement currents, become

$$\nabla\cdot\mathbf{B} = 0,\quad \nabla\times\mathbf{H} = 0 \tag{7a,b}$$

where the magnetic induction is given by

$$\mathbf{B} = \mu_0\left(\mathbf{M}+\mathbf{H}\right) \tag{8}$$

In general, the pressure of ferromagnetic fluid can distort an external magnetic field if magnetic interaction (dipole-dipole) takes place, but this is negligible for small particle concentration, as is assumed here. We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field, temperature and salinity, so that

$$\mathbf{M} = \frac{\mathbf{H}}{H} M(H,T,S) \tag{9}$$

The linearized magnetic equation is

$$M = M_0 + \chi(\mathrm{H} - H_0) - K(T-T_0) + K_2(S-S_0), \tag{10}$$

The basic state is assumed to be quiescent state and is given by

$$\left.\begin{aligned} &\mathbf{q} = \mathbf{q}_b = (0,0,0),\ T = T_b = T_0 - \beta_t z,\ S = S_b = S_0 - \beta_S z,\ \rho(z) = \rho_0[1+\alpha_t\beta_t z - \alpha_S\beta_S z],\\ &p = p_b(z),\ H_b(z) = \left[H_0 - \frac{K\beta_t z}{1+\chi} + \frac{K_2\beta_S z}{1+\chi}\right]\mathbf{k},\quad M_b(z) = \left[M_0 + \frac{K\beta_t z}{1+\chi} - \frac{K_2\beta_S z}{1+\chi}\right]\mathbf{k}. \end{aligned}\right\} \tag{11}$$

Let the component of the perturbed magnetization and the magnetic field be $(M_1^{'},\, M_2^{'},\, M_b(z) + M_3^{'})$ and $(H_1^{'},\, H_2^{'},\, H_b(z) + H_3^{'}),$ respectively. The perturbed viscosity and temperature are taken as $\mu_b(z) + \mu'$ and $T_b(z) + T',$ respectively. Moreover, the basic state is disturbed by an infinitesimal thermal perturbation and the basic state quantities are obtained by substituting the velocity of quiescent state in the governing Eqs. (1) – (4). The techniques of linearization and normal mode. [22,23] are used to finding the solutions of Eqs. (1) – (7). This can be written as

$$(w, T, \phi, S) = \left[w(z,t), T(z,t), \phi'(z,t), S'(z,t)\right] \exp\left(i(k_x x + k_y y)\right) \quad (12)$$

The vertical component of momentum equation can be written as

$$\begin{aligned}
&\rho_0 \frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial z^2} - k_0^2\right) w \\
&= \rho_0 g \alpha_S k_0^2 S' - \rho_0 g \alpha_t k_0^2 T' + K \beta_t k_0^2 \frac{\partial \phi'}{\partial z} - \frac{\mu_0 K^2 k_0^2 \beta_t (1-S_T)}{1+\chi} T' + \frac{\mu_0 K K_2 \beta_S k_0^2 (1-S_T)}{1+\chi} T' \\
&\quad + \frac{\mu_0 K K_2 \beta_t k_0^2}{1+\chi} S' - \frac{\mu_0 K^2 k_0^2 \beta_S}{1+\chi} S' + \mu_0 \left(\frac{\partial^2}{\partial z^2} - k_0^2\right)^2 w + \mu_0 K_2 \beta_S k_0^2 \frac{\partial \phi'}{\partial z} + \frac{\mu_b}{k_1} k_0^2 w \\
&\quad + \frac{\partial^2 \mu_b}{\partial z^2}\left(\frac{\partial^2}{\partial z^2} - k_0^2\right) w + 2 \frac{\partial \mu_b}{\partial z} \frac{\partial}{\partial z}\left(\left(\frac{\partial^2}{\partial z^2} - k_0^2\right) w\right) - \frac{1}{k_2} \frac{\partial \mu_b}{\partial z}\left(\frac{\partial w}{\partial z}\right)
\end{aligned} \quad (13)$$

The modified Fourier heat conduction equation is

$$\rho_0 C_{v,H} \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z}\right) = K_1 \left(\frac{\partial^2}{\partial z^2} - k_0^2\right)\theta + \left[\rho_0 c \beta_t - \left(\frac{\mu_0 K^2 T_0^2 \beta_t}{1+\chi}\right) + \left(\frac{\mu_0 K K_2 T_0 \beta_S}{1+\chi}\right)\right] w, \quad (14)$$

where $\rho_0 C = \rho_0 C_{v,H} + \rho_0 K H_0.$

The salinity equation is

$$\frac{\partial S}{\partial t} + \beta_S w = K_S \left(\frac{\partial^2}{\partial z^2} - k_0^2\right) S + S_T \left(\frac{\partial^2}{\partial z^2} - k_0^2\right)\theta. \quad (15)$$

Using the analysis similar to Sekar et al. [23] one gets

$$(1+\chi)\frac{\partial^2 \phi}{\partial z^2} + \left(1 + \frac{M_0}{H_0}\right) k_0^2 \phi - K \frac{\partial \theta}{\partial z} + K_2 \frac{\partial S}{\partial z} + S_T K \frac{\partial \theta}{\partial z} = 0. \quad (16)$$

where $\nabla_1^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ and $\nabla^2 = \nabla_1^2 + (\partial^2/\partial z^2).$

