Probability-possibility hybrid systems for merging technical indices

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ABSTRACT. The goal of any trader is to buy low and sell high, and thus make maximum revenue with minimum risk of loss. In the financial market, prices change on daily bases, leaving traders with confusion about what decision to take, hold, buy, or sell, and when to take it. Many market analysis techniques were introduced to help traders take a winning decision at the right time, one of which was technical analysis. Technical analysis uses indicators to forecast trend and price movements. Therefore this analysis technique aids traders with the decision making process. This analysis technique has shown great success, which made it the resort of most financial traders. However, the efficiency of this type of analysis is affected by many factors, putting into it a great deal of uncertainty, ambiguity and vagueness. In this study, three decision support systems based on a hybrid probabilistic-possibilistic general approach, are proposed and tested on historical prices of the EuroStoXX50, and the CAC40 indices. The following systems take advantage of the statistical claims of probability on historical data, the interpretability and uncertainty handling competences of possibility theory, and the foreseeing abilities of technical indicators, all merged together to arm traders with a reliable daily decision that assures risk-discounted revenue, with a contribution of efficiently taking advantage of multiple indicators.

RÉSUMÉ. Le but d’un gestionnaire de portefeuille est d’acheter bas et de vendre haut afin d’optimiser les rendements et de réduire les risques de perte. Face aux changements quotidiens, les gestionnaires doivent régulièrement prendre une décision de vendre, d’acheter ou de conserver leurs titres. De nombreuses techniques d’analyses ont été introduites afin de prendre la bonne décision. L’analyse technique, la plus utilisée, est basée sur des indicateurs financiers qui permettent de définir les tendances et de prévoir les variations des valeurs liquidatives des titres. Ces indicateurs financiers aident le gestionnaire dans la prise de décisions. Les succès effectifs ainsi que la simplicité de mise en œuvre de cette approche expliquent le fort intérêt suscité parmi les gestionnaires de portefeuilles. Cependant l’efficacité de cette approche est diminuée par plusieurs facteurs tels que l’incertitude,
l’ambiguïté ou l’imprécision de l’information fournie par ces indicateurs. Dans cet article, trois systèmes de décision basés sur une approche hybride probabilité-possibilité sont proposés et testés sur des données historiques de l’EuroStoXX50 et le CAC40. Le système proposé a pour avantage l’utilisation conjointe de l’extraction de l’information à partir des données historiques grâce aux probabilités et de la gestion de l’incertitude/imprécision de cette information grâce aux approches possibilistes. L’objectif est de fournir aux gestionnaires une décision plus robuste par fusion de plusieurs indicateurs que celle fournie par les indicateurs séparément.

KEYWORDS: technical analysis, technical indicators, probability, possibility fusion, probability-possibility Dubois-Prade transformation, Kullback Leibler divergence.

MOTS-CLÉS : analyse technique, indicateurs techniques, probabilités, fusion possibiliste, transformation probabilité possibilité de Dubois-Prade, Kullback Leibler.

1. Introduction

Alors que le débat sur la prédiction des marchés financiers est toujours en cours, prédire les mouvements futurs des prix a toujours été le désir de tous les cerveaux capables, des traders quotidiens aux analystes, des chercheurs et même des citoyens ordinaires. Pour se plonger dans le marché, il est nécessaire d’en analyser les tendances. Pour cette fin, deux approches très apparentes sont apparues au fil des ans : l’analyse fondamentale et l’analyse technique, comme le décrit Edwards et al., (2007). Depuis sa naissance, l’analyse technique est devenue un sujet d’intérêt pour de nombreux investisseurs, ce qui est la raison de sous-tendre ce travail. Malgré ses pouvoirs prémonitoires, l’analyse technique est souvent associée à la peur, car ses succès ne sont pas toujours assurés. Ce succès dépend souvent de la manière dont les signaux sont interprétés, des actions choisies et de l’exactitude avec laquelle les paramètres des indicateurs ont été sélectionnés, variant d’un indicateur à l’autre.

