STEADY MAGNETOHYDRODYNAMIC VISCO-ELASTIC HEAT GENERATING /ABSORBING SLIP FLOW THROUGH A POROUS MEDIUM WITH RADIATION EFFECT

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ABSTRACT
This paper investigates the combined effect of heat source/sink and radiative heat transfer to steady flow of a conducting optically thick visco-elastic fluid through a channel filled with saturated porous medium induced by non-uniform walls temperature. Analytical solutions of the nonlinear ordinary differential equations governing the flow are obtained using Adomian decomposition method. The effects of different parameters in the model on the temperature and velocity profiles are presented and discussed.
Key-words: - Navier slip, thermal radiation, heat generating/absorbing, Erying-Powell
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1. INTRODUCTION

The study of heat transfer to viscous flow has many important applications in engineering and geophysics. In recent times, quite a number of researches have been done on the heat transfer analysis to oscillatory flows through a channel filled with saturated porous medium. For example, [1] investigated the combined effect of a transverse magnetic field and radiative heat transfer to unsteady flow of a conducting optically thin fluid through a channel filled with saturated porous medium and non-uniform walls temperature. In a related study, [2] reported the effect of slip on the unsteady oscillatory flow of an incompressible viscous fluid through a planer channel filled with saturated porous medium. Other notable work on the convective heat flow based on Bejan's contructal theory includes [3]-[4].

Since the linear Newtonian shear-strain cannot predict the complex rheological behaviour of some complex fluids, over the years a number of models to study this nonlinear behaviour has been developed, one of these models is the Erying-Powel model, which has visco-elastic properties [5]-[6]. In a recent paper, the heat transfer to steady magnetohydrodynamic visco-elastic fluid flow with slip through a saturated porous medium in the presence of thermal radiation in the optically thin limit was studied in [7] using the Erying-Powell model.

In all the studies above, radiative heat transfer to steady hydromagnetic visco-elastic, electrically conducting optically thick fluid flowing through a channel with saturated porous medium in the presence of heat source/sink have not been investigated. However, the study is significant for flows occuring at a very high temperature, for instance in polymer melts and metal processing, various aspects of this radiative heat transfer are documented in [8] – [12].

The objective of the paper is to study the combined effect of thermal radiation in the optically thick limit and the heat source/sink on the conducting visco-elastic fluid flow through porous medium taking the effect of slip at the interface into consideration. In the following sections, the flow governing equations are formulated and non-dimensionalized. Section 3 of the paper deals with the analytical solution of the problem by using Adomian decomposition method [15] and section 4 presents the effect of each parameter in the model on both the temperature and velocity profiles while section 5 concludes the paper.

2. MODEL FORMULATION

We consider steady flow of an incompressible, visco-elastic, electrically conducting and heat-absorbing fluid flow through a saturated porous medium, which is subject to slip boundary condition at the interface of porous and fluid layers. A uniform transverse magnetic field of magnitude B_0 applied to the channel in the presence of thermal radiation, neglecting the induced magnetic field and the Hall effects. Take a Cartesian coordinate system (x, y) where the x-axis lies along the centre of the channel, y is the distance measured in the normal section as shown in figure 1 below.

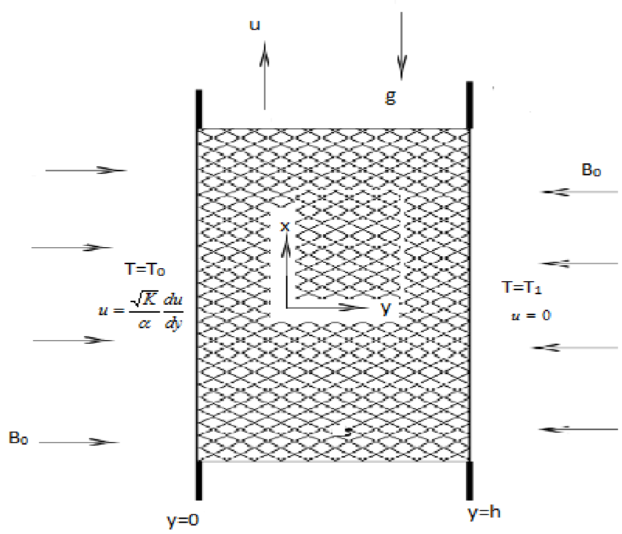

Figure 1:- Problem geometry

Then the governing equations for the fluid flow is given by

$$0=-\frac{1}{\rho}\frac{dp'}{dx'}+\frac{1}{\rho}\frac{d\tau_{xy}}{dy'}-\frac{\nu u'}{K}-\frac{\sigma_e B_0^2 u'}{\rho}+g\beta(T'-T_0'),\quad 0=k\frac{d^2T'}{dy'^2}+Q-\frac{dq_r}{dy'}. \tag{1}$$

