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UNSTEADY HYDROMAGNETIC NATURAL CONVECTION FLOW WITH HEAT AND MASS TRANSFER OF A THERMALLY RADIATING AND CHEMICALLY REACTIVE FLUID PAST A VERTICAL PLATE WITH NEWTONIAN HEATING AND TIME DEPENDENT FREE-STREAM

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ABSTRACT

Investigation of unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, optically thick radiating and chemically reactive fluid past an infinite vertical plate with Newtonian heating is carried out considering impulsive, accelerated and oscillatory movements of the free-stream. Exact analytical solutions for fluid temperature and species concentration are obtained using Laplace transform technique whereas solution for fluid velocity is obtained numerically using INVLAP routine of MATLAB software along with Laplace tranform technique. The numerical solutions for fluid velocity, fluid temperature and species concentration are displayed graphically whereas the numerical values of shear stress, rate of heat transfer and rate of mass transfer at the plate are presented in tabular form for various values of pertinent flow parameters. Comparison between the present numerical results and the earlier published results is made and there is an excellent agreement between these results.

Keywords: Hydromagnetic natural convection, Newtonian heating, thermal radiation, chemical reaction, free-stream.

1. INTRODUCTION

Effects of radiation on hydromagnetic natural convection flow is of considerable importance in many scientific and engineering applications such as high temperature casting and levitation processes, thermo-nuclear fusion, turbine blade heat transfer, furnace design, glass production, cosmical flight, propulsion systems of aircrafts and re-entry vehicles, solar power technology, heat transfer in thermophotovoltaics radiation filters (Tseng and Viskanta [1]) etc. Keeping in view the importance of such applications, Ogulu and Makinde [2] investigated unsteady hydromagnetic free convection flow of a dissipative and radiative fluid past a vertical plate with constant heat flux. Singh [3] performed numerical study of combined natural convection, conduction and surface radiation heat transfer in cavities. Seth *et al.* [4] discussed MHD natural convection flow with radiative heat transfer past an impulsively moving vertical plate with ramped wall temperature considering Rosseland approximation to radiative heat flux. Kumar *et al.* [5] investigated the effects of thermal radiation and mass transfer on unsteady MHD flow through a porous medium bounded by two porous horizontal parallel plates moving in opposite directions. Recently, Seth *et al.* [6] investigated unsteady hydromagnetic natural convection heat and mass transfer flow of a viscous, incompressible, electrically conducting and optically thick radiating fluid past an impulsively moving vertical plate taking Hall current and rotation into account.

Moreover, in many chemical engineering processes such as polymer production, manufacturing of ceramics or glassware etc. there may occur heterogeneous or homogeneous chemical reaction between foreign mass and ambient fluid. Therefore, the effects of chemical reaction cannot be neglected for those types of processes. Keeping in view of this fact, several researchers, namely, Afify [7], Palanimani [8], Chamkha *et al.* [9], Nandkeolyar *et al.* [10] and Seth *et al.* [11] investigated hydroromagnetic natural convection heat and mass transfer flow of a chemically reactive fluid past a body with different geometry considering various aspects of the problem.

It is well known that heat transfer process depends significantly on the specified thermal conditions. Natural convection flows are usually modelled by researchers under the assumptions of constant surface temperature or ramped wall temperature or constant surface heat flux. But, in many physical situations, the rate of heat transfer from the bounding surface is proportional to the local surface temperature. This type of heat transfer process is sometimes called conjugate convective flow or Newtonian heating. Such situations occur in many important engineering devices which include heat exchangers, gas turbines, seasonal thermal energy storage systems etc. Therefore, it seems to be significant to consider the effects of Newtonian heating in heat transfer process from practical point of view. The pioneering work on Newtonian heating condition is due to Merkin [12] who studied natural convection boundary layer flow over a vertical surface generated by Newtonian heating. Later on, Lesnic *et al.* [13] investigated free convection boundary layer flow along a vertical surface in a porous medium with Newtonian heating. Chaudhary and Jain [14] considered unsteady free convection flow of an incompressible fluid past an infinite vertical plate in which flow is generated by Newtonian heating as well as impulsive motion of the plate. Salleh *et al.* [15] studied mixed convection boundary layer flow over a horizontal circular cylinder with Newtonian heating boundary condition. Narahari and Dutta [16] analyzed the effects of thermal radiation and mass transfer on unsteady natural convection flow of a viscous, incompressible and optically thick radiating fluid near a vertical plate considering Newtonian heating thermal condition. Rajesh [17] investigated effects of mass transfer on flow past an impulsively started infinite vertical plate with Newtonian heating and first order chemical reaction. Recently, Hussanan *et al.* [18] studied unsteady natural convection flow of a viscous fluid past an oscillating plate considering the effects of Newtonian heating and thermal radiation whereas Seth and Sarkar [19] discussed unsteady hydromagnetic free convection flow past an impulsively moving vertical plate with Newtonian heating.