La finance comportementale décrit en profondeur l’intégration des émotions et de la psychologie humaine dans les marchés financiers. Kahneman et al., (1982) ont montré l’irrationalité qui accompagne les décisions humaines en temps de crise, laissant les marchés inefficaces à certains moments. Pour contourner ces biais humains, des méthodes de raisonnement et des techniques de l’intelligence artificielle ont longtemps été appliquées dans la finance pour la prédiction des marchés et l’évaluation des risques des portefeuilles. De nombreuses techniques allant des réseaux neuronaux, à la logique floue, à la probabilité et aux algorithmes génétiques ont été intégrées à la finance pour atteindre les objectifs les plus élevés qui sont d’atteindre le maximum de profit avec le minimum de perte.

Pour exemple, (Zhou et al., 2004) dans leur papier, ont intégré l’incertitude cognitive de l’analyse technique en utilisant un système basé sur la logique floue. Leur approche a démontré la capacité à déceler et interpréter les motifs techniques, comparé à des techniques d’analyse de motifs visuels utilisées par les experts. Dans son ouvrage (Hiemstra, 1994), l’auteur présente un système de prédiction du marché action et introduit la structure de l’architecture du système de support basé sur la logique floue. Un autre modèle de trading combinant la logique floue et l’analyse technique...
examined to help in finding patterns and trends in financial indices and was optimized using genetic algorithms, proposed by Cheung et al., (2007).

The fact that all reasoning methods have their limitations drove the motivation of researchers towards the world of hybrid intelligent systems. In their paper Abraham et al. (2000), the authors refer to Hybrid Intelligent Systems and try to overcome the limitations of each approach when individually applied. They propose integrating different learning and adaptation techniques to achieve synergetic effects through this hybridization. Many hybrid models and approaches have been proposed in the financial market world of research. Lin et al. (2002) developed a trading system model that predicts stock indices using a neuro-fuzzy framework. Their system gave evident high returns in comparison with other investment strategies like neural networks and linear regression models. A neuro-fuzzy system was also suggested in Pantazopoulos et al. (1998) for financial time-series prediction, and in Carlsson et al. (1996) for portfolios evaluation.

Probabilistic-fuzzy approaches have also been tackled. In their paper Van den Berg et al. (2004) combined the interpretability of fuzzy logic with the statistical properties of probabilistic systems through following the Takagi-Sugeno (TK) probabilistic fuzzy model to analyze financial markets. An additional probabilistic-fuzzy system was put to use by Xu et al., (2008), for estimating Value at Risk (VaR) that measures the expected loss of a portfolio.

Through this paper, multiple hybrid probabilistic-possibilistic (prob-poss) based decision support systems are described and tested on real time daily price data of the most traded indices in Europe the EuroStoXX50 and the CAC40, for 2004 till 2006, to provide traders with a multi indicator time saving system that mimics the decision making process with less risk of loss and more granted gain. The systems mechanism is lead by four of the most robust and predictive indicators. In the next section, an overview on technical analysis and the used Indicators is given.

In this paper, we propose multiple hybrid prob-poss decision support systems in order to study the effect of incorporating multiple Indicators instead of Individual ones and address the power of probability and possibility theories in that purpose. Section 2 is devoted to the selected technical indicators. The efficiency of the selected indicators is generally well admitted. Section 3 describes in details the proposed general prob-poss system which acts as a base to the three introduced decision support systems. Section 4 presents the three fusion system proposed in our study, the first decision support system is a majority vote; the second proposed is a possibility fusion system and the last one is also a possibility fusion including indicator’s reliability. A full testing, evaluation, and comparison of the three systems along with further proposed enhancements is detailed in sections 5 and 6. Finally, an overall conclusion of the work is delivered in section 7.

2. Technical Analysis

Technical analysis is the attempt to forecast a security future price movement, through analyzing its historical data. Technical analysts believe that the future can
thus be found in the past feature. They also assert that the fundamentals of a security’s value are all summed up by its price. Therefore, they resort to seeking patterns, trends, and some other price factors and accordingly taking their investment decisions as reminded by Achelis (2001).

Technical analysis had long been regarded with skepticism and doubt of its effectiveness. However, the accumulating evidence of market inefficiency caused a revival of academic interest in technical analysis’ claims. Since then, it has been showing great predictive power compared to other strategies and analysis as reminded by Lo et al. (1988), Neely et al. (1997), and Dempster et al. (2000).