The non-dimensional numbers can be written using

$$w^* = \frac{wd}{\nu},\ t^* = \frac{\nu t}{d^2},\ T^* = \left(\frac{K_1 a R^{1/2}}{\rho_0 C_{v,H} \beta_t \nu d}\right)\theta,\ \ \phi^* = \left(\frac{(1+\chi) K_1 a R^{1/2}}{\rho_0 C_{v,H} K \beta_t \nu d^2}\right)\phi,\ \ z^* = \frac{z}{d},\ \ a = k_0 d,$$

$$D = \frac{\partial}{\partial z^*},\ \ S^* = \left(\frac{K_S a R_S^{1/2}}{\rho_0 C_{v,H} \beta_S \nu d}\right) S,\ \ \nu = \frac{\mu}{\rho_0},\ \ k^* = \frac{k}{d^2},\ M_1 = \frac{\mu_0 K^2 \beta_t}{(1+\chi)\rho_0 g \alpha_t},\ \ M_2 = \frac{\mu_0 K^2 T}{(1+\chi)\rho_0 C_{v,H}},$$

$$M_3 = \frac{1 + (M_0/H_0)}{(1+\chi)},\ M_4 = \frac{\mu_0 K^2 \beta_S}{(1+\chi)\rho_0 g \alpha_S},\ \ M_5 = \frac{K_2 \beta_S}{K \beta_t},\ \ M_6 = \frac{K_S}{K_1}\ P_r = \frac{\mu C_{v,H}}{K_1},$$

$$R = \frac{\rho_0 C_{v,H} \beta_t \alpha_t g d^4}{\nu K_1},\ \ R_S = \frac{\rho_0 C_{v,H} \beta_S \alpha_S g d^4}{\nu K_S},\ \ \tau = \rho_0 C_{v,H}\left(\frac{K_S}{K_1}\right). \quad (17)$$

where R is the thermal Rayleigh number, R_S is the salinity Rayleigh number, P_r is the Prandtl number.

Then the Eqs. (13) – (16) become

$$\begin{aligned}
\frac{\partial}{\partial t^*}(D^2 - a^2) w^* &= a R^{1/2}\left[M_1 D\phi^* - (1 + M_1(1-S_T)T^*)\right] + M_1 M_5 a R^{1/2} D\phi^* - M_1 M_5 a R^{1/2}(1-S_T) T^* \\
&\quad + (D^2 - a^2)^2 w^* + a R_S^{1/2}\left[1 + M_4 + M_4 M_5^{-1}\right] S^* + (1 - Vz^{*2})(D^2 - a^2)^2 w^* \\
&\quad - \frac{(1 - Vz^{*2})}{k^*}(D^2 - a^2) w^* - 2V(D^2 - a^2) w^* - 4Vz^*(D^2 - a^2) Dw^* + \frac{2Vz^*}{k^*} Dw^*
\end{aligned} \quad (18)$$

$$P_r\left[\frac{\partial T^*}{\partial t^*} - M_2 \frac{\partial}{\partial t^*}(D\phi^*)\right] = (D^2 - a^2) T^* + a R^{1/2}(1 - M_2 - M_2 M_5) w^*, \quad (19)$$

$$P_r \frac{\partial S^*}{\partial t^*} = \tau (D^2 - a^2) S^* - a R_S^{1/2} M_6 w^* + S_T M_5 M_6^{-1}\left(R/R_S\right)^{1/2}(D^2 - a^2) T^*, \quad (20)$$

$$D^2 \phi^* - M_3 a^2 \phi^* - (1 - S_T) DT^* + M_5 M_6^{-1}\left(R/R_S\right)^{1/2} DS^* = 0, \quad (21)$$

where the following non-dimensional parameters are introduced.

3. EXACT SOLUTION FOR FREE BOUNDARIES USING GALERKIN TECHNIQUE

The simplest boundary conditions chosen, namely free-free, isothermal with infinite magnetic susceptibility χ in the perturbed field keep the problem analytically tractable and serve the purpose of providing a qualitative insight into the problem. The case of two free boundaries is of little physical interest, but it is mathematically important because one can derive an exact solution, whose properties guide our analysis. Thus the exact solution of the system subjected to the boundary conditions

$w^* = D^2 w^* = T^* = D\phi^* = S^* = 0$ at $z^* = -1/2$ and

$$z^* = +1/2. \quad (22)$$

is written in the form

$$w^* = A w_1(z) e^{\sigma t^*} \cos \pi z^*,\ \ T^* = B T_1(z) e^{\sigma t^*} \cos \pi z^*,\ \ S^* = C S_1(z) e^{\sigma t^*} \cos \pi z^*$$

$$D\phi^* = F \phi_1(z) e^{\sigma t^*} \cos \pi z^*,\ \ \phi^* = \frac{F}{\pi} \phi_1(z) e^{\sigma t^*} \sin \pi z^* \quad (23)$$