Under these assumptions, the appropriate boundary conditions for velocity involving slip flow and temperature fields are given as [2], [7] and [11]

$$u=\frac{\sqrt{K}}{\alpha}\frac{du}{dy}\quad T=T_0\quad y=0,\qquad u=0\quad T=T_1\quad y=h. \tag{2}$$

The Eyring-Powel visco-elastic model can be written as

$$\tau_{xy}=\mu\frac{du'}{dy'}+\frac{1}{\beta_0}Sinh^{-1}\left(\frac{1}{c}\frac{du'}{dy'}\right). \tag{3}$$

With

$$Sinh^{-1}\left(\frac{1}{c}\frac{du'}{dy'}\right)=\frac{1}{c}\frac{du'}{dy'}-\frac{1}{6}\left(\frac{1}{c}\frac{du'}{dy'}\right)^3\quad \left|\frac{1}{c}\frac{\partial u'}{\partial y'}\right|<1. \tag{4}$$

The heat source/sink parameter is given as [12]-[114] to be

$$Q=Q_0(T_0-T). \tag{5}$$

While the radiation heat flux q_r is obtained by the Rosseland approximation

$$q_r=-\frac{4\sigma'}{3k'}\frac{dT^4}{dy}. \tag{6}$$

Assuming that the temperature differences within the flow are such that the term T^4 can be expressed as a linear function of temperature. Therefore, by expanding T^4 in a Taylor series about T_0 and neglecting second and higher order terms, we get

$$T^4\cong 4T_0^3T-3T_0^4. \tag{7}$$

Substituting (3)-(7) in (1), we obtain

$$0=-\frac{1}{\rho}\frac{\partial p'}{\partial x'}+\frac{d}{dy'}\left(\mu\frac{du'}{dy'}+\frac{1}{\beta_0 c}\frac{du'}{dy'}-\frac{1}{6\beta_0}\left(\frac{1}{c}\frac{du'}{dy'}\right)^3\right)-\frac{\nu u'}{K}-\frac{\sigma_e B_0^2u'}{\rho}+g\beta(T'-T_0'),\quad 0=k\frac{d^2T'}{dy'^2}+Q_0(T_0-T)+\frac{16\sigma T_0^3}{3k'}\frac{d^2T'}{dy'^2}. \tag{8}$$

Where

u' is the axial velocity, T'- fluid temperature, p'- pressure, g -gravitational force, β_0, c-Erying–Powell parameters, ν-the kinematic viscosity, β-coefficient of volume expansion due to temperature, *k*-thermal conductivity *K*- porous medium permeability coefficient, B_0 -electromagnetic induction, Q_0 -heat source/sink parameter, σ_e -conductivity of the fluid, ρ- fluid density σ -Stefan–Boltzmann constant, k'-mean absorption coefficient, α -porous parameter.

Introducing the following non-dimensional quantities

$$a=1+\frac{1}{\nu\beta c},\ x=\frac{x'}{h},\ y=\frac{y'}{h},\ u=\frac{u'}{U},\ \theta=\frac{T'-T_0'}{T_w'-T_0'}\ \Pr=\frac{\rho C_p\nu}{k}$$
$$p=\frac{hp'}{\rho\nu U},\ Da=\frac{K}{h^2},\ Gr=\frac{g\beta(T_w'-T_0')}{\mu U},\ H^2=\frac{h^2\sigma_e B_0^2}{\mu},\ Ra=\frac{16\sigma T_0^3k}{3k'}$$
$$\delta=\frac{Q_0h^2}{\rho C_p\nu},\ b=\frac{U_0^2}{2\mu c^3\beta_0h^2},\ s^2=\frac{1}{Da},\ \lambda=-\frac{\partial p}{\partial x},\gamma=\frac{\sqrt{K}}{\alpha h}.$$

We have the dimensionless equation together with appropriate boundary conditions

$$0=\lambda+a\frac{d^2u}{dy^2}-b\left(\frac{du}{dy}\right)^2\frac{d^2u}{dy^2}-\left(s^2+Ha^2\right)u+Gr\theta. \tag{9}$$

Subject to

$$u(0)=\gamma\frac{du(0)}{dy},\ u(1)=0. \tag{10}$$

Together with

$$0=(1+Ra)\frac{d^2\theta}{dy^2}-\delta\Pr\theta, \tag{11}$$

$$\theta(0)=0,\ \theta(1)=1. \tag{12}$$

When $\delta<0$ implies heat absorption while $\delta>0$ heat emission. Da the Darcy number, δ- heat source/sink parameter, γ dimensionless Navier slip parameter, b is a parameter that measures the nonlinearity of the elastic part of the Erying-Powell model, s^2 is the porous medium shape factor parameter. The parameters Gr, H^2, Ra are the Grashof number, Hartmann's number and thermal radiation parameter respectively.

