CHEMICAL REACTION EFFECT ON MHD FREE CONVECTIVE MASS TRANSFER FLOW PAST AN IMPULSIVELY STARTED VERTICAL PLATE

N.Ahmed 1*, K.Kr. Das 2
* Professor1, Department of Mathematics, Gauhati University, Guwahati, Assam, Pin-781014, India. saheel_nazib@yahoo.com
2Research Scholar 2, Department of Mathematics, Gauhati University, Guwahati, Assam, Pin-781014, India kishoredas969@yahoo.in

ABSTRACT

This paper presents an analytical analysis to study the effect of chemical reaction on an unsteady MHD flow of an incompressible viscous electrically conducting fluid past an impulsively started vertical plate. A uniform magnetic field is assumed to be applied normal to the plate directed in to the fluid region. The temperature of the plate is made to rise linearly with time. The fluid considered is gray, absorbing, emitting-radiation but a non scattering medium. The resulting system of equations governing the flow is solved by adopting Laplace Transform technique in closed form. Detailed computations of the influence of Magnetic field, Radiation, Chemical reaction, Mass Grashof number and Schmidt number at the plate are demonstrated graphically and the results are physically interpreted.

The result shows that chemical reaction has significant effect on the flow and on the heat and mass transfer characteristics.

KEYWORDS: MHD, Chemical reaction, Skin friction, Radiation

1. INTRODUCTION

MHD is concerned with the study of the interaction of magnetic field and electrically conducting fluid in motion. There are numerous examples of applications of MHD principles including MHD generators, MHD pumps and MHD flow meters etc. Study of MHD flow with heat and mass transfer plays an important role in Biological Sciences. Effects of various parameters on human body can be studied and appropriate suggestions can be given to the persons working in hazardous areas having noticeable effects of magnetism and heat variation.

The natural flow arises in fluid when the temperature as well as species concentration change causes density variation leading to buoyancy forces acting on the fluid. Free convection is a process of heat or mass transfer in natural flow. The heating of rooms and buildings by use of radiator is an example of heat transfer by free convection. On the other hand, the principles of mass transfer are relevant to the working of systems such as a home humidifier and the dispersion of smoke released from a chimney into the environment. The evaporation of alcohol from a container is an example of mass transfer by free convection. Radiation is also a process of heat transfer through electromagnetic waves. Radiative convective flows are encountered in countless industrial and environment process like heating and cooling chambers, evaporation from large open water reservoirs, astrophysical flows and solar power technology. For natural convection, the existence of the large temperature difference between the surface and ambient causes the radiation effect to be important. At high temperature, thermal radiation can significantly affect the heat transfer and temperature distribution in the boundary layer flow of participating fluid. The study of radiative heat transfer flow is very important in manufacturing industry for the design of reliable equipments, nuclear power plants, gas turbines and various propulsion devices for air-craft, missiles, satellites and space vehicle. In view of importance of thermal radiation effect, several authors have carried out their research works to investigate the effect of it on some heat and mass transfer problems. Some of them are Hessain and Thakar [1], Rapits and Perdikis [2], Samad and Rahman [3], Muthucumaraswamy et al. [4], Reddy et al. [5], Ahmed and Sarmah [6], Rajput and Kumar [7], Ahmed et al. [8], Ahmed and Hazarika [9].

In many times it has been observed that foreign mass reacts with the fluid and in such a situation chemical reaction plays an important role in chemical industry. Theoretical descriptions of non-linear chemical dynamics have been presented by Epstein and Pojman [10] and Gray and Scott [11]. The effects of chemical reaction and mass transfer on MHD flow past a semi-infinite plate was analysed by Devi and Kandasamy [12]. The effects of mass transfer, Soret effect and chemical reaction on an oscillatory MHD free convective flow through a porous medium have been investigated by Ahmed and Kalita [13]. The flow problems concerning heat and mass transfer with thermal radiation and chemical reaction become interesting and fruitful from
practical point of view if the fluid is electrically conducting in presence of magnetic field. Owing to this, we have proposed in the present work is to investigate the chemical reaction effect on an MHD free convective mass transfer flow past an impulsively started vertical plate. Our work is a generalization to the work done by Rajput and Kumar [7]. In absence of chemical reaction, the solution obtained in the present paper is consistent with that of Rajput and Kumar [7].