In this paper, four best performing technical indicators were chosen (among the large amount of possible definitions) to be used in the system performance testing stage. Below is a brief overview on technical indicators, along with an introduction to the selected ones and the method to calculate them. In fact, a technical indicator is a series of data points derived by applying simple mathematical formulas, such as moving averages, means and standard deviations to past prices or volume data of a security. Following is an introduction to the selected indices which were well known with high predictive powers, and have successfully passed the test of time. Their aim is to distinguish between Bullish signals where the market is optimistic (i.e. buying signals) from the Bearish signal where the market is pessimistic (i.e. selling signals). Among the huge number of technical indicators, we have chosen the following four indicators for their success throughout time, and their reputed deployment among traders.

2.1. Commodity Channel Index (CCI)

CCI was originally developed by Donald Lambert in 1980. It aids traders in identifying cyclic patterns of securities. It typically oscillates between the levels of -100 and 100, enabling analysts to identify when the asset is overbought or oversold. A cross above the 100 level asserts that the security is being overbought, while a cross below the -100 levels asserts that it is being oversold. Refer to Achelis (2001).

\[
CCI = \frac{TP - MATP}{\sigma \times MD}
\]

Where \(TP\) is the typical Price which is the daily average of a security’s high, low and closing prices; \(MATP\) is the moving average of \(TP\) over \(N\)-period of time; \(MD\) is the mean deviation which is the average difference between \(TP\) and \(MATP\), and, \(\sigma\) is a constant with a default value of 0.015. Refer to Achelis (2001).

2.2. Relative Strength Index (RSI)

RSI is an oscillator type index that was first introduced by Welles Wilder in 1978. It is also used to identify where the security is oversold or overbought. It typically ranges between 0 and 100, with the level of 30 representing the oversold
extreme and the level 70 representing the overbought extreme, check Achelis (2001). Simply expressed, bullish signals are generated when the RSI signal crosses upwards the level of 70. Mathematically, it is defined as

$$\text{RSI} = 100 \frac{\text{RS}}{1 + \text{RS}}$$  \hspace{1cm} (2)$$

Where $\text{RS}$ is the ratio of total positive returns to the total negative returns in the last N number of days.

2.3. Moving Average Convergence Divergence (MACD)

MACD is a momentum indicator that follows trends. It was introduced by Gerald Appel in the late 1970s, refer to Achelis (2001). This indicator basically shows the relationship between two moving averages the MACD, which is a difference between 12-day Exponential Moving Average ($\text{EMA}_{12\ days}$) and the 26-day Exponential Moving Average ($\text{EMA}_{26\ days}$) and the signal line which is an exponential moving average of the MACD signal itself. EMA is similar to a simple moving average, except that more weight is given to the latest data.

$$\text{MACD Line} = \text{EMA}_{12\ days} - \text{EMA}_{26\ days}$$ \hspace{1cm} (3)$$

$$\text{Signal Line} = \text{EMA}(\text{MACD})_{9\ days}$$ \hspace{1cm} (4)$$

Bullish signals are generated when the MACD signal crosses the Signal line upwards while Bearish signals are generated when the MACD signal crosses the Signal line downwards.

2.4. Bollinger Bands (BB)

BB is volatility Indicator Introduced by John Bollinger in the 1980s. This indicator mainly includes three bands, the upper, middle, and lower bands. These bands almost act as moving average envelopes of the price as reminded by Achelis (2001). Bullish signals are generated when the Price signal crosses above the Upper Band, Middle Band, or Lower Band while Bearish signals are generated when the Price signal crosses below the Upper Band, Middle Band, or Lower Band. These bands are defined as:

$$\text{Middle Band} = \text{SMA}_{20\ days}$$ \hspace{1cm} (5)$$

$$\text{Upper Band} = \text{SMA}_{20\ days} + c\alpha$$ \hspace{1cm} (6)$$

$$\text{Lower Band} = \text{SMA}_{20\ days} - c\alpha$$ \hspace{1cm} (7)$$
Where SMA is a simple moving average over a period of time, c is a constant with a default value of 2, and α is the 20-day standard deviation of price.

Technical analysis and its Indicator can give vast returns when well interpreted. But it has a great factor of human emotions affecting it, which necessitates the resemblance of this human emotion incorporation. For satisfying this exact need one should take advantage of probability’s statistical claims and Possibility Theory’s reasoning approximation of uncertainty and vagueness. A full description of the general mechanism is given in section 3.