3. METHOD OF SOLUTION

Let us give a brief introduction of the method in its standard form, consider the standard operator [11]

$$Lw+Rw+Nw=m, \tag{13}$$

Where *w* is the unknown function, *L* is the highest order derivative that assumed to be easily invertible, *R* is a linear differential operator of order less than *L, Nu* represents the nonlinear terms, and *m* is the source term. Applying the inverse operator L^{-1} to both sides of (13) and using the given conditions we obtain

$$w=v-L^{-1}(Rw)-L^{-1}(Nw), \tag{14}$$

Where *v* represents the terms arising from integrating the source term *m* and from the auxiliary conditions, the standard ADM defines the solution *w* by the series

$$w=\sum_{n=0}^{\infty}w_n. \tag{15}$$

In addition, the nonlinear term series

$$Nw=\sum_{n=0}^{\infty}A_n. \tag{16}$$

Where A_n are the Adomian polynomials. Substituting (15) in (16) we get,

$$\sum_{n=0}^{\infty}A_n=N\left(\sum_{n=0}^{\infty}w_n\right). \tag{17}$$

By Taylors' series expansion, for any nonlinear function $f(w)$, then the Adomian polynomials (17) are given by

$$\begin{aligned}A_0&=f(w_0)\\A_1&=w_1f^{(1)}(w_0)\\A_2&=w_2f^{(1)}(w_0)+\frac{1}{2!}w_1^2f^{(2)}(w_0)\\A_3&=w_3f^{(1)}(w_0)+w_1w_2f^{(2)}(w_0)+\frac{1}{3!}w_1^3f^{(3)}(w_0)\end{aligned} \tag{18}$$

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The components $w_0, w_1, w_2,\ldots$ are then determined recursively by using the relation

$$\begin{cases}w_0=v,\\ w_{k+1}=-L^{-1}Rw_k-L^{-1}A_k, & k\ge 0.\end{cases} \tag{19}$$

Where w_0 is referred to as the zeroth component. An n-components truncated series solution is thus obtained as

$$S_n = \sum_{i=0}^{\infty} w_i \; . \tag{20}$$

Now, a direct integration of (11) gives

$$\theta(y) = \theta(0) + \int_0^y \left(\frac{d\theta(0)}{dy}\right) dx + \left(\frac{\delta \Pr}{1+Ra}\right) \int_0^y \int_0^y \theta(x)\, dxdx, \tag{21}$$

using the boundary condition (12), we obtain

$$\theta(y) = e\ y + \left(\frac{\delta \Pr}{1+Ra}\right) \int_0^y \int_0^y \theta(x) dxdx. \tag{22}$$

Where $e = \dfrac{d\theta(0)}{dy}$ is to be determined later by using the other boundary condition in (12).

Also integrating (9), we get

$$u(y) = b + \int_0^y c\ dx + \left(\frac{1}{a}\right) \int_0^y \int_0^y \left(-(\lambda + Gr\theta(x)) + b\left(\frac{du(x)}{dy}\right)^2 \frac{d^2u(x)}{dy^2} + \left(s^2 + H^2\right) u(x) \right) dxdx, \tag{23}$$

where $b = u(0),\ c = \dfrac{du(0)}{dy}$ are to be determined using the boundary conditions (10).

To obtain the solution of the integral equations (22) - (23) we assume a series solution of the form

$$\theta(y) = \sum_{n=0}^{\infty} \theta_n(y), \tag{24}$$

$$u(y) = \sum_{n=0}^{\infty} u_n(y). \tag{25}$$

Now, substituting (24) in the integral equation (22) gives

$$\sum_{n=0}^{\infty} \theta_n(y) = e\ y + \left(\frac{\delta \Pr}{1+Ra}\right) \int_0^y \int_0^y \left(\sum_{n=0}^{\infty} \theta_n(x)\right) dxdx. \tag{26}$$