2. MATHEMATICAL ANALYSIS

We consider an unsteady MHD flow of an incompressible viscous electrically conducting fluid past an impulsively started vertical plate. Introduce a coordinate system (x, y, z) with the x-axis is taken along the plate in the upward direction, y-axis is taken normal to the plate directed in to the fluid region and z-axis along the width of the plate. A magnetic field of uniform strength Bo is applied normal to the plate. The magnetic Reynolds number is assumed to be so small that the induced magnetic field produced by the electrically conducting fluid can be neglected in comparison to the applied magnetic field. The viscous and Ohmic dissipations of energy are neglected. Initially the fluid and the plate were at the same temperature Te and concentration Ce in the stationary condition. At time t > 0, the plate starts to move suddenly with velocity U = U0 in its own plane and the temperature of the plate as well as the concentration level near the plate is raised linearly with time.

The equations governing the flow are

MHD momentum equation:

$$\frac{\partial u}{\partial t} - g\beta(T - T_e) + g\beta' (C - C_e) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_y^2 u}{\rho}$$

Energy equation:

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_y}{\partial y}$$

Species Continuity equation:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} + K(T_e - T)$$

The relevant initial and boundary conditions are:

$$u = 0 , \ T = T_e , \ C = C_e , \ \forall y , \ t \leq 0$$

$$u = U_0 , \ T = T_e + (T_e - T_e) A_1 ,$$

$$C = C_e + (C_e - C_e) A_1$$

at \( y = 0 , t > 0 \)

$$u \rightarrow 0 \ T \rightarrow T_e , \ C \rightarrow C_e \ \text{at} \ y \rightarrow \infty$$

where, \( U_0 = (A V_0)^\frac{1}{2} \)

The local radiant for the case of optically thin gray gas is expressed by

$$\frac{\partial q_y}{\partial y} = -4 a' \sigma (T_e^4 - T^4)$$

(5)

Consider the temperature difference within the flow sufficiently small, \( T^4 \) can be expressed as the linear function of temperature. By expanding \( T^4 \) in a Taylor series about \( T_e \) and neglecting higher order terms, we obtain

$$T^4 = T_e^4 + 3 T_e^3 (T - T_e)$$

(6)

On using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + 16 a' \sigma T_e^3 (T_e - T)$$

(7)

We introduce the following non-dimensional quantities:

$$\tilde{u} = \frac{u}{u_0} , \ \tilde{y} = \frac{y u_0}{\nu} , \ \tilde{T} = \frac{(T - T_e)}{(T_e - T_e)} , \ \tilde{C} = \frac{(C - C_e)}{(C_e - C_e)} , \ \tilde{\theta} = \frac{\theta}{\theta_0}$$

$$\tilde{C} = \frac{C}{C_e} , \ \tilde{q}_y = \frac{\sigma B_y^2 u_0^3}{\rho u_0^2} , \ R = \frac{16 a' \nu^2 \sigma T_e^3}{k u_0^2}$$

(8)

The non dimension form of equations (1), (7) and (3) are as follows:

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} = Gr \tilde{\theta} + Gm \tilde{C} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} - M \tilde{u}$$

$$\frac{\partial \tilde{\theta}}{\partial \tilde{t}} = \frac{1}{Pr} \frac{\partial^2 \tilde{\theta}}{\partial \tilde{y}^2}$$

$$\frac{\partial \tilde{C}}{\partial \tilde{t}} = \frac{1}{Sc} \frac{\partial^2 \tilde{C}}{\partial \tilde{y}^2} - K \tilde{C}$$

(9)

(10)

(11)

The initial and boundary conditions in dimensionless form are as follows:

$$\tilde{u} = 0 , \ \tilde{\theta} = 0 , \ \tilde{C} = 0 \ \forall \ \tilde{y} , \ \tilde{t} \leq 0$$

$$\tilde{u} = 1 , \ \tilde{\theta} = 1 , \ \tilde{C} = 1 \ \text{at} \ \tilde{y} = 0 , \ \tilde{t} > 0$$

Dropping bars in the above equations, we have

$$\frac{\partial \bar{u}}{\partial t} = Gr \bar{\theta} + Gm \bar{C} + \frac{\partial^2 \bar{u}}{\partial y^2} - M \bar{u}$$

$$\frac{\partial \bar{\theta}}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \bar{\theta}}{\partial y^2}$$

$$\frac{\partial \bar{C}}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \bar{C}}{\partial y^2} - K \bar{C}$$