So far, in the above mentioned literature, it is noticed that the unsteadiness in the flow-field is induced due to the time-dependent movement of the plate. But, it was Lighthill [20] who gave the idea of time-dependent movement of free stream in his remarkable article on the response of laminar skin friction and heat transfer due to fluctuations in the stream velocity. He considered two-dimensional laminar boundary layer flow about a cylindrical body when the velocity of the free stream relative to the body oscillates in magnitude but not in direction. Later on, this idea was successfully exploited into several directions of fluid dynamics by many renowned researchers. Mention may be made of the research studies of Lin [21], Warsi [22], Messiha [23], Soundalgekar [24], Raptis and Perdikis [25], Kim [26] and Gorla [27]. Recently, Das *et al.* [28] studied unsteady mixed convective flow of a viscous and incompressible fluid past an infinitely long vertical plate with Newtonian heating considering impulsive, accelerated and oscillatory movements of the free-stream. However, investigation of the effects of an externally imposed magnetic field on the combined heat and mass transfer flow of a heat radiating fluid past a vertical plate with Newtonian heating in the presence of chemical reaction and time dependent movement of the free stream remains untreated so far.

Aim of the present investigation is to study unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, optically thick radiating and chemically reactive fluid past an infinite vertical plate with Newtonian heating and time dependent moving free-stream. For practical interest, three different types, namely, (i) impulsive (ii) accelerated and (iii) oscillatory movements of the free-stream are considered. This type of study may find applications in many areas of science and technology such as solar energy collection systems, fire dynamics in insulations, geothermal energy systems, catalytic reactors, nuclear power reactors, recovery of petroleum products, plasma physics, cosmical flight, nuclear waste repositories etc.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, optically thick radiating and chemically reactive fluid past an infinite vertical flat plate in the presence of thermal and mass diffusions. x'-axis is taken along the vertical plate in upward direction and y'-axis is normal to the plane of the plate. Fluid is permeated by a uniform transverse magnetic field B_0 which is applied in a direction parallel to y'-axis. Initially, i.e. at time $t' \le 0$, both the fluid and plate are at rest and maintained at uniform temperature T'_∞. Also the level of concentration of fluid is maintained at uniform concentration C'_∞. At time $t' > 0$, it is assumed that the rate of heat transfer from the plate is proportional to the fluid temperature T' and free-stream starts to move with a velocity $U(t')$. At the same time level of concentration at the surface of the plate is raised to uniform concentration C'_w. It is assumed that there exists a homogeneous chemical reaction of first order with uniform rate K' between the diffusing species and fluid. Geometry of the problem is presented in Figure 1.

Since plate is of infinite extent along x' and z' directions and is electrically non-conducting, all the physical quantities except pressure depend on y' and t' only. It is assumed that the magnetic Reynolds number is very small so that the induced magnetic field produced by fluid motion is neglected in comparison to applied one. This assumption is justified because magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in industrial processes [29]. Also no external electric field is applied into the flow-field so that the effect of polarization of fluid is negligible. This corresponds to the case where no energy is added or extracted from the fluid by electrical means [29].

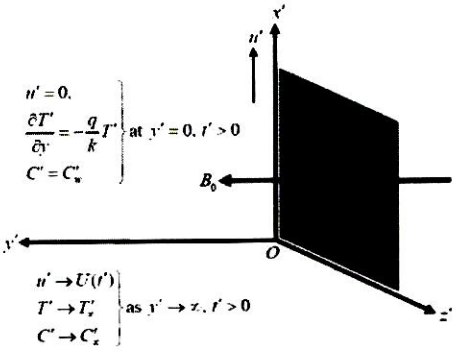

Fig. 1: Geometry of the problem

Taking into consideration the assumptions made above, the governing equations for the natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, optically thick radiating and chemically reactive fluid, under Boussinesq approximation, are given by

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p}{\partial x'} + \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta'\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) - \frac{\sigma B_0^2}{\rho}u', \quad (1)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y'}, \quad (2)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'}, \quad (3)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K'\left(C' - C'_\infty\right), \quad (4)$$

where u', T', C', ρ, υ, σ, g, β', β^*, k, c_p, D, q_r and p are, respectively, fluid velocity, fluid temperature, species concentration, fluid density, kinematic coefficient of viscosity, electrical conductivity, acceleration due to gravity, coefficient of thermal expansion, coefficient of expansion of species concentration, thermal conductivity of fluid, specific heat at constant pressure, chemical molecular diffusivity, radiative heat flux and fluid pressure.

Initial and boundary conditions for the problem are specified as

$$t' \le 0 : u' = 0, T' = T'_\infty, C' = C'_\infty \text{ for all } y', \quad (5a)$$

$$t' > 0 : u' = 0, \frac{\partial T'}{\partial y'} = -\frac{q}{k}T', C' = C'_w \text{ at } y' = 0, \quad (5b)$$

$$: u' \to U(t'),\ T' \to T'_\infty,\ C' \to C'_\infty \text{ as } y' \to \infty, \quad (5c)$$

where q is the constant heat flux.