Substituting Eq. (23) in linearized perturbation dimensionless equations (Eqs 18-21), we get the following equations

$$\begin{aligned}
&\left\{\begin{aligned} &\sigma (D^2 - a^2)^2 w_1(z) - (1 - Vz^2)(D^2 - a^2)^2 w_1(z) \\ &+2V(D^2 - a^2)\left[1 + 2zDw_1(z)\right] + \left(\frac{(1 - Vz^2)\mathrm{a}^2}{k_1}\right) w_1(z) - \frac{1}{k_2} 2vzDw_1(z) \end{aligned}\right\} A \\
&+ aR^{1/2}\left\{1 + M_1(1 + M_5)(1 - S_T) T_1(z)\right\} B - aR_S^{1/2}(1 + M_4 + M_4 M_5^{-1}) S_1(z) C \\
&+ aR^{1/2}\ M_1(1 + M_5) D\phi_1(z) F = 0
\end{aligned} \quad (24)$$

$$aR^{1/2}(1 - M_2 - M_2 M_5) w_1(z) A + (D^2 - a^2 - P_r \sigma) T_1(z) B + P_r \sigma M_2 D\phi_1(z) F = 0, \quad (25)$$

$$-aR_S^{1/2} M_6 w_1(z) A + S_T M_5 M_6^{-1}\left(\frac{R}{R_S}\right)^{1/2}(D^2 - a^2) T_1(z) B + \left\{\tau (D^2 - a^2) - \sigma P_r\right\} S_1(z) C = 0, \quad (26)$$

$$-R_S^{1/2} \pi^2 (1 - S_T) DT_1(z) B + R^{1/2}\ M_5 M_6^{-1} DS_1(z) C + R_S^{1/2}(D^2 - a^2 M_3)\phi_1(z) F = 0, \quad (27)$$

For existence of non-trivial solutions, the determine of the coefficients of A, B, C and F must vanish. This determinant on simplification yields

$$X\sigma^3 - Y\sigma^2 + Z\sigma + W = 0. \tag{28}$$

where,

$$X = -c_5 P_r \left\langle S_1 \ \left\langle P_r T_1 \ \ c_1 w_1 \right\rangle \phi_1 \right\rangle$$

$$Y = c_1 c_5 \left\langle T_1 \ \left\langle P_r\, S_1 \ \ c_1 w_1 \right\rangle \ \phi_1 \right\rangle - c_5 P_r \tau c_1 \left\langle T_1 \ \left\langle S_1 \ \ w_1 \right\rangle \phi_1 \right\rangle$$

$$\begin{aligned} Z = &+\frac{a^2}{k_1} c_5 \left\{ \left\langle \phi_1 \ \left\langle P_r\, T_1 \ \ \tau c_1 S_1 \right\rangle (1-Vz^2) w_1 \right\rangle - \left\langle \phi_1 \ \left\langle c_1 T_1 \ \ P_r S_1 \right\rangle \ (1-Vz^2) w_1 \right\rangle \right\} \\ &+\frac{2}{k_2} c_5 \left\{ \left\langle VzDw_1 \ \left\langle c_1 T_1 \ \ P_r S_1 \right\rangle \ c_5 \phi_1 \right\rangle - \left\langle Dw_1 \ \left\langle P_r\, T_1 \ \ \tau c_1 S_1 \right\rangle \ \phi_1 \right\rangle \right\} - c_1^2 c_5 \left\langle (1-Vz^2) w_1 \left\langle P_r\, T_1 \ \ \tau b_1 S_1 \right\rangle \ \phi_1 \right\rangle \\ &a^2 R \left\langle c_2 D\phi_1 \ \left\langle (1-S_T) DT_1 \ \ w_1 \right\rangle \ P_r S_1 \right\rangle - c_1 c_5 \left\langle T_1 \ \left\langle c_1 w_1 \ \ \tau c_1 S_1 \right\rangle \ \phi_1 \right\rangle + a^2 R c_2 c_4 \left\langle D\phi_1 \ \left\langle P_r T_1 \ \ M_6 w_1 \right\rangle \ DS_1 \right\rangle \\ &+ a^2 R_S c_3 c_5 \left\langle S_1 \ \left\langle P_r T_1 \ \ M_6 w_1 \right\rangle \ \phi_1 \right\rangle + \left\langle (1-Vz^2) c_1^2\, w_1 \ \left\langle c_1 T_1 \ \ P_r S_1 \right\rangle \ c_5 \phi_1 \right\rangle \\ &- \left\langle 2Vc_1 (1+2zDw_1) \ \left\langle c_1 T_1 \ \ P_r S_1 \right\rangle \ c_5 \phi_1 \right\rangle + a^2 R \left\langle (1 + c_2 (1-S_T) T_1) \ \left\langle w_1 \ \ P_r S_1 \right\rangle \ c_5 \phi_1 \right\rangle \\ &+ c_1 c_5 \left\langle \phi_1 \ \left\langle P_r\, T_1 \ \ \tau c_1 S_1 \right\rangle \ 2V(1+2zDw_1) \right\rangle \end{aligned}$$

