NUMERICAL STUDY OF MHD, THERMAL RADIATION FREE COVOLUTION HEAT AND MASS TRANSFER FROM VERTICAL SURFACES IN POROUS MEDIA CONSIDERING SORTE AND DUFOUR EFFECTS

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ABSTRACT

In the present approach, a two dimensional steady MHD, thermal radiation free convection flow of heat and mass transfer from a vertical surface in porous media has been analyzed numerically by considering Soret and Dufour effects. The governing non-linear partial differential equations have been transformed by a similar transformation into a system of ordinary differential equations, which are solved numerically by using implicit finite difference scheme. The dimensionless velocity, temperature and concentration profiles are displayed graphically showing the effects for the different values of the Lewis number, soret number, magnetic number and radiation parameter.

Keywords: free convection, porous medium, MHD, Dufour, Soret, thermal radiation and finite difference method.

1. INTRODUCTION

Several studies have been found to analyze the influence of the combined heat and mass transfer process by natural convection in a thermal and/or mass stratified porous medium, owing to its wide applications, such as development of advanced technologies for nuclear waste management, hot dike complexes in volcanic regions for heating of ground water, separation process in chemical engineering, etc. Here stratified porous medium means that the ambient concentration of dissolved constituent and/or ambient temperature is not uniform and varies as a linear function of vertical distance from the origin.

When heat and mass transfer occur simultaneously in a moving fluid, the relation between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradient and this is the Soret or thermal-diffusion effect. These effects are considered as second-order phenomena, on the basis that they are of smaller order of magnitude than the effects described by Fourier’s and Fick’s laws, but they may become significant in areas such as geosciences or hydrology.

The study of magneto hydrodynamics (MHD) flows have stimulated extensive attention due to its significant applications in three different subject areas, such as a astrophysical, geophysical and engineering problems. Free convection in electrically conducting fluids through an external magnetic field has been a subject of considerable research interest of a large number of scholars for a long time due to its diverse applications in the fields such as nuclear reactors, geothermal engineering, liquid metals and plasma flows, among the others. Fluid flow control under magnetic force is also applicable in magneto hydrodynamics generators and a host of magnetic devices used in industries. Steady and transient free convection Coupled heat and mass transfer by natural convection in a fluid-saturated porous medium has attracted considerable attention in the last years, due to many important engineering and geophysical applications. Recent books by Nield and Bejani[1] and Ingham and Pop[2, 3] present a comprehensive account of the available information in the field. The MHD free-convection and Mass Transfer flow with Hall current, viscous dissipation, Joule heating and Thermal diffusion is studied by Singh, A.K[4]. Kishan et al [5] studied the MHD free convection heat and mass transfer in a doubly stratified Darcy porous medium considering soret and Dufour effect with viscous dissipation. Effect of doubly stratification on free convection in Darcian porous medium have been studied by Murthy et al [6], Vidyasagar et al [7] studied the Mass Transfer effects on radiative MHD flow over a non isothermal stretching sheet and embedded in a porous medium. Srinivasulu and Bhaskar Reddy [8] studied the Thermo-diffusion and Diffusion-thermo effects on MHD boundary layer flow past an exponential stretching sheet with thermal radiation and viscous dissipation.

Lakshmi Narayana and Murthy [9] investigated the effects of Soret and Dufour on free convection heat and mass transfer from a vertical surface in a doubly stratified Darcy

The objective of this paper is to study simultaneous heat and mass transfer by natural convection from a vertical surface embedded in a fluid saturated Darciian porous medium under the influence of magnetic field including Soret and Dufour effect in the presence of thermal radiation.

2. PROBLEM FORMULATION

Consider the natural convection in a porous medium saturated with a Newtonian fluid on a vertical flat plate. The x-coordinate is measured along the surface and the y-coordinate normal to it (see Figure 1). The temperature of the ambient medium is $T_{\infty}$ and the wall temperature is $T_{w}$.

The flow along the vertical flat plate contains a species A slightly soluble in the fluid B, the concentration at the plate surface is $C_{w}$ and the solubility of A in B far away from the plate is $C_{\infty}$.

Several assumptions are used throughout the present paper: (a) the fluid and the porous medium are in local thermodynamic equilibrium; (b) the flow is laminar, steady-state and two-dimensional; (c) the porous medium is isotropic and homogeneous; (d) the properties of the fluid and porous medium are constant; (e) the Boussinesq approximation is valid and the boundary layer approximation is applicable; (f) the concentration of dissolved A is small enough.