It is noticed from equation (2) that pressure p is independent of y' i.e. pressure is same within the boundary layer as well as in the free-stream.
Making use of boundary conditions (5c), pressure p is obtained which is given by

$$-\frac{1}{\rho}\frac{\partial p}{\partial x'}=\frac{dU}{dt'}+\frac{\sigma B_0^2}{\rho}U. \tag{6}$$

Using equation (6), equation (1) assumes the form

$$\frac{\partial u'}{\partial t'}=\frac{dU}{dt'}+\upsilon\frac{\partial^2 u'}{\partial y'^2}+g\beta'(T'-T'_\infty)+g\beta^*(C'-C'_\infty)-\frac{\sigma B_0^2}{\rho}(u'-U). \tag{7}$$

Now, for an optically thick gray fluid the radiative heat flux q_r is approximated by Rosseland approximation as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'}, \tag{8}$$

where σ^* and k^* are Stefan Boltzmann constant and mean absorption coefficient respectively.

To linearize T'^4 it is assumed that there is a small temperature difference between the fluid within the boundary layer region and free-stream. With this assumption T'^4 is expanded in Taylor series about $T'=T'_\infty$ and after neglecting the second and higher order terms, we obtain

$$T'^4\cong 4T_\infty'^3T'-3T_\infty'^4. \tag{9}$$

Using equations (8) and (9) in equation (3), we obtain

$$\rho c_p\frac{\partial T'}{\partial t'}=\left(k+\frac{16\sigma^* T_\infty'^3}{3k^*}\right)\frac{\partial^2 T'}{\partial y'^2}. \tag{10}$$

We introduce the following non-dimensional quantities to present equations (4), (7) and (10) along with the initial and boundary conditions (5a) to (5c) in non-dimensional form

$$\left.\begin{array}{l}\eta=U_0y'/\upsilon,\ t=t'U_0^2/\upsilon,\ u=u'/U_0,\ U(t')=U_0f(t),\\ T=(T'-T'_\infty)/T'_\infty,\ C=(C'-C'_\infty)/(C'_w-C'_\infty),\end{array}\right\} \tag{11}$$

where $U_0=q\upsilon/k$ is the characteristic velocity.

Making use of (11), equations (7), (10) and (4), in non-dimensional form, become

$$\frac{\partial u}{\partial t}=\frac{df}{dt}+\frac{\partial^2 u}{\partial \eta^2}+G_rT+G_cC-M^2(u-f), \tag{12}$$

$$\frac{\partial T}{\partial t}=(1+N_r)\frac{1}{P_r}\frac{\partial^2 T}{\partial \eta^2}, \tag{13}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial \eta^2}-KC, \tag{14}$$

where $G_r=\upsilon g\beta' T_\infty'^3/U_0^3$ is thermal Grashof number, $G_c=\upsilon g\beta^*(C'_w-C'_\infty)/U_0^3$ is solutal Grashof number, $M^2=\sigma B_0^2\upsilon/\rho U_0^2$ is magnetic parameter, $N_r=16\sigma^* T_\infty'^3/3kk^*$ is radiation parameter, $P_r=\rho\upsilon c_p/k$ is Prandtl number, $S_c=\upsilon/D$ is Schmidt number and $K=\upsilon K'/U_0^2$ is chemical reaction parameter.

Initial and boundary conditions (5a) to (5c), in non-dimensional form, become

$$t\le 0\ :\ u=0,\ T=0,\ C=0 \text{ for all } \eta, \tag{15a}$$

$$t>0\ :\ u=0,\ \frac{\partial T}{\partial \eta}=-(1+T),\ C=1 \text{ at } \eta=0, \tag{15b}$$

$$:u\to f(t),\ T\to 0,\ C\to 0 \text{ as } \eta\to\infty. \tag{15c}$$

Equations (12)-(14), after taking Laplace transform and using initial conditions (15a), reduce to

$$\frac{d^2\bar{F}}{d\eta^2}-(s+M^2)\bar{F}+G_r\bar{T}+G_c\bar{C}=0, \tag{16}$$

$$\frac{d^2\bar{T}}{d\eta^2}-\lambda s\bar{T}=0, \tag{17}$$

$$\frac{d^2\bar{C}}{d\eta^2}-S_c(s+K)\bar{C}=0, \tag{18}$$

where $\lambda=P_r/(1+N_r)$, $\bar{F}(\eta,s)=\bar{u}(\eta,s)-\bar{f}(s)$,

$\bar{u}(\eta,s)=\int_0^\infty F(\eta,t)e^{-st}dt$, $\bar{T}(\eta,s)=\int_0^\infty T(\eta,t)e^{-st}dt$,

$\bar{C}(\eta,s)=\int_0^\infty C(\eta,t)e^{-st}dt$, $s>0$ is the Laplace transform parameter.