While upon substitution of (25) in (23), we get

$$\sum_{n=0}^{\infty} u_n(y) = u(0) + \int_0^y \frac{du(0)}{dy} dx + \left(\frac{1}{a}\right) \int_0^y \int_0^y \left(-(\lambda + Gr\theta(x)) + b\sum_{n=0}^{\infty} A_n(x) + \left(s^2 + H^2\right) \sum_{n=0}^{\infty} u_n(x) \right) dxdx. \tag{27}$$

where A_n represent the nonlinear term in (23), by using (18) we get

$$A_0 = \left(\frac{du_0}{dy}\right)^2 \frac{d^2u_0}{dy^2},$$
$$A_1 = 2\frac{du_0}{dy}\frac{du_1}{dy}\frac{d^2u_0}{dy^2} + \left(\frac{du_0}{dy}\right)^2 \frac{d^2u_1}{dy^2},$$
$$A_2 = \left(\frac{du_1}{dy}\right)^2 + 2\left(\frac{du_0}{dy}\right)\left(\frac{du_2}{dy}\right)\frac{d^2u_0}{dy^2} + 2\left(\frac{du_0}{dy}\right)\left(\frac{du_1}{dy}\right)\frac{d^2u_1}{dy^2} + \frac{d^2u_0}{dy^2}\frac{d^2u_2}{dy^2}, \tag{28}$$

From (26) we obtain the recursive relations for the temperature field as

$$\theta_0(y) = e\ y,$$
$$\theta_{n+1}(y) = \left(\frac{\delta \Pr}{1+Ra}\right) \int_0^y \int_0^y (\theta_n(x)) dxdx. \tag{29}$$

While the recursive relation for the series solution is given by

$$u_0(y) = u(0) + \int_0^y \frac{du(0)}{dy} dx + \left(\frac{1}{a}\right) \int_0^y \int_0^y (-(\lambda + Gr\theta(x))) dxdx,$$
$$u_{n+1}(y) = \left(\frac{1}{a}\right) \int_0^y \int_0^y \left(bA_n(x) + \left(s^2 + H^2\right) u_n(x)\right) dxdx. \tag{30}$$

Obtaining few terms of the (29) and (30) we obtain the partial sum

$$u(y) = \sum_{n=0}^{k} u_n(y), \tag{31}$$

$$\theta(y) = \sum_{n=0}^{k} \theta_n(y). \tag{32}$$

The accuracy of the solution (31) and (32) can be improved by obtaining more components of the recursive relations.

4. DISCUSSION OF RESULTS

Using mathematica we obtain the following results for the partial sum (31) – (32) and the result are presented in figures (1) – (13). In figure 1, it is observed that for negative value of the heat source/sink parameter there is a rise in the fluid temperature however for positive value of the same parameter in there is a decrease in fluid temperature as observed in figure 2.