3. Probability Possibility General System

The proposed general probability possibility system is basically divided into three modules: Technical Indicator Module (TMI), Probability Module (PM), and Transformation Module (TM). The system is fed up with time series of daily security prices to be examined, and it generates an output of possibility distribution functions representing the possibility distributions of the decisions (Buy, Hold, and Sell) for the respective number of indicators used. This system is not a decision support system. It acts as the bedrock to the following proposed decision support systems in this paper. The system architecture is illustrated in Figure 1.

![Flow Diagram of the Probability-Possibility General System](image)

Below is a detailed explanation of each module.

3.1. Technical Indicators Module

The Indicator Module is the first module in the described system; it takes an input of N-period daily price data of a security, and estimates from those prices the values of the four used Indicators.

![TIM input-output](image)
The mathematical formulas used to estimate each of the four Indicators, are given by Eq 1 to 7.

3.2. Probability Module

This module takes as input the previously estimated indicator values at historically known winning dates of buying, holding and selling. Those dates are derived by subtracting the closing price at date \( \mathbf{D}_t \) from the price after 5 days \( \mathbf{D}_{t+5} \). If the difference is above a winning threshold (2% in the results presented below), then, this date is labeled as a winning buy date. On the other hand, when the difference is below a threshold (also 2% in the results presented below) then, the day is labeled as a winning sell date. Elsewhere, it is considered as a hold date.

![Figure 3. Probability Module I/O](image)

Having the winning dates of buying, holding and selling from historical data and the estimated daily indicator values from the previous module, one can easily generate, for each indicator, the probability distribution functions of its values at the specified set of dates for the three decisions. A non-parametric density estimation technique was chosen since the used indicators are not of a preset form. As for the use of histograms, it has been eliminated for its lack of continuity, which is important for the proposed fusion processes. Therefore, the following widely used formula for kernel density estimate was used.

\[
f(x) = \frac{1}{n} \sum_{i=1}^{n} K(x - x_i)
\]

Where \( K \) is the kernel density estimator of data sample \( (x_1, \ldots, x_n) \). It is a symmetric function that integrates to one, and \( n \) is the number of data samples used.

3.3. Transformation Module

In this module, a probability to possibility transformation is applied using Dubois-Prade symmetric transformation techniques introduced in (Dubois et al., 2004). Following those techniques, the module transforms all indicators’ BUY,
HOLD and SELL probability distributions into possibility distributions as illustrated in figure 4. The purpose of this transformation is to deduce daily degrees of membership, from each indicator, to the three decisions, hence preparing the data for further fusion process.

The symmetric probability-possibility transformation \( p \rightarrow \pi \) suggested by Dubois et al., was adopted in this module. It is defined by:

\[
\pi_i = \sum_{j=1}^{n} \min(p_i, p_j)
\]  

(9)

Where \( n \) is the number of Indicator values data samples forming the probability distribution, and \( p_i \) and \( p_j \) are the probability estimates at indices \( i, j \).

This transformation allows mapping any indicator value into a degree of membership on a scale from 0 to 1 for each decision. Notice that, the obtained possibility distribution depends on the considered indicator delivered by the technical Indicators’ Module. In other words, the generated possibility distribution may correspond to one or several indicator(s).

4. Decision Support Systems

This section presents three tested decision systems, followed by a comparison of efficiency and performance in the following section. The first is a decision fusion system. The second is a possibility based fusion system, also the last system is possibility-based but it integrates reliability factors to the fusion and decision making process.
4.1. Majority Vote Decision Support System

As its name indicates, this system simply uses the majority vote of decisions recommended by N (4 in our case) indicators used to aid the trader in making a decision at a certain date. It uses the modules of the general prob-poss system with an added Module for Majority Vote.

![Figure 5. Flow Diagram of the Majority Vote System](image)

The system workflow is as follows:

– Applying the steps of the probability-possibility general system described above to obtain the possibility distributions of buying, holding, and selling for all the indicators selected.

– Mapping for each indicator its daily values to the possibility distribution, and attain at that date the possibility degrees on a scale from 0 to 1 for buying holding and selling.