$$\begin{aligned} W = &a^2 R c_4 c_2 \left\langle D\phi_1 \ \left\langle w_1 \ \ S_T c_1 c_4 T_1 \right\rangle \ DS_1 \right\rangle + c_1^2\, c_5 \left\langle (1-Vz^2) w_1 \left\langle c_1 T_1 \ \ \tau c_1 S_1 \right\rangle \phi_1 \right\rangle \\ &- \left\langle 2Vc_1 (1+2zDw_1) \left\langle c_1 T_1 \ \ \tau c_1 S_1 \right\rangle c_5 \phi_1 \right\rangle \\ &+ \frac{2}{k_2} \left\langle VzDw_1 \ \left\langle c_1 T \ \ c_5 \phi_1 \right\rangle \ \tau c_1 S_1 \right\rangle - \frac{a^2}{k_1} c_5 \left\langle (1-Vz^2) w_1 \ \left\langle c_1 T_1 \ \ \tau c_1 S_1 \right\rangle \ \phi_1 \right\rangle \\ &+ a^2 R c_5 \left\langle (1 + c_2 (1-S_T) T_1) \ \left\langle w_1 \ \ \tau c_1 S_1 \right\rangle \ \phi_1 \right\rangle + a^2 R_S \left\langle c_3 S_1 \ \left\langle w_1 \ \ S_T c_1 c_4 T_1 \right\rangle \ c_5 \phi_1 \right\rangle \\ &+ a^2 R \tau c_1 c_2 \left\langle D\phi_1 \ \left\langle w_1 \ \ (1-S_T) DT_1 \right\rangle \ S_1 \right\rangle \\ &+ a^2 R c_2 c_4 \left\langle D\phi_1 \ \left\langle c_1 T_1 \ \ M_6 w_1 \right\rangle \ DS_1 \right\rangle + a^2 R_S c_3 c_5 \left\langle S_1 \ \left\langle P_r T_1 \ \ M_6 w_1 \right\rangle \ \ \phi_1 \right\rangle \end{aligned}$$

For obtaining stationary instability, the time-dependent term T_4 is equal to zero. From Eq. (28) it is easy to obtain the eigenvalue R_{sc} and upon using $k_2 = \varepsilon k_1$, where ε is non – dimensional parameter governing anisotropy.

$$R_{sc} = \frac{x_1 - a^2 R_S \left(x_2 S_T + x_3 \right)}{\tau x_4 + M_1 (1+M_5) \left[\left(1-S_T\right) \tau x_5 + x_6 S_T + \tau \left(1-S_T\right) x_7 + x_8 \right]}$$

where

$$\begin{aligned} x_1 = &\left\langle (1-Vz^2) c_1^2\, w_1 \ \left\langle \tau c_1 S_1 \ \ c_1 T_1 \right\rangle \ c_5 \phi_1 \right\rangle - \frac{a^2}{k_1} \left\langle (1-Vz^2) w_1 \ \left\langle c_1 T_1 \ \ \tau c_1 S_1 \right\rangle \ c_5 \phi_1 \right\rangle \\ &+ \frac{2}{\varepsilon k_1} \left\langle VzDw_1 \ \left\langle c_1 T \ \ c_5 \phi_1 \right\rangle \ \tau c_1 S_1 \right\rangle - \left\langle c_5 \phi_1 \ \left\langle c_1 T_1 \ \ \tau c_1 S_1 \right\rangle \ 2Vc_1 (1+2zDw_1) \right\rangle \end{aligned}$$

$$x_2 = c_3 c_5 \left\langle \phi_1 \ \ \left\langle w_1 \ \ c_1 c_4 T_1 \right\rangle \ S_1 \right\rangle$$

$$x_3 = c_3 c_5 \left\langle S_1 \ \left\langle P_r T_1 \ \ M_6 w_1 \right\rangle \ \ \phi_1 \right\rangle$$

$$x_4 = \left\langle 1 \ \left\langle w_1 \ \ \tau c_1 S_1 \right\rangle \ \ c_5 \phi_1 \right\rangle$$

$$x_5 = \tau \left\langle c_1 S_1 \ \left\langle c_5 \phi_1 \ \ T_1 \right\rangle \ w_1 \right\rangle$$

$$x_6 = c_4 \left\langle DS_1 \ \left\langle w_1 \ \ c_1 c_4 T_1 \right\rangle \ D\phi_1 \right\rangle$$

$$x_7 = c_1 \left\langle D\phi_1 \ \left\langle w_1 \ \ DT_1 \right\rangle \ S_1 \right\rangle$$

$$x_8 = c_4 \left\langle D\phi_1 \ \left\langle c_1 T_1 \ \ M_6 w_1 \right\rangle \ DS_1 \right\rangle$$

$$c_1 = D^2 - a^2,\ c_2 = M_1 (1+M_5),\ c_3 = 1 + M_4 + (M_4 / M_5),$$
$$c_4 = (M_5 / M_6) \text{ and } c_5 = D^2 - a^2 M_3.$$

where $\langle u,\ v \rangle = \int_{-1/2}^{1/2} uv\, dz$ and w_1, T_1, ϕ_1 and S_1 are trial functions that satisfy the boundary conditions. The above choice of trigonometry function tacitly implies the use of a higher order Galerkin method. For very large M_1, one gets the results for the magnetic mechanism, and the critical thermomagnetic Rayleigh number for stationary mode is calculated using

$$N_{sc} = M_1 R_{sc} = \frac{x_1 - a^2 R_S \left(x_2 S_T + x_3 \right)}{(1+M_5) \left[\left(1-S_T\right) \tau \left(x_5 + x_7 \right) + x_6 S_T + x_8 \right]}$$