Corresponding boundary conditions for $\bar{F}$, $\bar{T}$ and $\bar{C}$ are given by

$$\left.\begin{array}{lll}\bar{F}=-\bar{f}(s), & \dfrac{d\bar{T}}{d\eta}=-\left(\dfrac{1}{s}+\bar{T}\right), & \bar{C}=\dfrac{1}{s} \text{ at } \eta=0,\\ \bar{F}\to 0, & \bar{T}\to 0, & \bar{C}\to 0 \text{ as } \eta\to\infty.\end{array}\right\} \tag{19}$$

Solutions of equations (16) to (18) subject to the boundary conditions (19) are given by

$$\bar{T}(\eta,s)=\frac{e^{-\eta\sqrt{\lambda s}}}{s\left(\sqrt{\lambda s}-1\right)}, \tag{20}$$

$$\bar{C}(\eta,s)=\frac{e^{-\eta\sqrt{S_c(s+K)}}}{s}, \tag{21}$$

$$\bar{u}(\eta,s)=\begin{cases}\bar{f}(s)\left[1-e^{-\eta\sqrt{s+M^2}}\right]+\dfrac{G_r^*\left(e^{-\eta\sqrt{\lambda s}}-e^{-\eta\sqrt{s+M^2}}\right)}{s(s+\lambda_1)\left(\sqrt{\lambda s}-1\right)}\\ +\dfrac{G_c^*\left(e^{-\eta\sqrt{S_c(s+K)}}-e^{-\eta\sqrt{s+M^2}}\right)}{s(s-\lambda_2)} & \text{for } \lambda\ne 1 \text{ and } S_c\ne 1,\\ \bar{f}(s)\left[1-e^{-\eta\sqrt{s+M^2}}\right]+\dfrac{G_r\left(e^{-\eta\sqrt{s}}-e^{-\eta\sqrt{s+M^2}}\right)}{M^2s\left(\sqrt{s}-1\right)}\\ +\dfrac{G_c\left(e^{-\eta\sqrt{(s+K)}}-e^{-\eta\sqrt{s+M^2}}\right)}{(M^2-K)s} & \text{for } \lambda=1 \text{ and } S_c=1.\end{cases} \tag{22}$$

where $\lambda_1=M^2/(1-\lambda)$, $\lambda_2=(S_cK-M^2)/(1-S_c)$, $G_r^*=G_r/(1-\lambda)$ and $G_c^*=G_c/(1-S_c)$.
Taking inverse Laplace transforms of equations (20) and (21) and using the boundary conditions (19), the exact solutions

for fluid temperature $T(\eta,t)$ and species concentration $C(\eta,t)$ are obtained and are presented in the following form:

$$T(\eta,t)=e^{\frac{t}{\lambda}-\eta}\,erfc\left(\frac{\eta}{2}\sqrt{\frac{\lambda}{t}}-\sqrt{\frac{t}{\lambda}}\right)-erfc\left(\frac{\eta}{2}\sqrt{\frac{\lambda}{t}}\right), \quad (23)$$

$$C(\eta,t)=\frac{1}{2}\left[e^{\eta\sqrt{S_c K}}\,erfc\left(\frac{\eta}{2}\sqrt{\frac{S_c}{t}}+\sqrt{K t}\right)+e^{-\eta\sqrt{S_c K}}\,erfc\left(\frac{\eta}{2}\sqrt{\frac{S_c}{t}}-\sqrt{K t}\right)\right]. \quad (24)$$

The expressions for rate of heat transfer at the plate i.e. $(\partial T/\partial\eta)_{\eta=0}$ and rate of mass transfer at the plate i.e. $(\partial C/\partial\eta)_{\eta=0}$ are presented in the following form:

$$-(\partial T/\partial\eta)_{\eta=0}=e^{t/\lambda}\left[1+erf\left(\sqrt{t/\lambda}\right)\right], \quad (25)$$

$$-(\partial C/\partial\eta)_{\eta=0}=\sqrt{S_c/\pi t}\,e^{-Kt}+\sqrt{S_c K}\,erf\left(\sqrt{Kt}\right). \quad (26)$$

An exact inverse Laplace transform of equation (22) can be obtained only when $M^2=0$ i.e. in the absence of applied magnetic field (Das *et al.* [28]). Therefore, the presence of magnetic field in equation (22) causes the task to obtain analytical solution for fluid velocity impossible. To the best of our knowledge no researcher has yet obtained a closed form analytical solution of the present problem.

3. NUMERICAL SOLUTION

Exact inverse Laplace transform of equation (22) is not possible due to the reasons mentioned above. The numerical solution for fluid velocity is obtained by computing inverse Laplace transform of equation (22) with the help of INVLAP routine (Hollenbeck [30]) of MATLAB software by considering three types of movement of free-stream which are given below:

I. Impulsive movement of free-stream.
In this case, $f(t)=1$ i.e. $\bar{f}(s)=1/s$.

II. Accelerated movement of free-stream.
In this case, $f(t)=t$ i.e. $\bar{f}(s)=1/s^2$.

III. Oscillatory free-stream.
In this case, $f(t)=\cos\omega t$ i.e. $\bar{f}(s)=s/(s^2+\omega^2)$, where $\omega=\omega'\upsilon/U_0^2$ is frequency parameter and ω' is frequency of oscillations.