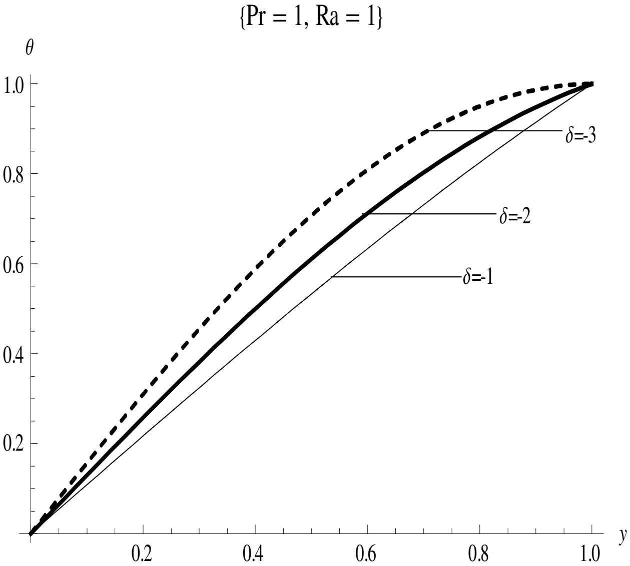

Figure 1: - Temperature profile for variations in heat source parameter

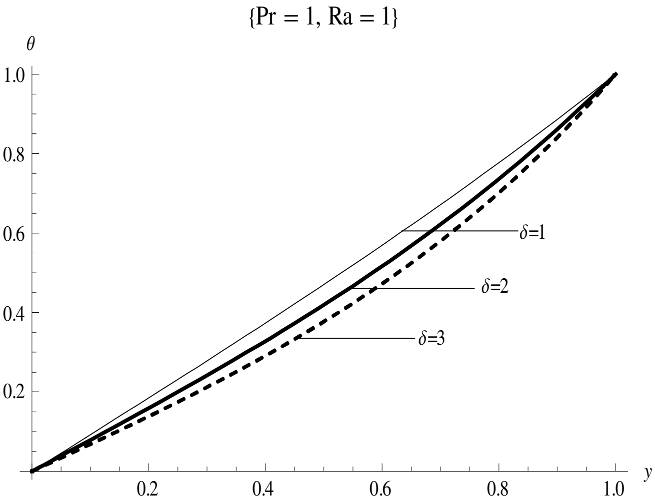

Figure 2: - Temperature profile for variations in heat sink parameter

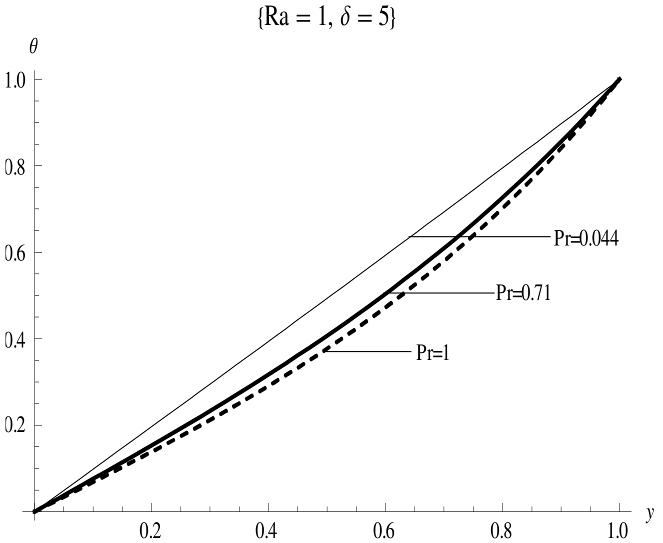

Figure 3: - Temperature profile for variations in Prandtl number

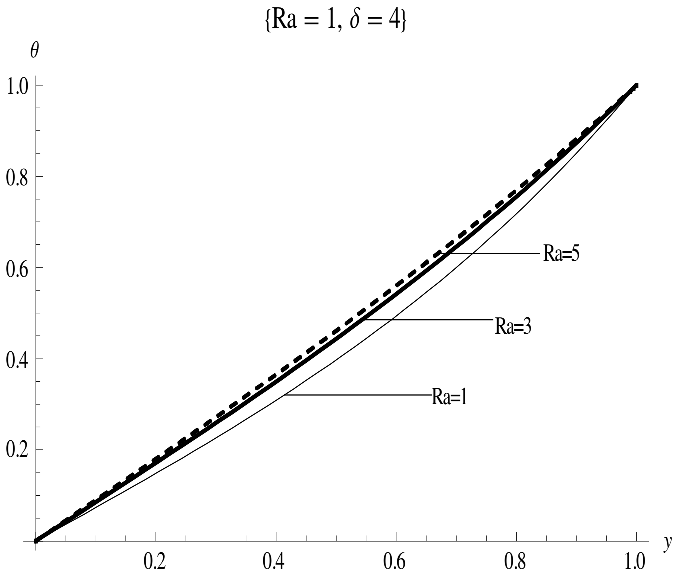

Figure 4: - Temperature profile for variations in radiation parameter

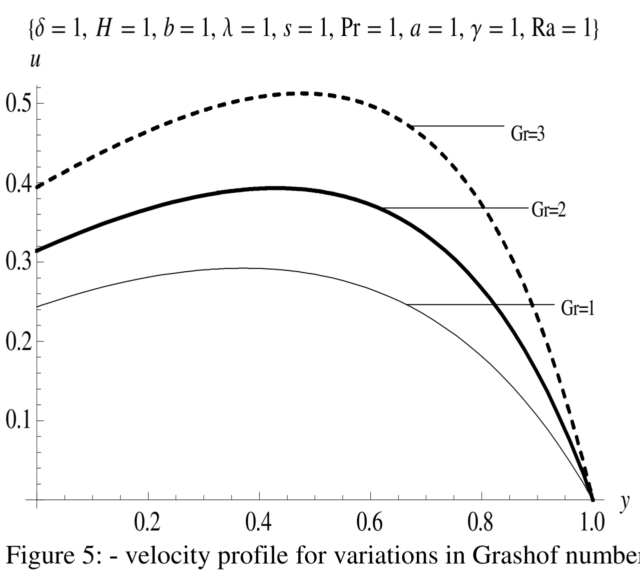

Figure 5: - velocity profile for variations in Grashof number

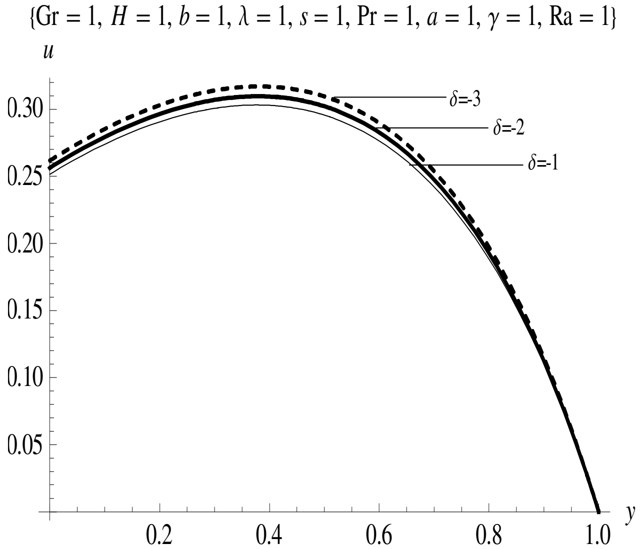

Figure 6: - velocity profile for variations in heat source parameter

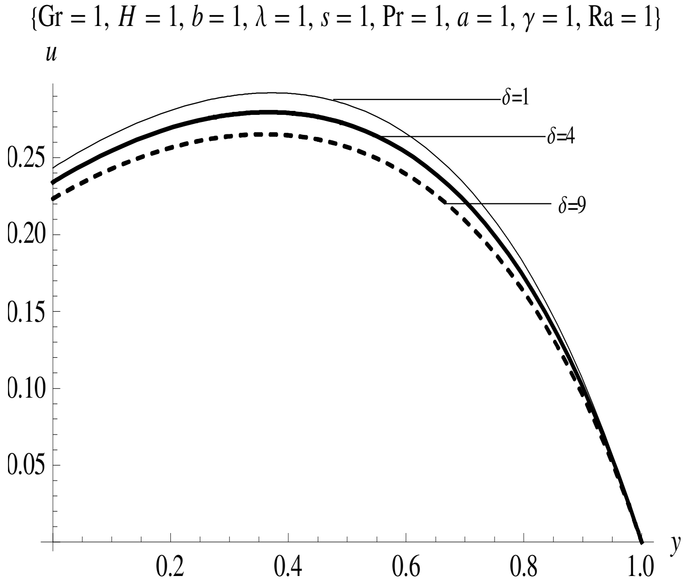

Figure 7: - velocity profile for variations in heat sink parameter

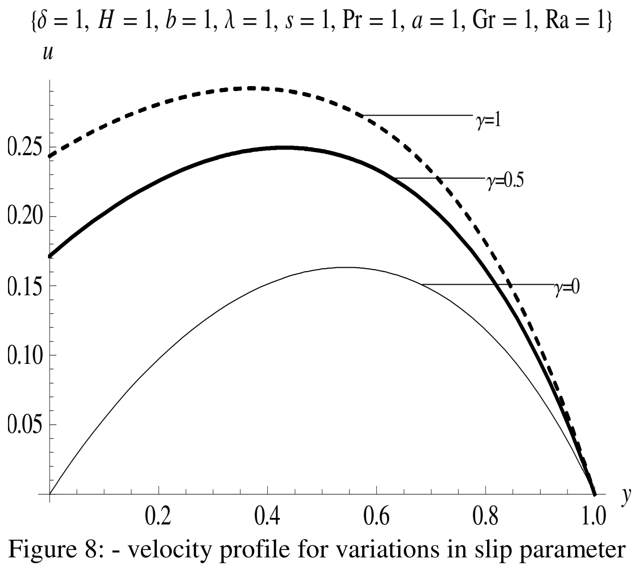

Figure 8: - velocity profile for variations in slip parameter

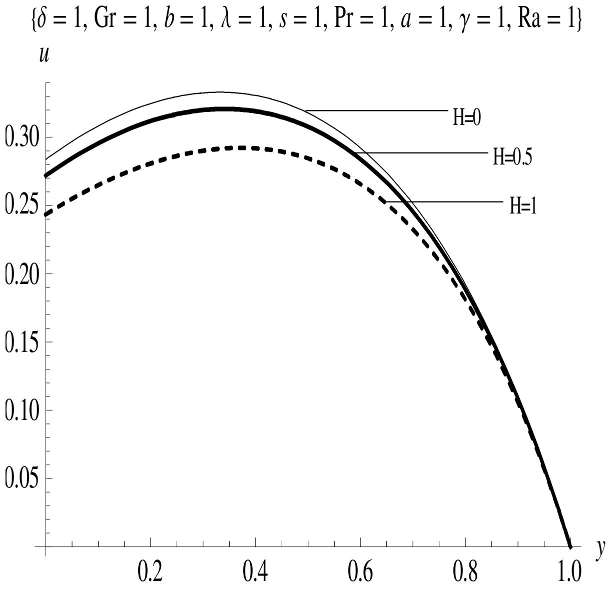

Figure 9: - velocity profile for variations in Hartmann's number

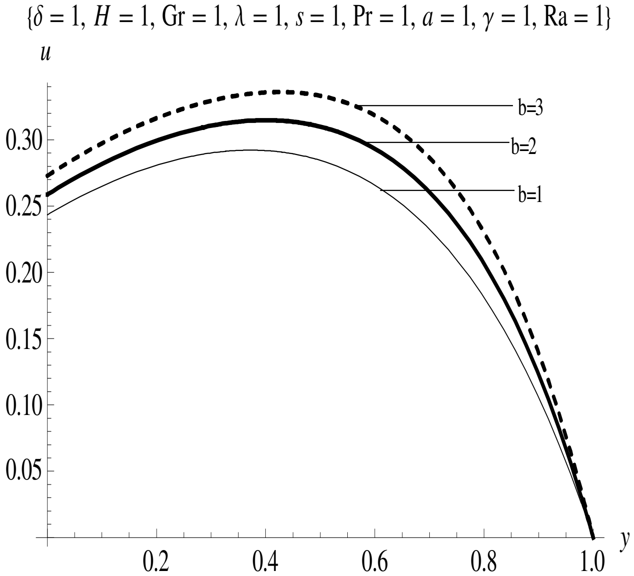

Figure 10: - velocity profile for variations in non-Newtonian parameter