(13)

(14)

(15)

with boundary conditions:

$$\bar{u} = 0 , \ \bar{\theta} = 0 , \ \bar{C} = 0 \ \forall \ y , \ \bar{t} \leq 0$$

$$\bar{u} = 1 , \ \bar{\theta} = 1 , \ \bar{C} = 1 \ \text{at} \ \bar{y} = 0 , \ \bar{t} > 0$$

(16)

(17)

3. METHOD OF SOLUTIONS

On taking Laplace Transform of the equations (13) to (15), the following differential equations are derived.

$$\frac{d^2 \bar{u}}{d \bar{y}^2} - (M + s) \bar{u} = -Gr \bar{\theta} - Gm \bar{C}$$

(17)
\[
\frac{d^2 \bar{v}}{dy^2} - (R + s Pr) \bar{v} = 0 \\
\frac{d^2 \bar{C}}{dy^2} - Sc (K + s) \bar{C} = 0
\]

(18)  
(19)

Subject to boundary conditions:
\[\bar{u} = \frac{1}{s}, \quad \bar{\theta} = \frac{1}{s^2}, \quad \bar{C} = \frac{1}{s}, \quad \bar{v} = 0 \text{ at } y = 0 \]
\[\bar{u} = 0, \quad \bar{\theta} = 0, \quad \bar{C} = 0 \text{ at } y \to \infty \]

where,
\[\bar{u} = L \{u(y,t)\}, \quad \bar{T} = L \{T(y,t)\}, \quad \bar{C} = L \{C(y,t)\}\n
The solution of the equations (17) to (19) under the conditions (20) are
\[\bar{u} = \left[ \frac{1}{s} \left( \frac{a_1}{s^2} \left( s - a \right) - \frac{a_2}{s^2} \left( s - b \right) \right) e^{\sqrt{(M+1)} y} + \frac{a_1}{s^2} e^{\sqrt{(Pr+A)}} + \frac{a_2}{s^2} e^{\sqrt{(Sc+B)}} \right] \frac{1}{s^2} e^{\ln(Sc+B) y} \]
(21)
\[\bar{\theta} = \left( \frac{a_1}{s^2} \right) e^{\ln(Pr+A) y} \]
(22)
\[\bar{C} = \left( \frac{a_1}{s^2} \right) e^{\ln(Sc+B) y} \]
(23)

Taking inverse Laplace transforms of the equations (21) to (23), we derive the expressions for the representative temperature, concentration and the velocity fields as follows:
\[\bar{u}(y,t) = \bar{u} \]
(24)
\[\bar{\theta}(y,t) = \bar{\theta} \]
(25)
\[\bar{C}(y,t) = g_2 \]
(26)

with
\[a = \frac{R - M}{1 - Pr}, \quad a_1 = \frac{Gr}{1 - Pr}, \quad a_2 = \frac{M - K \cdot Sc}{1 - Sc - 1}, \quad b = \frac{R}{Pr} \]

where,
\[\omega_1 = \omega (1, M, 1), \quad \omega_2 = \omega (1, M + a, t), \quad \omega_3 = \omega (Pr, c + a, 1), \quad \omega_4 = \omega (1, M + b, t), \quad \omega_5 = \omega (Sc, b + K, t), \quad \omega_6 = \omega (Pr, e + c, 1), \quad \omega_7 = \omega (Sc, K + t) \]

\[\Psi(x,y,t) = \sqrt{\frac{x}{\pi t}} e^{-x^2} \cdot \sqrt{\frac{y}{\pi t}} e^{-y^2} \]
(27)