In-line with these assumptions, the governing equations describing the conservation of mass, momentum, energy and concentration can be written as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{2cm} (1)

$$u = \frac{gK}{\nu} \left[ \beta_{T} (T - T_{\infty}) + \beta_{C} (C - C_{\infty}) \right] - \frac{\sigma}{\rho} \left( B_{0}^{2} \right) u$$  \hspace{2cm} (2)

$$\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{T}}{C_{p} \rho} \frac{\partial^{2} T}{\partial y^{2}} - 1 \frac{1}{\rho C_{p}} \frac{\partial q_{r}}{\partial y}$$  \hspace{2cm} (3)

$$\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{D_{m}}{C_{p}} \frac{\partial^{2} C}{\partial y^{2}} + \frac{D_{m}k_{T}}{C_{p}} \frac{\partial^{2} T}{\partial y^{2}}$$  \hspace{2cm} (4)

Where all quantities are defined in the nomenclature.

The boundary conditions of the problem are

$$y = 0 : v = 0, T = T_{w}, C = C_{w}$$ \hspace{2cm} (5a)

$$y \to \infty : u \to 0, T \to T_{\infty}, C \to C_{\infty}$$ \hspace{2cm} (5b)

Where $T_{w}, T_{\infty}, C_{w}$ and $C_{\infty}$ have constant values.

Using the Rosseland approximation for radiation Chen [19], radiative heat flux is simplified as

$$q_{r} = - \frac{4\sigma_{s} \frac{\partial T^{4}}{\partial y}}{3}$$ \hspace{2cm} (6a)
Where \( \sigma \) and \( \alpha^* \) are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow are such that the term \( T^4 \) may be expressed as a linear function of temperature. Hence, expanding \( T^4 \) in a Taylor series about \( T_\infty \) and neglecting higher order terms we get:

\[
T^4 \approx 4T^3T_\infty - 3T^2_\infty
\]  

(6b)

Using equations (6a) and (6b) equation (3) becomes:

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} + \frac{16 \sigma T^3_\infty}{3 \alpha^* \rho C_p} \frac{\partial^2 T}{\partial y^2}
\]

(7)

"Equations (1), (2), (4), (5), (7)" are now nondimensionalized using the following quantities:

\[
\psi = \alpha_m Ra_x^{1/2} f(\eta), \quad \theta = (T - T_\infty) / (T_w - T_\infty),
\]

\[
\phi = (C - C_\infty) / (C_w - C_\infty),
\]

\[
\eta = \frac{y}{Ra_x^{1/2}},
\]

(8)

Where the stream function \( \psi \) is defined in the usual way

\[
u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}
\]

and \( Ra_x = g \beta(\theta_w - \theta_\infty) x / (\nu \alpha_m) \) is the local Rayleigh number. The governing equations become

\[
f'(1 + M) = \theta + N \phi
\]

(10)

\[
\left(1 + \frac{16}{3R}\right) \theta'' - f \theta' + D_f \phi'' = 0
\]

(11)

\[
\frac{1}{Le} \phi' + f \phi' + S \theta' = 0
\]

(12)

Where \( Le, D_f, S_f, R \) and \( N \) are Lewis, Dufour, Soret numbers, Radiation and sustentation parameter, respectively

\[
Le = \frac{\alpha_m}{D_m}, \quad D_f = \frac{D_m k_f (C_w - C_\infty)}{C_s C_p \alpha_m (T_w - T_\infty)}
\]

\[
S_f = \frac{D_m k_f (T_w - T_\infty)}{C_s C_p \alpha_m (C_w - C_\infty)}, \quad N = \frac{\beta_c (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}
\]

(13)

We notice that \( N \) is positive for thermally assisting flows, negative for thermally opposing flows and zero for thermal-driven flows. The transformed boundary conditions are

\[
f(0) = 0, \theta(0) = 1, \phi(0) = 1
\]

\[
\theta \to 0, \phi \to 0 \text{ as } y \to \infty
\]

(14)

The parameters of engineering interest for the present problem are the local Nusselt number and local Sherwood number, which are given by the expressions

\[
Nu_x / Ra_x^{1/2} = -\theta'(0), \quad Sh_x / Ra_x^{1/2} = -\phi'(0).
\]

(15)

3. MATHEMATICAL SOLUTION

The set of non-linear ordinary differential equations (10) - (12) with boundary conditions (14) have been solved numerically, by using Crank Nicolson implicit finite difference method. A step size of \( \Delta \eta = 0.01 \) was selected to be satisfactory for a convergence criteria of \( 10^{-5} \) in all cases. The value of \( \eta_\infty \) was found to each iteration loop by the statement \( \eta = \eta + \Delta \eta \). In order to see the effect of step size \( \Delta \eta \) we ran the code for our model with two different step sizes \( \Delta \eta = 0.01, \Delta \eta = 0.001 \) and each case we found very good agreement between them.