Shear stress τ at the plate is given by

$$\tau=(\partial u/\partial\eta)_{\eta=0}. \quad (27)$$

4. RESULTS AND DISCUSSION

In order to validate the correctness of the present numerical results, we have compared the numerical values of fluid temperature $T(\eta,t)$ obtained numerically using INVLAP routine of MATLAB software when radiation parameter $N_r=0$ (i.e. in absence of thermal radiation) with those computed from the exact analytical solution by Choudhary and Jain [14] and Das *et al.* [28] for various values of time t taking $P_r=0.71$ which is shown in Figure 2. It is observed from Figure 2 that the present results are in excellent agreement with the results obtained by Chaudhary and Jain [14] and Das *et al.* [28]. Similarly, Figure 3 presents a comparison between the numerical values of species concentration $C(\eta,t)$ obtained numerically using INVLAP routine of MATLAB software with those computed from the exact analytical solution (24) and the solution obtained by Nandkeolyar *et al.* [10] in the absence of suction/blowing using INVLAP routine of MATLAB software for various values of time t when $S_c=0.6$ and $K=2.5$. From Figure 3, it is noticed that the present results are in excellent agreement with the results obtained from the exact analytical solution (24) and the solution obtained by Nandkeolyar *et al.* [10] in the absence of suction/blowing using INVLAP routine of MATLAB software. Therefore, the numerical results obtained in this paper are correct and justified.

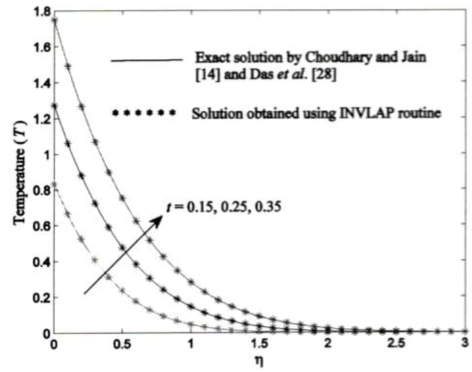

Fig. 2: Temperature profiles when $N_r=0$.

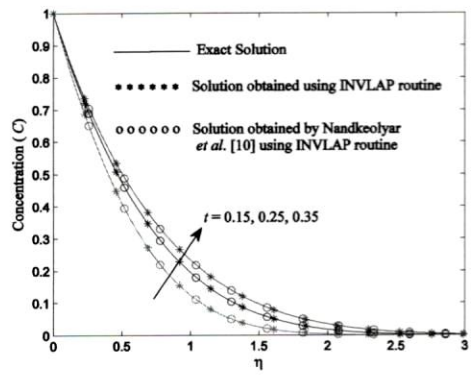

Fig. 3: Concentration profiles when $K=2.5$.

To study the effects of magnetic field, thermal buoyancy force, concentration buoyancy force, chemical reaction, thermal radiation, time and frequency of oscillations on the flow-field, numerical values of fluid velocity u within the boundary layer region are displayed graphically versus boundary layer coordinate η through Figures 4 to 10 for several values of magnetic parameter M^2, thermal Grashof number G_r, solutal Grashof number G_c, chemical reaction parameter K, thermal radiation parameter N_r, time t and frequency parameter ω taking Prandtl number $P_r=0.71$ and Schmidt number $S_c=0.6$. It is evident from Figure 4 that fluid velocity u decreases on increasing M^2 for impulsive, accelerated and oscillatory movements of the free-stream. This implies that magnetic field tends to retard fluid flow within the boundary layer region for impulsive, accelerated and oscillatory movements of the free-stream. This is due to the fact that the application of a magnetic field to an electrically conducting fluid gives rise to a force, called Lorentz force, which has the tendency to resist the fluid motion in the flow-field.

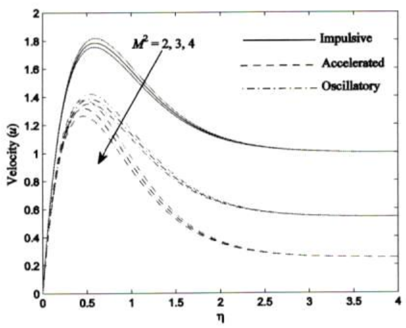

Fig. 4: Velocity profiles when $G_r = 6, G_c = 5$, $N_r = 2$, $K = 2.5$, $\omega = 4$ and $t = 0.25$.

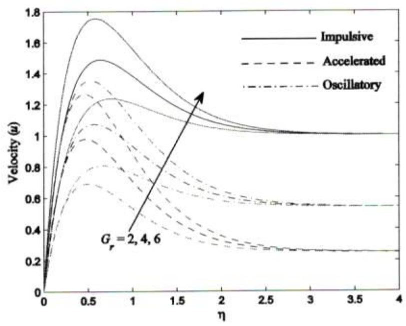

Fig. 5: Velocity profiles when $M^2 = 4, G_c = 5$, $N_r = 2$, $K = 2.5$, $\omega = 4$ and $t = 0.25$.

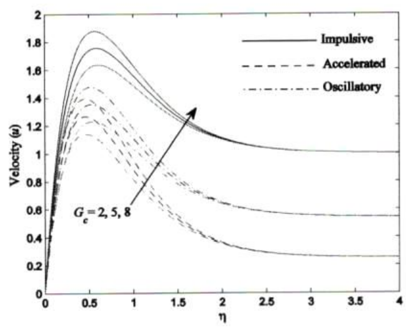

Fig. 6: Velocity profiles when $M^2 = 4, G_r = 6$, $N_r = 2$, $K = 2.5$, $\omega = 4$ and $t = 0.25$.