7. RESULTS AND DISCUSSIONS

In order to get clear insight of the physical problem, numerical computations from the analytical solutions for the representative velocity field, concentration field, and the coefficient of skin friction, the rate of heat transfer and rate of mass transfer at the plate have been carried out by assigning some arbitrary chosen specific values to the physical parameters like Hartmann number (M), Radiation parameter (R), Chemical reaction parameter (K), Grashof number of mass transfer (Gm), Schmidt number (Sc) and normal coordinate y. Throughout our investigation, the values of Prandtl number (Pr) and Grashof number of heat transfer (Gr) have been kept fixed at 0.71 (corresponding to air) and 5 (0 to 5, externally cooled case) respectively as the numerical computations are concerned. In our present investigation, the values of Schmidt number are chosen as .30, .60, .78 respectively which corresponds to Helium, steam and Ammonia at 25°C and 1 atmospheric pressure.
The behaviors of the velocity field \( u \) versus normal coordinate \( y \) under the influence of chemical reaction \( K \), Hartmann number \( M \), Schmidt number \( Sc \) and Grashof number for mass transfer \( Gm \) are depicted respectively in figures 1, 2, 3 and 4. It is marked in the above figures that the fluid velocity increases from \( u = 1 \) (at the plate) in a very thin layer of the fluid adjacent to the plate and after this fluid layer the fluid velocity asymptotically decreases to its zero value as \( y \to \infty \). This phenomenon is clearly supported by the physical reality, since the buoyancy force effect is significant near the heated plate surface, which results in a sudden rise in the fluid velocity adjacent to the plate surface. Further, it is also observed that the velocity \( u \) falls due to increasing values of Schmidt number \( Sc \) and chemical reaction parameter \( K \), whereas it rises under the effects of Hartmann number \( M \) and Grashof number for mass transfer \( Gm \). In other words, the velocity field is accelerated due to imposition of the transverse magnetic field and under the effect of buoyancy force caused due to composition gradient, whereas this motion is retarded under the effect of Schmidt number and chemical reaction.

Figures 5 and 6 correspond to the temperature distribution \( \theta \) against normal coordinate \( y \) under the influence of radiation parameter \( R \) and time \( t \). It is observed from figure 5 that temperature \( \theta \) decreases as the radiation parameter \( R \) increases. This result qualitatively agrees with expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, there by decreasing the temperature of the fluid. On the other hand temperature \( \theta \) rises with increasing time \( t \).
Further it is seen from both figures 5 and 6 that the temperature asymptotically falls from its maximum value \( \theta = t \) at \( y = 0 \) to its minimum value \( \theta = 0 \) at \( y \rightarrow \infty \).

![Fig-5 Temperature profile versus y for t = .2](image1)

![Fig-6 Temperature profile versus y for R = 4](image2)

Figures 7 and 8 exhibit the variations in species concentration \( C \) against normal coordinate \( y \) under the influence of Schmidt number \( Sc \) and chemical reaction parameter \( K \). These figures clearly show that a rise in Schmidt number or chemical reaction parameter causes a fall in the concentration level of the fluid. Like temperature field, the concentration field also asymptotically decreases from its maximum value \( C = t \) at \( y = 0 \) to minimum value \( C = 0 \) at \( y \rightarrow \infty \).

![Fig-7 Concentration profile versus y for K = 5, t = .2](image3)

![Fig-8 Concentration profile versus y for M = 5, t = .2](image4)

The effects of Hartmann number \( M \) on skin-friction \( \tau \) against time \( t \) is presented in figure 9. It is observed that the shear stress at the plate is considerably reduced with an increase in the strength of an applied magnetic field. As such strong magnetic field may be applied in operations to successfully inhibit the drag force along the plate.

Figures 10, 11 and 12 demonstrate the behaviour of the skin-friction \( \tau \) versus time \( t \) under the influence of chemical reaction parameter \( K \), Schmidt number \( Sc \) and Grashof number for mass transfer \( Gm \). It is clear from these figures that drag force \( \tau \) rise under the effect of chemical reaction parameter \( K \) and Schmidt number \( Sc \), whereas falls due to Grashof number for mass transfer \( Gm \).
Fig-9 Skin friction versus $t$ for $K = 5$, $Sc = .6$, $Gm = 10$

Fig-10 Skin friction versus $t$ for $M = 5$, $Sc = .6$, $Gm = 10$

Fig-11 Skin friction versus $t$ for $K = 5$, $M = 5$, $Gm = 10$

Fig-12 Skin friction versus $t$ for $K = 5$, $M = 5$, $Sc = .6$

The effect of radiation parameter $R$ on the coefficient of rate of heat transfer in terms of Nusselt number $Nu$ has been displayed in figure 13. Here it is seen that $Nu$ increases due to increasing values of time $t$ and it rises under radiation effect.