4. RESULTS AND DISCUSSIONS

Numerical Calculations were carried out for different values of \( D_f, S_r, M \) \( Le, R \) and \( N \). For the purpose of discussing the effect of various parameters on the flow behavior some numerical calculations have been carried out for non dimensional velocity profiles \( f' \), temperature profiles \( \theta \), and concentration profiles \( \phi \).

In Figure 2, the velocity profile \( D_f, S_r, M \) presented for fixed values of \( D_f, S_r, M \). The non dimensional velocity \( f' \) increases with the increasing of \( N \). In Figure 3, we plotted velocity profile \( f' \) for different values of
$D_f, S_r, M, N$. From Figure 3 we observed that the velocity profile increases with the increasing of Lewis number $Le$. From Figure 4 we observed that the velocity profile $f'$ increases with the increase of magnetic parameter $M$.

Figure 5 shows that the velocity profile increases with increase of radiation parameter $R$. Figure 6 shows that the temperature profile increases with the increase of Soret number $S_r$. In Figure 7, the temperature profile plotted for different magnetic parameter $M$. From this figure we observed that the temperature profile decreases with increase of magnetic parameter $M$. Figure 8 plotted for different Radiation parameter $R$. From this figure we observed that the temperature profile increases with the increase of Radiation parameter $R$. Figure 9 shows the effect of Lewis number $Le$ on the concentration. From Figure 9 we observed that the concentration profile decreases with the increase of Lewis number $Le$.

Figure 2. Velocity profiles for different values of $N$ with $S_r = 0.001, D_f = 10.0, M = 0$

Figure 3. Velocity profiles for different $Le$ with $S_r = 0.001, D_f = 10.0, M = 0, N = 1$.

Figure 4. Velocity profiles for different Magnetic parameter with $S_r = 0.001, D_f = 10.0$

Figure 5. Velocity Profiles for different values of $R$ with $S_r = 0.001, D_f = 10.0, M = 0, N = 1$

Figure 6. Temperature profiles for different Soret parameters with $D_f = 10.0, M = 0, N = 1$.

Figure 7. Temperature profiles for different magnetic parameters with $S_r = 0.001, D_f = 10.0$.
5. CONCLUSIONS

In this paper we discussed MHD, Free convection heat and mass transfer from vertical surfaces in porous media with soret, dufour effects under the influence of thermal radiation. Using the similarity transformations a set of ordinary differential equations has been derived for the conservation of mass, momentum and species diffusion. These non linear, coupled differential equations have been solved under valid boundary conditions using implicit finite difference method.

The conclusions of this paper are

- The velocity profile increases with increase of Magnetic parameter and Lewis number.
- The velocity and temperature profiles increases with the increase of Radiation Parameter.
- As increasing of Soret number the temperature profile increases where as the Concentration profile decreases with increase of Lewis number.

6. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$S_r$</td>
<td>Soret number</td>
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<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>Mean absorption coefficient</td>
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<tr>
<td>$R$</td>
<td>Thermal radiation parameter</td>
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<tr>
<td>$x, y$</td>
<td>Cartesian co-ordinates along and normal to the surface, respectively</td>
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<tr>
<td>$\alpha_m$</td>
<td>Thermal diffusivity</td>
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<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzman constant</td>
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<tr>
<td>$C$</td>
<td>Concentration</td>
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<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Concentration susceptibility</td>
</tr>
<tr>
<td>$Ra_s$</td>
<td>Local Rayleigh number</td>
</tr>
<tr>
<td>$u, v$</td>
<td>Darcian velocities in the x and y directions</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Dimensionless temperature</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
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<tr>
<td>$\psi$</td>
<td>Stream function</td>
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<tr>
<td>$\beta_T$</td>
<td>Coefficient of thermal expansion</td>
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<tr>
<td>$\beta_C$</td>
<td>Coefficient of concentration expansion</td>
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<tr>
<td>$\phi$</td>
<td>Dimensionless concentration</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Similarity variable</td>
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<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
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<tr>
<td>$D_m$</td>
<td>Mass diffusivity</td>
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<td>$f$</td>
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<td>$K$</td>
<td>Darcy permeability</td>
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<td>$k_T$</td>
<td>Thermal diffusion ratio</td>
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7. REFERENCES