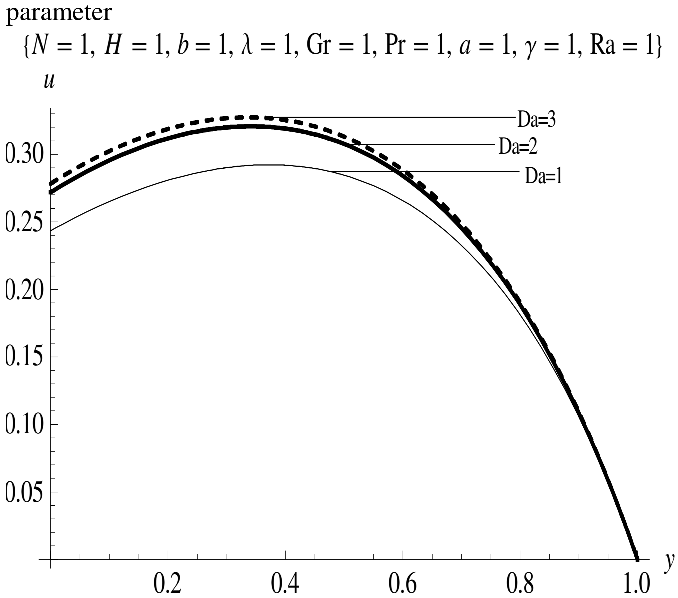

Figure 11: - velocity profile for variations in Darcy number

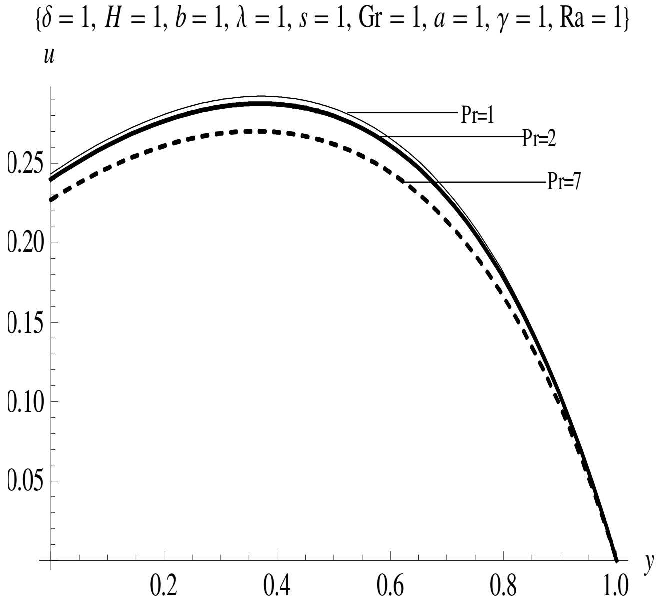

Figure 12: - velocity profile for variations in Prandtl number

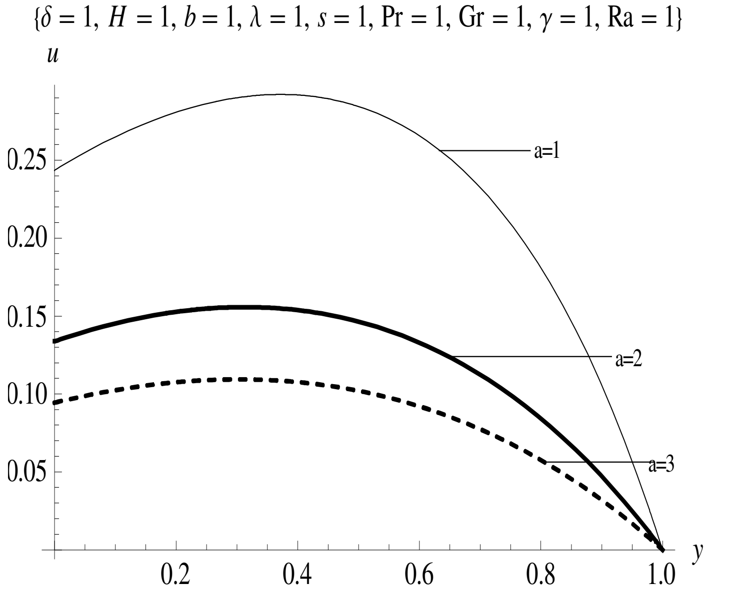

Figure 13: - velocity profile for variations in viscosity parameter

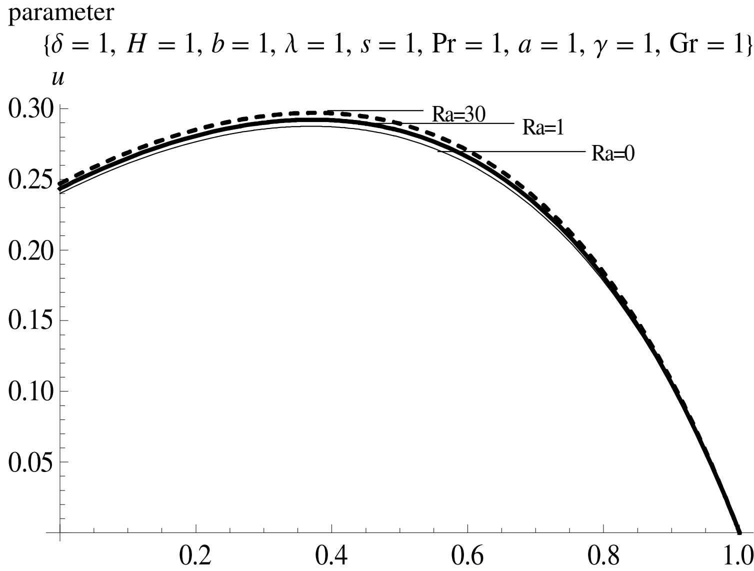

Figure 14: - velocity profile for variations in Radiation parameter

Moreover, figure 3 shows the plot of temperature against the Prandtl number, it is observed here that as the Prandtl number increases there is decrease in the fluid temperature due to decrease in the fluid thermal diffusivity. Figure 4 represents the effect of thermal radiation on the temperature profile, as observed from the graph that as the fluid radiation parameter increases the fluid temperature also rises this is because the fluid absorbs heat from the heated wall. The effect of increase in the Grashof number on the flow is observed in figure 5, and it shows that increase in the Grashof number leads to an increase in the flow velocity