– Considering at the level of each indicator the decision with the maximum possibility as the recommended decision by that indicator. Note that in case two decisions have the same degree of membership, the recommended decision in that case becomes holding.

– Choosing the most frequent decision among the recommended decisions as the majority vote to be taken at that that date. Note that in case of equal frequency of decisions by indicators, Holding is considered as the action to be taken.

### Table 1. Majority Vote Illustration Example

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Buy</th>
<th>Hold</th>
<th>Sell</th>
<th>Recommended Decision (Max Degree)</th>
<th>Majority Vote Is</th>
<th>Buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSI</td>
<td>0.2</td>
<td>0.6</td>
<td>0.8</td>
<td>Sell</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CCI</td>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
<td>Buy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BB</td>
<td>0.8</td>
<td>0.6</td>
<td>0.4</td>
<td>Buy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MACD</td>
<td>0.4</td>
<td>0.8</td>
<td>0.7</td>
<td>Hold</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 illustrates an example the majority vote decision at a certain date. Each indicator value at that date was mapped to the possibility distribution functions.
generated by the prob-poss general system, reaching by that the possibility degrees of memberships displayed in the table. The decision with the highest degree was selected as the recommended decision by the indicator, illustrated by the grey highlighted cells in the table. In the above example RSI suggests Selling, both CCI and BB suggests Buying, and MACD suggests Holding, therefore Buying is the most frequently recommended decision among Indicators, which makes it the majority vote at that date. The testing and performance of the system on real time historical prices is described in section 5.

4.2. Possibility Fusion Decision Support System

Similar to the majority vote system, this system uses as a base to its work the modules of the prob-poss general system figure 6. The difference between this system and the majority vote is that, instead of following a decision recommended by one of the indicators, it employs three fusion techniques on the decisions possibility distributions of multiple indicators.

The systems workflow is as follows:

– Applying the steps of the probability-possibility general system described above to obtain the possibility distributions of buying, holding, and selling for all the indicators selected.

– Mapping for each indicator its daily values to the possibility distribution, and attain at that date the possibility degree of membership on a scale from 0 to 1 for buying, holding, and selling.

– Computing for each decision the Maximum, Average, and Minimum possibility degrees of membership among the degrees of the different indicators in use.

– Performing the three fusion techniques: Maximum of Maximums (MoMax), Maximum of Averages (MoAvg), and Maximum of Minimums (MoMin). Each technique will suggest a daily decision individually.

  - MoMax technique is applied by choosing the decision with the maximum degree among the maximums estimated in step 3.

  - MoAvg technique is applied by choosing the decision with the maximum degree among the averages estimated in step 3.

  - MoMin technique is applied by choosing the decision with the maximum degree among the minimums estimated in step 3.
Table 2. Possibility Fusion Illustration Example

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Buy</th>
<th>Hold</th>
<th>Sell</th>
<th>Fusion Techniques Decisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>MACD</td>
<td>0.2</td>
<td>0.6</td>
<td>0.9</td>
<td>---</td>
</tr>
<tr>
<td>RSI</td>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
<td>---</td>
</tr>
<tr>
<td>CCI</td>
<td>0.8</td>
<td>0.6</td>
<td>0.4</td>
<td>---</td>
</tr>
<tr>
<td>BB</td>
<td>0.4</td>
<td>0.7</td>
<td>0.6</td>
<td>---</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.8</td>
<td>0.7</td>
<td>0.9</td>
<td>MoMax: Sell</td>
</tr>
<tr>
<td>Average</td>
<td>0.4</td>
<td>0.6</td>
<td>0.5</td>
<td>MoAvg: Hold</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.2</td>
<td>0.3</td>
<td>0.1</td>
<td>MoMin: Hold</td>
</tr>
</tbody>
</table>

The fusion with its three techniques is illustrated in Table 2, where the degrees of memberships of the decisions by each indicator are shown above. Similar to the majority vote system; daily prices are delivered to the prob-poss general system, where daily degrees of membership to decisions are obtained from each indicator. Then, for each decision the Maximum, Average, and Minimum degrees were computed. Taking as an example the Buy decision in the above table, it has a Maximum of 0.8 degree proposed by CCI, an average of \((0.2+0.5+0.8+0.4)/4=0.4\), and a Minimum of 0.2 coming from MACD. Finally the daily decision for each technique was considered to be the maximum degree. Thus, in the above example MoMax proposed a Sell, while both MoAvg and MoMin proposed a Hold at that date.