4. DISCUSSION OF RESULTS

The effect of temperature dependent viscosity on Soret driven thermohaline convection in a ferromagnetic fluid layer heated from below and salted from above saturating an anisotropic porous medium subjected to a transverse uniform magnetic field has been considered by using the Brinkman model and the linear stability analysis. The small thermal perturbation technique is used and normal mode technique is applied for the perturbation quantities. Here, the free-free boundary conditions are used. The present investigation is carried out through stationary instability.

Before we discuss the important results of the system, we turn our attention to the possible range of values of different parameters arising in the study. The range of values of the temperature dependent viscosity parameter V is assumed from 0.1 to 0.5 [12]. The value of anisotropic effect is considered from 0.03 to 3.1 [22]. The buoyancy magnetization parameter M_1, is assumed to be 1000 [6]. For these type of fluids M_2 will have a negligible value and hence taken to be zero. The range of Salinity Rayleigh number R_S is between -500 and 500 and Soret parameter S_T ranges from -0.002 to 0.002 [22]. The Brinkman model has been used for permeability k ranges from 0.1to 0.9 [20] and the non-buoyancy magnetization parameter M_3 is taken to have from 5 to 25 [22]. The Prandtl number P_r is taken to be 0.01 [20] and the magnetic number M_4, M_5 and M_6 are taken to be 0.1 [18]. The ratio of mass transport to heat transport τ is assumed from 0.03 to 0.011 [20].

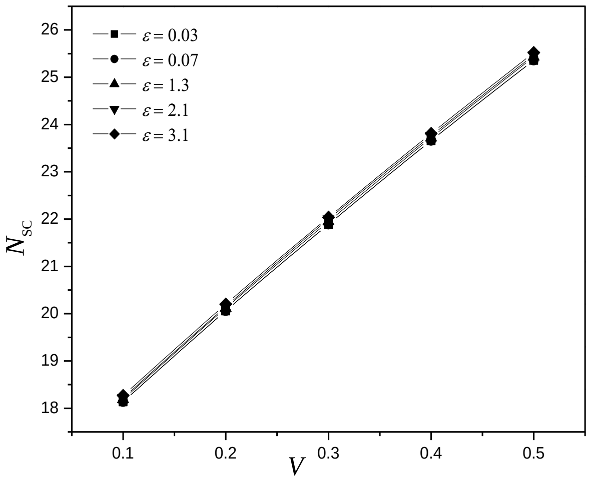

Figure 1. Marginal instability curve for variation of N_{sc} versus V for various values of anisotropy effect ε, R_S = -500, S_T = -0.002, k = 0.1, M_3 = 5 and $\tau = 0.03$

Figure 1 gives the critical magnetic thermal Rayleigh number N_{sc} versus the temperature dependent viscosity V for different values of an anisotropy parameter ε. It is observed from Figure 1 that the temperature dependent viscosity V has a stabilizing on the system when V increases, N_{sc} increases and this stabilizing effect of V is much pronounced. Figure 2 represents the plots of critical magnetic thermal Rayleigh number N_{sc} with respect to non-buoyancy magnetization effect M_3 for various values of temperature dependent viscosity V, S_T = -0.002, R_S = -500, $\tau = 0.05,\ \varepsilon = 0.03$ and k = 0.1. When M_3 and V increases, N_{sc} gets decreasing values. Therefore, the convective system has a destabilizing effect, which is not much pronounced. This is because variation in magnetization releases extra energy which adds up to the thermal energy to destabilize the system.

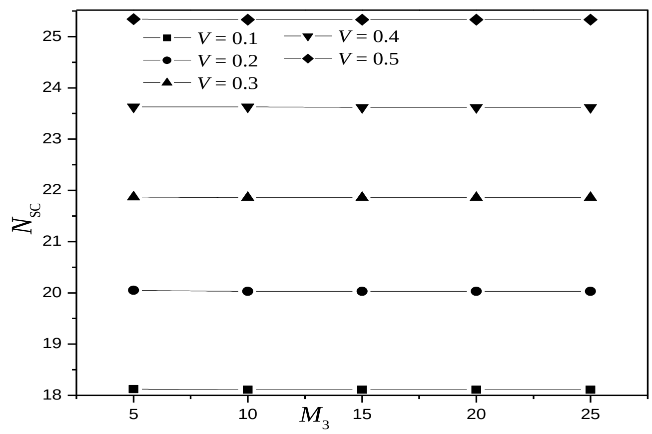

Figure 2. Marginal instability curve for variation of N_{sc} versus M_3 for various values of temperature dependent viscosity effect V, $\varepsilon = 0.03,$ R_S = -500, S_T = -0.002, k = 0.1 and $\tau = 0.03$