Figures 14 and 15 exhibit the behaviour of Sherwood number $Sh$ at the plate under the effect of chemical reaction parameter $K$ and Schmidt number $Sc$. These figures indicate that a rise in the chemical reaction parameter $K$ and Schmidt number $Sc$ leads to a rise the Sherwood number $Sh$. 
Fig. 13 Nusselt number versus t

Fig. 14 Sherwood number versus t for Sc = .6

Fig. 15 Sherwood number versus t for K = 5

Table 1: Velocity profile versus y for M=5, Sc = .6, K = 5, Gm = 10, t = .2

<table>
<thead>
<tr>
<th>y</th>
<th>R = 4</th>
<th>R = 6</th>
<th>R = 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.4</td>
<td>1.63338</td>
<td>1.63211</td>
<td>1.63103</td>
</tr>
<tr>
<td>.8</td>
<td>.843189</td>
<td>.842396</td>
<td>.841744</td>
</tr>
<tr>
<td>1.2</td>
<td>.30057</td>
<td>.300297</td>
<td>.300081</td>
</tr>
<tr>
<td>1.6</td>
<td>.0818722</td>
<td>.0818151</td>
<td>.0817661</td>
</tr>
<tr>
<td>2</td>
<td>.0176437</td>
<td>.0176326</td>
<td>.0176245</td>
</tr>
</tbody>
</table>

It is inferred from Table 1 that an increase in the radiation parameter R leads to fall in velocity.

8. CONCLUSION

- Magnetic field accelerates the fluid motion although the chemical reaction as well as Schmidt number retards the fluid flow.
- The fluid temperature decreases as the radiation parameter increases.
- The concentration of the fluid falls as each of Schmidt number and chemical reaction parameter increases.
• The viscous drag at the plate in the direction of the buoyancy force may be successfully inhibited on application of strong magnetic field in operations.
• An increase in chemical reaction and Schmidt number results in a growth in the drag force and it falls under the effect of Grashof number for mass transfer.
• The rate of heat transfer (from plate to the fluid) rises due to the absorption of thermal radiation.
• The rate of mass transfer increases due to the chemical reaction as well as Schmidt number.

REFERENCES

NOMENCLATURE
\[ B_0 \] Strength of the applied magnetic field; Tesla
\[ C \] Species concentration of the fluid; KMOL
\[ \bar{C} \] Dimensionless concentration; KMOL
\[ C_p \] Specific heat at constant pressure; \( \frac{\text{Joule}}{\text{Kg} \times \text{K}} \)
\[ C_w \] Concentration of the fluid; KMOL
\[ C_{\infty} \] Concentration of the fluid far away from the plate; KMOL
\[ D \] Chemical molecular diffusivity
\[ g \] Acceleration due to gravity m/s²
\[ a^* \] Absorption coefficient kg m²/s
\[ Gr \] Thermal Grashof number
\[ Gm \] Mass Grashof number
\[ k \] thermal conductivity; \( \frac{\text{W}}{\text{mK}} \)
\[ M \] Hartmann number
\[ \rho \] Fluid pressure; Newton/m²
\[ Pr \] Prandtl number
\[ R \] Radiation parameter
\[ q_r \] Radiative heat flux in the y direction
\[ Sc \] Schmidt number
\[ T \] temperature of the fluid near the plate; Kelvin
\[ T_w \] temperature of the fluid; Kelvin
\[ T_{\infty} \] Fluid temperature far away from the plate; Kelvin
\[ t \] Time; second
\[ u \] Velocity of the fluid in X direction; m/s
\[ \bar{u} \] Dimensionless velocity
\[ u_0 \] Velocity of the fluid
\[ \gamma \] Co-ordinate axis normal to the plate
\[ \bar{\gamma} \] Dimensionless co-ordinate axis normal to the plate
\[ \Lambda \] constant
\[ erf \] Error function
\[ erf_c \] Complementary error function

GREEK SYMBOLS
\[ \beta \] Volumetric coefficient of thermal expansion; \( \frac{1}{\text{K}} \)
\[ \beta^* \] Volumetric coefficient of expansion with concentration
\[ \mu \] Coefficient of viscosity; Kg/ms
\[ \theta \] Non dimensional temperature
\[ \sigma \] Electrical conductivity; (ohm × meter)⁻¹
\[ \sigma^* \] Stefan-Boltzmann constant
\[ \nu \] Kinematic viscosity; m²/s⁻¹
\[ \rho \] Fluid density; Kg/m³