4.3. Possibility Fusion with Reliability Decision Support System

This system uses as a base the Possibility fusion system modules with additional steps to the probability and Fusion modules. The difference between this system and the above introduced one is that instead of giving all indicators the same importance upon fusion, a reliability factor is allocated for each indicator according to its robustness, and then incorporated in the fusion process, figure 7.

Figure 7. Flow Diagram of the Reliable Possibility Fusion System
To estimate the reliability of each indicator, first in the probability module two steps are added after estimating the probability distributions of decisions for each indicator, and one step was added to the Possibility Fusion Module where the reliability is introduced to the fusion process.

1st Step: It is noted that for any indicator, the farther the indicators probability distributions of buying and selling are from each other, the easier it is to distinguish the decisions of that indicator and the higher its reliability becomes. On the other hand, the more the distributions overlap the lower becomes the indicator’s reliability. For achieving that purpose, the Kullback Leibler Divergence (distance) between the BUY and SELL probability density functions is calculated for each indicator. The Kullback-Leibler divergence, also known as the relative entropy, is a measure of how different two probability distributions (over the same event space) are from each other, introduced by Kullback et al. (1951). The KL divergence of probability distributions B, S on a finite set x is defined as:

$$D_{KL}(B|S) = \int_{-\infty}^{\infty} \ln \left( \frac{b(x)}{s(x)} \right) b(x) \, dx$$  \hspace{1cm} (10)

Where $b(x)$ and $s(x)$ denote the densities of B and S.

2nd Step: Interpreting the Kullback Leibler distance ($D_{KL}$) to a reliability factor between 0 and 1 $0 < \beta_i < 1$, in order to be used by the fusion module. For reaching that goal, Sigmoid function is applied to the Kullback Leibler distances from the earlier step.

$$\beta_i = \frac{1}{1 + e^{-\gamma D_{KL}}}$$  \hspace{1cm} (11)

Where $D_{KL}$ refers to the Kullback Leibler Divergence measures, and $\gamma$ could be any constant value as long as it is equally chosen for all Indicator reliability factors. Notice the two extreme conditions:

When $\beta_i \approx 0$ \hspace{1cm} $\mu_i = 1 \hspace{1cm} \forall i$ \hspace{1cm} $\mu_i' = 1$

When $\beta_i = 1$ \hspace{1cm} $\mu_i = \mu_i \hspace{1cm} \forall i$ \hspace{1cm} $\mu_i' = \mu_i$

For reaching this purpose the following formula of sigmoid is applied on the earlier calculated Kullback Leibler distance measures.

3rd Step: Before applying the same three fusion techniques of the earlier introduced system MoMax, MoAvg, and MoMin, for each indicator the maximum between its decision degrees and the level $1 - \beta_i$ is calculated, and then the same fusion techniques are applied on the values of $\text{Max}(\ln \alpha_i, 1 - \beta_i)$. The purpose of this added step is basically discarding all non efficient decision degrees of membership of an indicator by considering the level $1 - \beta_i$ as the new base for the indicator’s distribution, thus applying the fusion to the more efficient decisions.
Table 3. Possibility Fusion with Reliability Illustration Example

<table>
<thead>
<tr>
<th>Indicators</th>
<th>1-(\beta_i)</th>
<th>Buy</th>
<th>Hold</th>
<th>Sell</th>
<th>Fusion Techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max(Macd,1-(\beta_{Macd}))</td>
<td>0.3</td>
<td>0.3</td>
<td>0.6</td>
<td>0.9</td>
<td>---</td>
</tr>
<tr>
<td>Max(RSI,1-(\beta_{RSI}))</td>
<td>0.4</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>---</td>
</tr>
<tr>
<td>Max(CCI,1-(\beta_{CCI}))</td>
<td>0.3</td>
<td>0.8</td>
<td>0.6</td>
<td>0.4</td>
<td>---</td>
</tr>
<tr>
<td>Max(BB,1-(\beta_{BB}))</td>
<td>0.5</td>
<td>0.5</td>
<td>0.7</td>
<td>0.6</td>
<td>---</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.8</td>
<td>0.7</td>
<td>0.9</td>
<td></td>
<td>MoMax: Sell</td>
</tr>
<tr>
<td>Average</td>
<td>0.5</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>MoAvg: Hold</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.3</td>
<td>0.4</td>
<td>0.4</td>
<td></td>
<td>MoMin: Hold</td>
</tr>
</tbody>
</table>