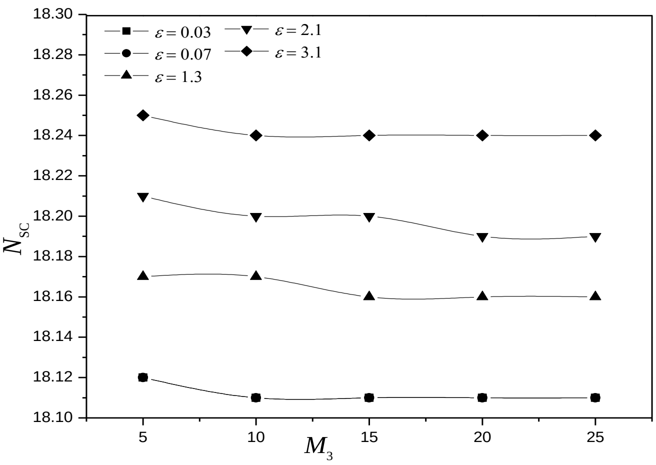

Figure 3. Marginal instability curve for variation of N_{sc} versus M_3 for various values of anisotropy effect $\varepsilon,$ R_S = -500, S_T = -0.002, k = 0.1, V = 0.1 and $\tau = 0.03$

In figure 3, the varation of N_{sc} versus M_3 for different values of anisotropy effect of ε, S_T = -0.002, R_S = -500, $\tau = 0.05$ and k = 0.1. It gives that the non-buoyancy magnetization parameter M_3 has a destabilizing behavior. Also, in the value $\varepsilon = 0.03$ and $0.07,$ the N_{sc} gets an exact same effect on the convective system. There is no much variations in N_{sc} due to the increasing values of M_3 and ε, which is depicted in figure 3. But, there are variations in N_{sc} due to the increasing values of M_3 and V, which is depicted in figure 2. It seems that M_3 has little effect on the stability. Figure 4 represents the plots of N_{sc} versus the salinity Rayleigh number R_S for different values of temperature dependent viscosity V, S_T = -0.002, $\tau = 0.05,\ \varepsilon = 0.03$ and k = 0.1. It shows that the salinity Rayleigh number R_S has a stabilizing behavior on the convective system. The increasing effect of salt on the system, the system gets more thermal energy and it has stabilizing effect.

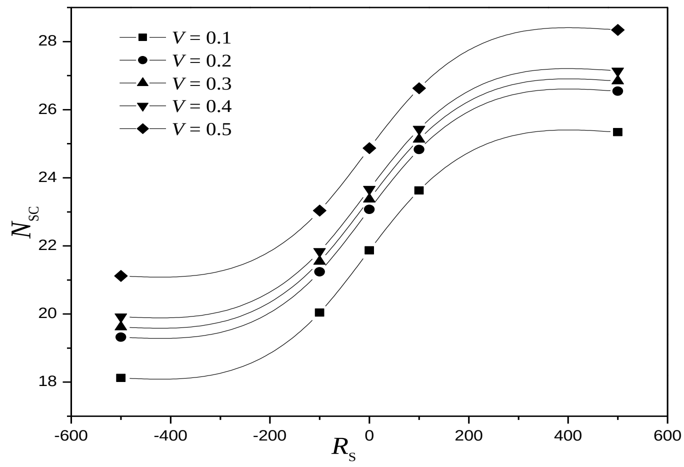

Figure 4. Marginal instability curve for variation of N_{sc} versus R_S for various values of temperature dependent viscosity effect V, $\varepsilon = 0.03,$ M_3 = 5, S_T = -0.002, k = 0.1 and $\tau = 0.03$

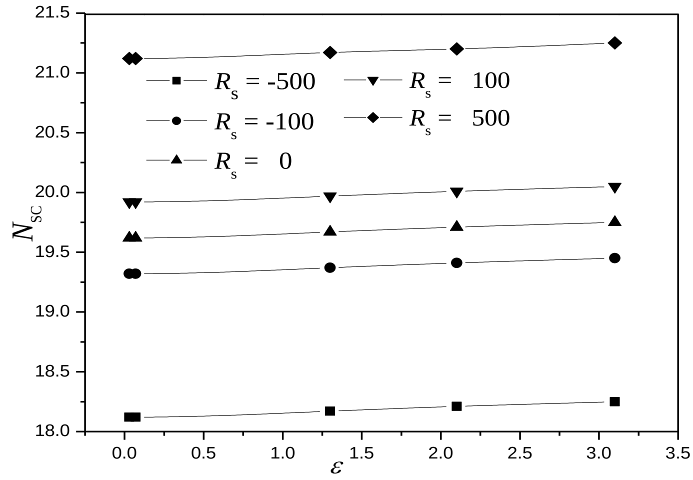

Figure 5. Marginal instability curve for variation of N_{sc} versus ε for various values of salinity Rayleigh number R_S, S_T = -0.002, k = 0.1, M_3 = 5, V = 0.1, and $\tau = 0.03$

In figure 5, the variation of N_{sc} versus ε for different salinity Rayleigh number R_S is investigated. This figure exhibits a stabilizing effect is not much pronounced because the presence of salinity Rayleigh number R_S increase from -500 to 500, N_{sc} increases. Figure 6 indicates the variation of N_{sc} versus the interdiffusion of heat and mass, namely Soret effect S_T for different V. This figure gives as increase of S_T, increase of N_{sc}. This leads to stabilizing effect is much pronounced. But, introducing and increasing of ε on the convective system has a stabilizing behavior is not pronounced much which is depicted in figure 7.