It is revealed from Figures 5 and 6 that fluid velocity u increases on increasing G_r and G_c for impulsive, accelerated and oscillatory movements of the free-stream. Since G_r measures the relative strength of thermal buoyancy force to viscous force and G_c measures the relative strength of concentration buoyancy force to viscous force, an increase in G_r and G_c lead to an increase in thermal buoyancy force and concentration buoyancy force respectively. This implies that thermal buoyancy force and concentration buoyancy force tend to accelerate fluid flow within the boundary layer region for impulsive, accelerated and oscillatory movements of the free-stream. It is perceived from Figure 7 that fluid velocity u increases on increasing N_r. This implies that thermal radiation tends to accelerate fluid flow within the boundary layer region for impulsive, accelerated and oscillatory movements of the free-stream. It is observed from Figure 8 that fluid velocity u decreases on increasing K for impulsive, accelerated and oscillatory movements of the free-stream.

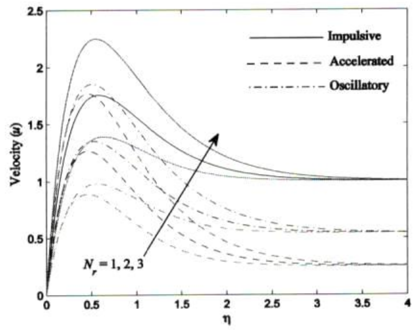

Fig. 7: Velocity profiles when $M^2 = 4, G_r = 6, G_c = 5$, $K = 2.5$, $\omega = 4$ and $t = 0.25$.

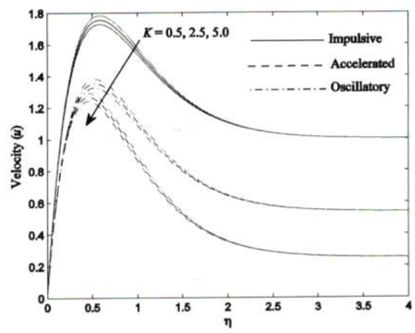

Fig. 8: Velocity profiles when $M^2 = 4, G_r = 6, G_c = 5$, $N_r = 2$, $\omega = 4$ and $t = 0.25$.

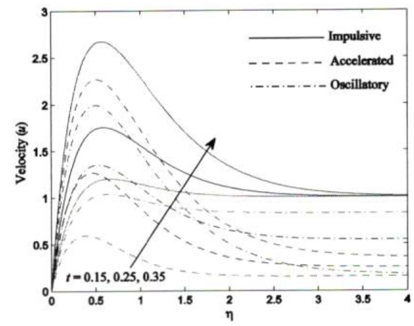

Fig. 9: Velocity profiles when $M^2 = 4, G_r = 6, G_c = 5$, $N_r = 2$, $K = 2.5$ and $\omega = 4$.

This implies that chemical reaction tends to decelerate fluid flow within the boundary layer region for impulsive, accelerated and oscillatory movements of the free-stream. It is noticed from Figure 9 that fluid velocity u increases on increasing t for impulsive, accelerated and oscillatory movements of the free-stream. This implies that, with the progress of time, fluid-flow is getting accelerated within the boundary layer region for impulsive, accelerated and oscillatory movements of the free-stream. It is evident from Figure 10 that fluid velocity u decreases on increasing ω. This implies that frequency of oscillations tends to decelerate fluid flow within the boundary layer region for oscillatory movement of the free-stream.

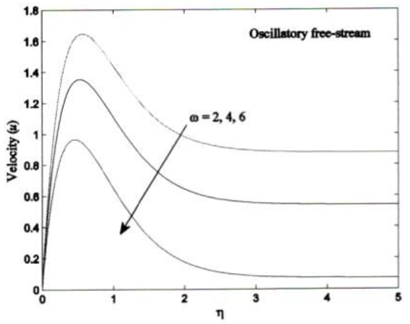

Fig.10: Velocity profiles when $M^2 = 4, G_r = 6, G_c = 5, N_r = 2, K = 2.5$ and $t = 0.25$.

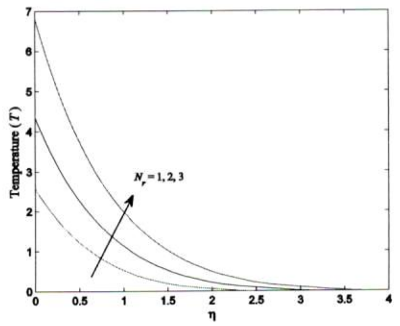

Fig.11: Temperature profiles when $t = 0.25$.

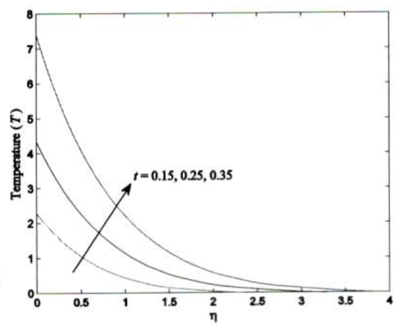

Fig.12: Temperature profiles when $N_r = 2$.