Table 3 demonstrates the three fusion techniques with incorporating, for each indicator, the reliability factor as mentioned in the above steps description. The degrees of membership in the above table are the result of the choosing the maximum value between the level 1 - \(\beta_i\) and the original decisions’ possibility degrees. The affected degrees are marked in red in the above table, it is noted that the most affected fusion technique with the reliability incorporation was the MoMin, the MoAvg is also affected, and as for MoMax it showed almost a negligible effect on its decisions’ degrees. It is also evident in this example that, the overall decisions of the three techniques did not change at that date. A more detailed testing is addressed in section 5.

5. Systems Performances Comparison and Evaluation

This section includes a detailed view on the performance of the three introduced decision support systems and individual indicators. Each of the systems was tested using real time daily historical prices of two of the most commonly traded European indices, the CAC40 and the EUROSTOXX50, from the year 2004 to the year 2006. The number of shares considered for testing was maximum one share at all times. The return or gain in Price unit and in percentage was calculated upon each Sell action preceded by a Buy action throughout the whole testing period, using the Rate on Investment and the average return in percentage formulas, figure7.

\[
RoI_{total} = \sum_{D=1}^{n} Selling\ cost_{D} - Buying\ cost_{D-\alpha}
\]

\[
Average\ return\ % = \frac{1}{n} \sum_{D=1}^{n} \frac{Selling\ cost_{D} - Buying\ cost_{D-\alpha}}{Buying\ cost_{D-\alpha}} \times 100
\]
Where, **Rate on Investment (ROI)** is the price of the tested Index at the date of selling, and **Buying cost** is the price of the tested index at the date of buying.

The Hit Ratio in percentage of each decision support system and individual indicators were computed according to the following formula.

\[
\text{Hit Ratio} = \frac{N_{\text{winning action}}}{N_{\text{total actions}}} \times 100
\]

Where, \(N_{\text{winning action}}\) is the number of gained trades, and \(N_{\text{total actions}}\) is the total number of trades made through the testing period of time.

### 5.1. System Gain

#### 5.1.1. Majority Vote System Performance

Figure 8, shows a plot of Majority vote system cumulative gain versus that of individual indicators, over three years [2004-2006] of testing on the EuroStoXX50. It is obvious that all indicators along with the majority vote show an increased gain throughout the whole period. This implies no overall loss is marked. Although this is an interesting outcome, it does not fully satisfy the intended work motivation, since two indicators CCI and RSI marked higher return than the Majority vote system. For that purpose, more systems were developed adding the effect of fusion and leaving majority vote as a reference for results’ comparison.
5.1.2. Possibility Fusion System performance

Figure 9, includes two plots. Plot1 showing the % cumulative gain of the three possibility fusion techniques. It can be noticed that MoMax marked the highest return among the three techniques. Plotting the % cumulative gain of MoMax versus individual indicators in plot2 shows a noticeably better performance than the majority vote, where MoMax marked higher return than individual indicators. Despite the remarked success of this system, it treats indicators equally with respect to their efficiency in the system, which is not the case in real time. Therefore, it was mandatory to check the effect of reliability of indicators on the fusion, for a possibility of giving even more promising results.

![Figure 9. Plot1 - % Cumulative Gain of MoMax Vs MoAvg Vs MoMin, Plot2 - % Cumulative Gain of MoMax Vs Individual Indicators](image)

5.1.3. Possibility Fusion with Reliability performance

Figure 10 shows three plots comparing the three fusion techniques with and without reliability. It is evident that the most effect is on the MoMin fusion technique, where the fusion with reliability MoMin shows a remarkably higher return than the fusion without reliability. A slight effect is noted on the MoAvg fusion technique, while a negligible effect is marked with the MoMax fusion technique. Over all, it could be said that the reliability does have a positive effect with increasing the returns of the system.
5.2. Comparison and Evaluation

According to formulas (13-14), the % Average Return and the % Hit Ratio are calculated respectively, for all indicators individually and for the applied systems. The results are included in table 4.