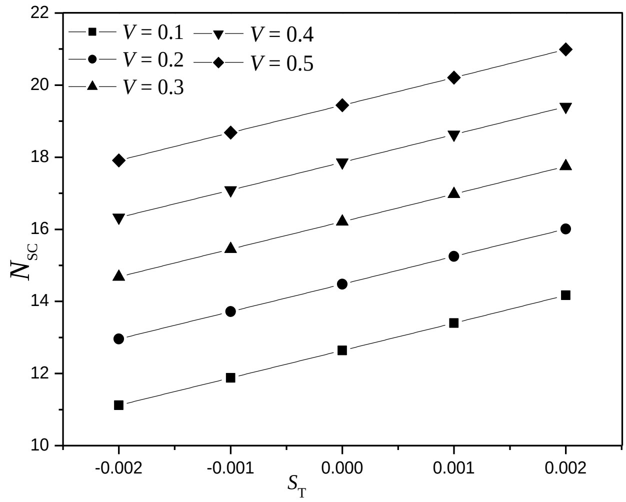

Figure 6. Marginal instability curve for variation of N_{sc} versus S_T for various values of temperature dependent viscosity effect V, $\varepsilon = 0.03$, $M_3 = 5$, $S_T = -0.002$, $k = 0.1$ and $\tau = 0.03$

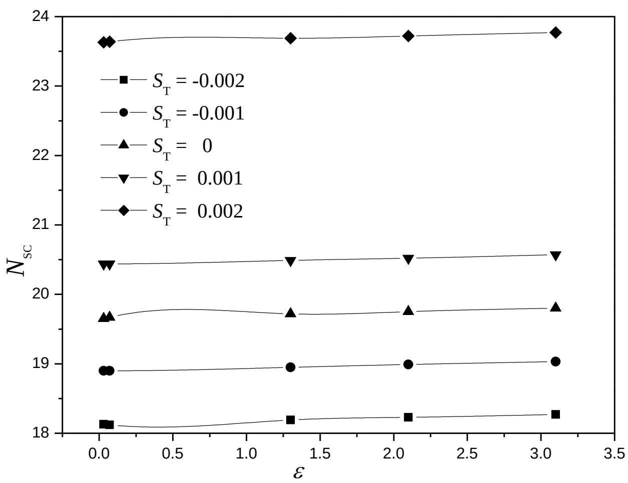

Figure 7. Marginal instability curve for variation of N_{sc} versus ε for various values of Soret coefficient S_T, $R_S = -500$, $k = 0.1$, $M_3 = 5$, $V = 0.1$ and $\tau = 0.03$

It is observed from figure 8 that the increase in the ratio of mass transport to the heat transport τ shows a uniform stabilizing behavior, for increasing value of V and ε, $S_T = -0.002$, $R_S = -500$, $\tau = 0.05$ and $k = 0.1$. In this figure, the system is analyzed for stabilizing effect and when increasing of ε from 0.03 to 3.1, the N_{sc} has same effect. This is because the increase in mass transport adds up to the system to be top heavy. Figure 9 illustrates that N_{sc} versus V for different values of permeability of the porous medium k. When increasing of V from 0.1 to 0.5 and k from 0.1 to 0.9, N_{sc} is increased. It seems that the system stabilization and which is plotted form positive value of R_S (= 500) and S_T (= 0.002) and negative range of R_S (= -500) and S_T (= -0.002), there is no change on the situation of the system. It is also observed from the figures that the increase in pore size make the fluid flow easy to cause convection early.

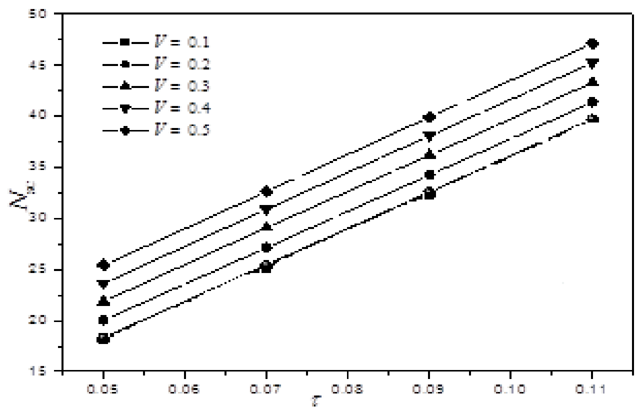

Figure 8. Marginal instability curve for variation of N_{sc} versus τ for various values of V, $S_T = -0.002$, $R_S = -500$, $k = 0.1$, $\varepsilon = 0.03$ and $M_3 = 5$