The numerical values of fluid temperature T, computed from the analytical solution (23), are displayed graphically versus boundary layer coordinate η in Figures 11 and 12 for various values of N_r and t taking $P_r = 0.71$. It is revealed from Figures 11 and 12 that fluid temperature T increases on increasing N_r and t. This implies that, throughout the boundary layer region, thermal radiation tends to enhance fluid temperature and fluid temperature is getting enhanced with the progress of time.

The numerical values of species concentration C, computed from the analytical solution (24), are displayed graphically versus boundary layer coordinate η in Figures 13 and 14 for various values of K and t taking $S_c = 0.6$. It is noticed from Figures 13 and 14 that species concentration C decreases on increasing K whereas it increases on increasing t. This implies that chemical reaction tends to reduce species concentration whereas species concentration is getting enhanced with the progress of time throughout the boundary layer region.

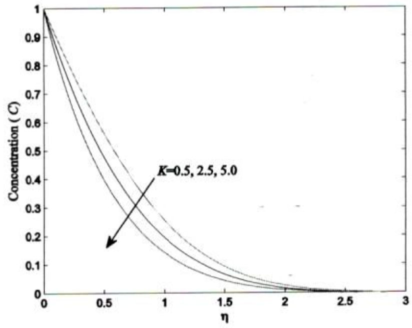

Fig.13: Concentration profiles when $t = 0.25$.

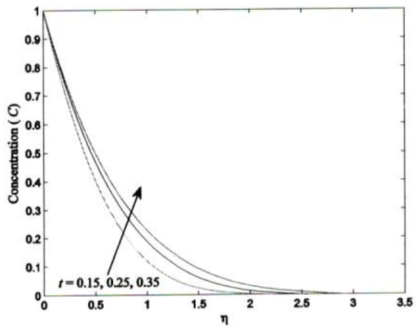

Fig.14: Concentration profiles when $K = 2.5$.

The numerical values of shear stress τ at the plate for impulsive, accelerated and oscillatory movements of the free-stream are displayed in tabular form through Tables 1 to 4 for various values of M^2, G_r, G_c, K, N_r and t taking $P_r = 0.71$ and $S_c = 0.6$. It is evident from Table 1 that τ increases on increasing M^2 for the impulsive and oscillatory movements of the free-stream whereas it decreases increasing M^2 for the accelerated movement of the free-stream. Also, τ increases on increasing G_r for impulsive, accelerated and oscillatory movements of the free-stream. This implies that magnetic field tends to enhance shear stress at the plate for the impulsive and oscillatory movements of the free-stream whereas it has a reverse effect on the shear stress at the plate for the accelerated movement of the free-stream. Thermal buoyancy force has tendency to reduce shear stress at the plate for impulsive, accelerated and oscillatory movements of the free-stream. It is revealed from Table 2 that τ decreases on increasing K whereas it increases on increasing G_c for impulsive, accelerated and oscillatory movements of the free-stream. This implies that chemical reaction tends to reduce shear stress at the plate whereas concentration buoyancy force has a reverse effect on it for impulsive, accelerated and oscillatory movements of the free-stream. It is noticed from Table 3 that τ increases on increasing N_r and t for impulsive, accelerated and oscillatory movements of the free-stream. This implies that thermal radiation tends to enhance shear stress at the plate whereas shear stress at the plate is getting enhanced with the progress of time for impulsive, accelerated and oscillatory movements of the free-stream. It is perceived from Table 4 that, for oscillatory movement of free-stream, τ decreases on increasing ω. This implies that, for oscillatory movement of the free-stream, frequency of oscillations tends to reduce shear stress at the plate.

Table 1: Shear stress τ when $G_c = 5, K = 2.5, N_r = 2, t = 0.25$ and $\omega = 4$.

	Movement of free-stream	$M^2 \downarrow G_r \rightarrow$	2	4	6
τ	Impulsive	2	4.9402	6.8915	8.8429
		3	5.0659	6.9619	8.8580
		4	5.1851	7.0299	8.8748
	Accelerated	2	3.9442	5.8955	7.8468
		3	3.8787	5.7748	7.6708
		4	3.8204	5.6652	7.5101
	Oscillatory	2	3.4645	5.4159	7.3672
		3	3.5255	5.4215	7.3176
		4	3.5820	5.4268	7.2717

Table 2: Shear stress τ when $M^2 = 4, G_r = 6, N_r = 2, t = 0.25$ and $\omega = 4$.

	Movement of free-stream	$G_c \downarrow K \rightarrow$	0.5	2.5	5.0
τ	Impulsive	2	8.1637	8.1310	8.0973
		5	8.9566	8.8748	8.7907
		8	9.7495	9.6186	9.4841
	Accelerated	2	6.7990	6.7663	6.7326
		5	7.5919	7.5101	7.4260
		8	8.3848	8.2539	8.1194
	Oscillatory	2	6.5606	6.5278	6.4942
		5	7.3535	7.2717	7.1876
		8	8.1464	8.0155	7.8809

Table 3: Shear stress τ when $M^2 = 4, G_r = 6, G_c = 5, K = 2.5$ and $\omega = 4$.