due to increase in the buoyancy force. Figures 6 and 7 shows the influence of heat generation and absorption on the velocity profile; as observed from the graph when the fluid absorbs heat there is increase in fluid velocity while emission of heat reduces the fluid velocity as seen in figure 7. In figure 8, the increase in the fluid slip parameter enhances fluid flow velocity at the lower wall. The plot of the flow velocity against the magnetic field parameter as observed in figure 9 shows that as the Hartmann's number increases the fluid velocity also decreases, this true due to the presence of Lorentz force in the magnetic field. The force has a retarding effect on the flow when the magnetic field is placed across the channel. In figure 10, the fluid velocity increases with increases in the non-Newtonian parameter this is due to the rise in the elasticity of the fluid. It is important to note here that if the negative part of the absolute value in (4) is in use then any increase in the non-Newtonian parameter will reduce the flow velocity. Figure 11, shows that increase in channel porous permeability leads to an increase in the fluid flow velocity. In figure 12, the fluid velocity is observed to drop with increase in Prandtl number so also in figure 13 it is observed that increase in fluid viscosity reduces the flow velocity due to increase in frictional force within the fluid particles. Finally as observed from figure 14, it is observed that increase in thermal radiation improves the flow velocity.

5. CONCLUSION

The combined effects of thermal radiation and heat source and sink on non-Newtonian Magneto hydrodynamics visco-elastic fluid flow through a saturated porous medium with slip at the cold plate has been investigated. Using Adomian decomposition method, analytical solutions of momentum and energy equations are obtained. The results show that increase in both radiation and heat absorption parameters increase the fluid temperature and flow velocity of the non-Newtonian fluid within the channel while increase in the heat emission reduce both the fluid temperature and velocity profiles.

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