Analyzing the results in table 4, one can notice the coherence of the results with the reasoning of the plots presented earlier. For Majority Vote, both its return and hit ratio indicate better results than some indicators, but not all other indicators.

With the Possibility Fusion without reliability, the MoMax fusion technique marks a higher return than the other two Fusion techniques, higher than all indicators individually, and higher than that of Majority Vote System. The Reliability Fusion marks even higher results with MoMax whether with respect to return or hit ratio than all other techniques, also the reliability effect on MoMin is the highest where, there is a noted increase in gain with the use of reliability factors. This paper includes a comparison in testing the proposed systems with respect to each other, while the current work motivation in progress is comparing and
evaluating the system with respect to using other reasoning methods and fusion techniques, such as Bayesian network based probability fusion.

### Table 4. Return of Individual Indicators Vs Decision Support Systems

<table>
<thead>
<tr>
<th>Individual Indicators</th>
<th>EUROSTOXX50</th>
<th>CAC40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% Average Return</td>
<td>% Hit Ratio</td>
</tr>
<tr>
<td>RSI</td>
<td>0.346%</td>
<td>60.8%</td>
</tr>
<tr>
<td>CCI</td>
<td>0.481%</td>
<td>61.8%</td>
</tr>
<tr>
<td>BB</td>
<td>0.212%</td>
<td>61.1%</td>
</tr>
<tr>
<td>MACD</td>
<td>0.192%</td>
<td>50.7%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision Support Systems</th>
<th>Majority Vote</th>
<th>Possibility Fusion</th>
<th>Reliable Possibility Fusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV</td>
<td>0.315%</td>
<td>59.3%</td>
<td>0.595%</td>
</tr>
<tr>
<td>MoMax</td>
<td>0.493%</td>
<td>62.9%</td>
<td>0.869%</td>
</tr>
<tr>
<td>MoAvg</td>
<td>0.394%</td>
<td>62.8%</td>
<td>0.634%</td>
</tr>
<tr>
<td>MoMin</td>
<td>0.261%</td>
<td>54.8%</td>
<td>0.418%</td>
</tr>
<tr>
<td>MoMax</td>
<td>0.494%</td>
<td>63.4%</td>
<td>0.876%</td>
</tr>
<tr>
<td>MoAvg</td>
<td>0.412%</td>
<td>63%</td>
<td>0.631%</td>
</tr>
<tr>
<td>MoMin</td>
<td>0.423%</td>
<td>58.63%</td>
<td>0.577%</td>
</tr>
</tbody>
</table>

### 6. Further Proposed Enhancements

Despite the proven success of the introduced hybrid prob poss systems, current and future aims of enhancing the performance, developing and testing new approaches will always stay a priority. Our current continued work plan includes a hopefully even better performing system, where the constant reliability factors are replaced by dynamic reliabilities that change over time, giving updated reliability factors to the fusion at all times. It is performed by distributing the period of study into equal time frames, where the same reliable possibility fusion system is applied on each frame. An initial explanatory schema is shown in figure 11.

Some other purposed steps forward would be testing the systems on multiple time periods to study any effect of this change on the performance. What would also add value to the work plan is including a portfolio allocation module that guides traders in their investments with single or multiple asset allocation decisions. From the point of view of portfolio management, apart the return, investors are interested in the volatility of the portfolio. Thus, we have to introduce some well know metric of the pair return/volatility such as the Sharpe ratio for instance.
7. Conclusion

Three probability-Possibility based trading systems have been proposed to aid traders in taking less risky and more guided decisions and help them fulfill their aims of making profit. They were designed to act as decision support mechanisms that take advantage of technical Indicator prediction powers, the statistical abilities that probability offers, along with possibility theory claims of interpretability and uncertainty handling. All systems showed evident success when tested on real time historical prices, especially the systems using Possibility Fusion, where the Fusion with reliability being the best preferment among all the considered systems. Many ideas on current systems enhancement and new developed approaches with different reasoning methods are still being applied and continued. The field of reasoning methods application in finance could be considered as a mine of research, where lies an unlimited horizon of innovation.

References