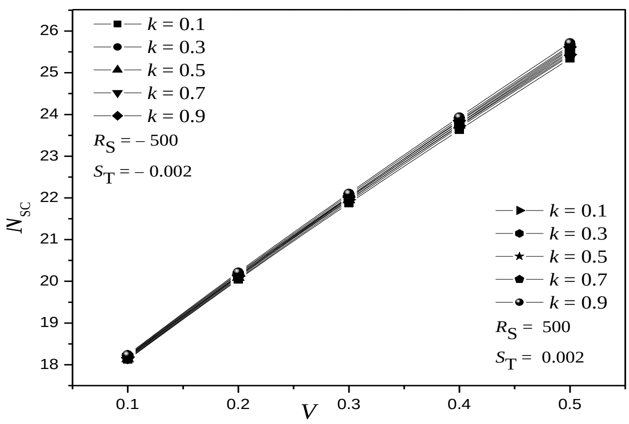

Figure 9. Marginal instability curve for variation of N_{sc} versus V for various values of k ($R_S = -500$, $S_T = -0.002$ and $R_S = 500$, $S_T = 0.002$), $\varepsilon = 0.03$, $M_3 = 5$ and $\tau = 0.03$

5. CONCLUSION

Ferro thermoconvective instability of magnetic fluid layer heated from below and salted from above in the presence of an anisotropic porous medium and temperature field dependent viscosity suspended to a transverse uniform magnetic field has been investigated using Brinkman model with Soret effect. The computational Galerkin method is used. In this analysis, we have analyzed the effect of various parameters like medium permeability k, anisotropic parameter ε, ratio of mass transport to heat transport τ, non- buoyancy magnetization parameter M_3, temperature dependent viscosity parameter V, Soret coefficient S_T and Salinity Rayleigh number R_S.

The destabilizing behavior is analyzed for non-buoyancy magnetization parameter M_3 on the convective system. The stabilizing effect is investigated for the temperature dependent viscosity parameter V in a very small and large value of salinity concentration, which is depicted in figure 9. In this moment, the system gets slight variation in convection process. Moreover, in some investigations, the porous medium k and anisotropy effect ε are analyzed for destabilizing effect [22]. But the introduction of temperature dependent viscosity V, the permeability of the porous medium k and anisotropy effect ε have a stabilizing effect.

Thus, from above analysis, one can conclude that the non-buoyancy magnetization parameter, temperature gradient, porous medium, anisotropic porous medium and salinity gradient have a profound influence on the onset of convection.

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NOMENCLATURE

B	Magnetic induction (T)
C_{vH}	Effective heat capacity at constant volume and magnetic field ($kJm^{-3}K^{-1}$)
d	Thickness of the fluid layer (m)
D/Dt	Convective derivative [$D/Dt = \partial/\partial t + \vec{q}.\nabla$](s^{-1})
g	Gravitational acceleration (0,0-g) (ms^{-2})

H	Magnetic field (Am^{-1})
k_0	Resultant wave number [$k_0 = \sqrt{k_x^2 + k_y^2}$] (m^{-1})
k_1	Permeability of the porous medium
K_1	Thermal diffusivity ($Wm^{-1}K^{-1}$)
K_2	Salinity magnetic coefficient [$\equiv (\partial M / \partial S)_{H_0, T_0}$]
kx ,k_y	Wave number in the x and y direction m^{-1}
K_s	Concentration diffusivity ($Wm^{-1}k^{-1}g^{-1}$)
k	Unit vector in vertical direction
K	Pyromagnetic coefficient [$\equiv -(\partial M / \partial T)_{H_0, T_0}$]
M	Magnetization (Am^{-1})
M_0	Mean value of the magnetization at H = H_0 and T = T_0.
P	Hydrodynamic pressure (Nm^{-2})
q	Velocity of the ferrofluid (u
v	w) (ms^{-1})
S	Solute concentration (kg)
S_0	Average salinity
S_T	Soret coefficient
T	Temperature (K)
T	Time (s)
T_0	Average temperature
ε	Anisotropy parameter

α_t	Coefficient of thermal expansion (K^{-1})
α_s	Solvent coefficient of expansion (K^{-1})
β_t	Uniform temperature gradient (Km^{-1})
β_s	Uniform concentration gradient (kgm^{-1})
δ	Small positive quantity
μ_0	Magnetic permeability of vacuum (NA^{-2})
μ_1	Reference viscosity at T = T_0
μ	Dynamic viscosity ($kgm^{-1}s^{-2}$)
ρ_0	Mean density of the clean fluid (kgm^{-3})
ρ	Density of the fluid (kgm^{-3})
σ	Growth rate (s^{-1})
φ	Viscous dissipation factor containing second order terms in velocity
ϕ	Magnetic scalar potential (A)
θ	Perturbation in temperature (K)
χ	Magnetic susceptibility [$\equiv (\partial M / \partial H)_{H_0, T_0}$]
∇	Vector different operator [$\equiv \vec{i}(\partial / \partial x) + \vec{j}(\partial / \partial y) + \vec{k}(\partial / \partial z)$]