	Movement of free-stream	$N_r \downarrow t \rightarrow$	0.15	0.25	0.35
τ	Impulsive	1	4.9068	6.5705	8.8104
		2	5.8100	8.8748	13.6056
		3	6.8662	11.9869	21.0622
	Accelerated	1	3.1736	5.2058	7.7065
		2	4.0768	7.5101	12.5017
		3	5.1330	10.6222	19.9583
	Oscillatory	1	4.1572	4.9673	6.2740
		2	5.0604	7.2717	11.0692
		3	6.1166	10.3838	18.5258

Table 4: Shear stress τ when $M^2 = 4, G_r = 6, G_c = 5, K = 2.5$ and $N_r = 2$.

	τ		
	Oscillatory movement of free-stream		
$\omega \downarrow t \rightarrow$	0.15	0.25	0.35
2	5.6170	8.4401	12.8624
4	5.0604	7.2717	11.0692
6	4.2057	5.7380	9.2956

The numerical values of rate of heat transfer at the plate i.e. $(\partial T/\partial\eta)_{\eta=0}$, computed from the analytical expression (25), are presented in tabular form in Table 5 for various values of N_r and t taking $P_r = 0.71$. It is evident from Table 5 that $(\partial T/\partial\eta)_{\eta=0}$ increases on increasing N_r and t. This implies that thermal radiation has tendency to enhance rate of heat transfer at the plate. As time progresses, rate of heat transfer at the plate is getting enhanced.

The numerical values of rate of mass transfer at the plate i.e. $(\partial C/\partial\eta)_{\eta=0}$, computed from the analytical expression (26), are presented in tabular form in Table 6 for various values of K and t taking $S_c = 0.6$. It is revealed from Tables 6 that $(\partial C/\partial\eta)_{\eta=0}$ increases on increasing K whereas it decreases on increasing t. This implies that chemical reaction tends to enhance rate of mass transfer at the plate. As time progresses, rate of mass transfer at the plate is getting reduced.

Table 5: Rate of heat transfer $(\partial T/\partial\eta)_{\eta=0}$.

$N_r \downarrow t \rightarrow$	$-(\partial T/\partial\eta)_{\eta=0}$		
	0.15	0.25	0.35
1	3.0065	3.8586	4.9468
2	3.6269	5.2661	7.7015
3	4.3677	7.2247	12.1128

Table 6: Rate of mass transfer $(\partial C/\partial\eta)_{\eta=0}$.

$K \downarrow t \rightarrow$	$-(\partial C/\partial\eta)_{\eta=0}$		
	0.15	0.25	0.35
0.5	1.2120	0.9811	0.8643
2.5	1.5269	1.3698	1.3050
5.0	1.8828	1.7853	1.7541

5. CONCLUSIONS

An investigation of unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, optically thick radiating and chemically reactive fluid past an infinite vertical plate with Newtonian heating considering impulsive, accelerated and oscillatory movements of the free-stream is carried out. Significant findings are:

- Magnetic field and chemical reaction tend to retard fluid flow within the boundary layer region for impulsive, accelerated and oscillatory movements of the free-stream whereas thermal buoyancy force, concentration buoyancy force and thermal radiation have reverse effects on it.
- With the progress of time, fluid-flow is getting accelerated within the boundary layer region for impulsive, accelerated and oscillatory movements of the free-stream.
- For oscillatory movement of the free-stream, frequency of oscillations tends to decelerate fluid flow within the boundary layer region.
- Magnetic field tends to enhance shear stress at the plate for the impulsive and oscillatory movements of the free-stream whereas it has a reverse effect on the shear stress at the plate for the accelerated movement of the free-stream.
- Thermal buoyancy force and chemical reaction have tendency to reduce shear stress at the plate for impulsive, accelerated and oscillatory movements of the free-stream whereas concentration buoyancy force and thermal radiation have reverse effects on it.
- For oscillatory movement of the free-stream, frequency of oscillations tends to reduce shear stress at the plate.

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Nomenclature

B_0 : uniform magnetic field,
C : Species concentration,
c_p : specific heat at constant pressure,
D : chemical molecular diffusivity,
g : accleration due to gravity,
G_r : thermal Grashof number,
G_c : solutal Grashof number,
K : chemical reaction parameter,
k : thermal conductivity,
k^* : mean absorption coefficient,
M^2 : magnetic parameter,
N_r : radiation parameter,
p : fluid pressure,
P_r : Prandtl number,
q : constant heat flux,
q_r : radiative heat flux,
S_c : Schmidt number,
t : time,
T : fluid temperature,
u : fluid velocity,
U_0 : characteristic velocity,

Greek Symbols

β' : coefficient of thermal expansion,
β^* : coefficient of expansion of species concentration,
υ : kinematic coefficient of viscosity,
ρ : density,
σ : electrical conductivity,
σ^* : Stefan Boltzmann constant,
ω : frequency parameter.